Robotics: Basics and Selected Advanced Concepts
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Lecture – 13
Mobility of Parallel Manipulators

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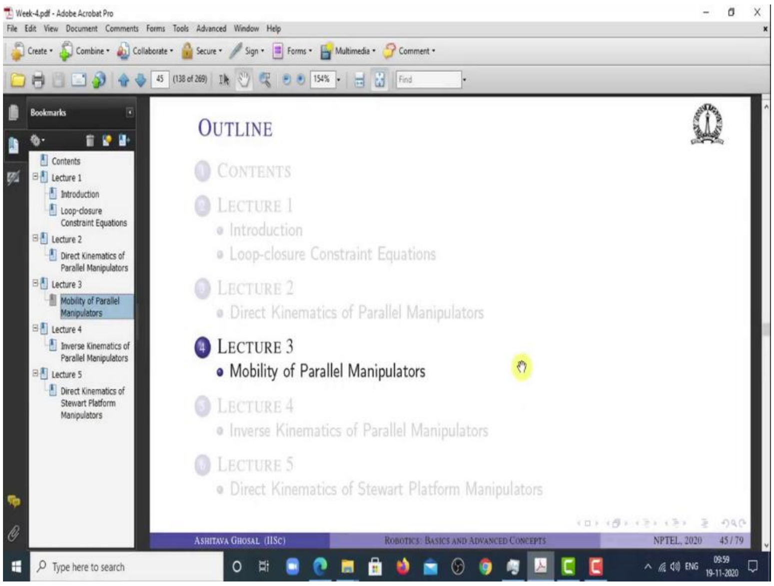

Welcome to this NPTEL lectures on Robotics Basics and Advanced Concepts. In this lecture, we will look at Mobility of Parallel Manipulators.

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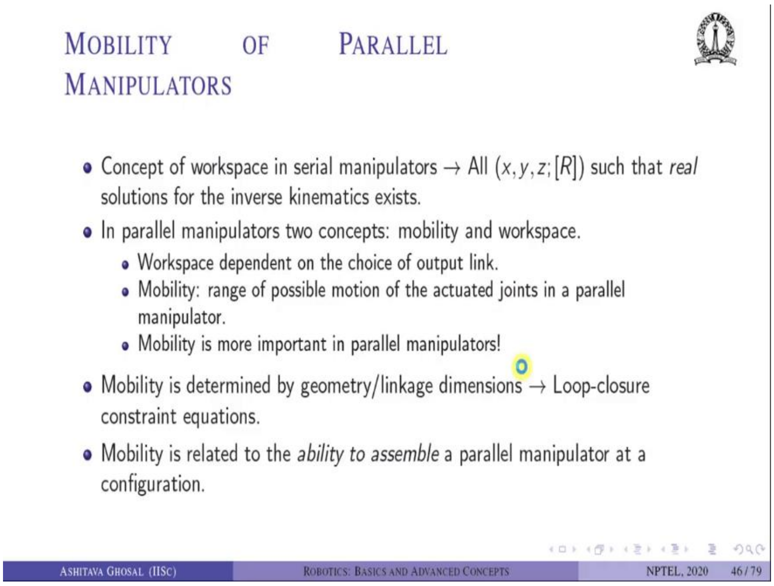

So, to introduce this concept of mobility, we can contrast with the concept of workspace in serial manipulators ok. All in a serial manipulator the workspace is defined as all x y z and rotation matrices of the end effector such that the real solutions for the inverse kinematics exist ok.

So, basically we gave the position and orientation of the end effector and then we solved the inverse kinematics which basically means solve for the joint variables which will lead to that position and orientation of the end effector. If there are real values of the joint variables and the inverse kinematic solution exists, then we say that the given point x, y, z and the rotation matrix of the end effector are inside the workspace.

In parallel manipulator, there are two concepts; first is mobility and the second is workspace ok. So, remember the output link is not natural or, but obvious in a parallel robot. So, the workspace depends on the choice of the output link. The mobility on the other hand is the range of possible motions of the actuator joint in a parallel manipulator.

So, we clearly know which are the actuator joints and as you will see later that not all possible values of the actuator joints can be achieved ok. So, in a sense mobility is more important in parallel manipulators. So, the mobility is determined by geometry and linkage dimensions or in other words it is determined by the loop closure constraint equations which are used to solve the direct kinematics ok.

Mobility is related to the ability to assemble the parallel manipulator at a given configuration. So, if I give you values of theta and phi actuated and passive joint variables, can I assemble the manipulator in that configuration; that is what mobility is related to.

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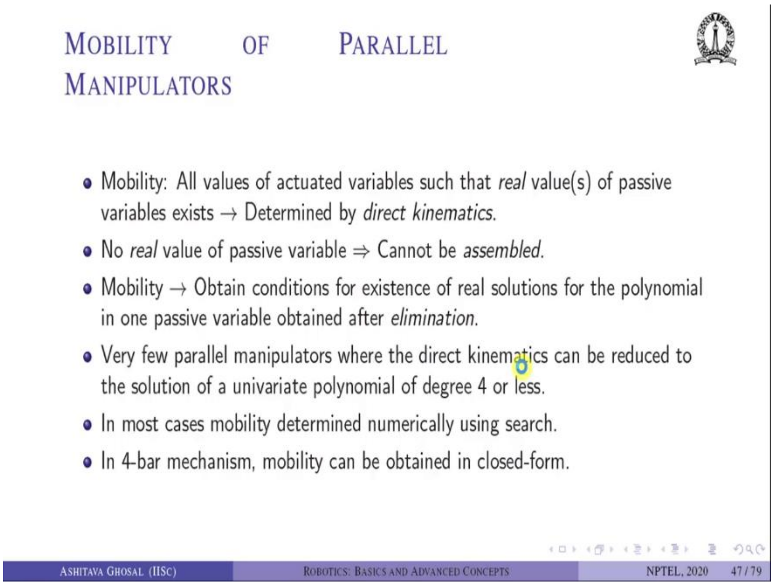

So, in definition mobility implies all values of actuated variables such that there are real values of passive joint variables and it is determined by direct kinematics. If there are no real values of passive joint variables, the mechanism or the parallel manipulator cannot be assembled at that configuration. So, mobility means obtain conditions for the existence of real solutions for the polynomial in one passive variable obtained after elimination.

So, remember in the direct kinematics of parallel robots, we obtained a single polynomial in one of the passive joint variables. And if that polynomial does not give real values ok, then the manipulator cannot be assembled at that configuration ok. And again as you can remember elimination was used to obtain that polynomial in one passive joint variable.

There are very few parallel manipulators where the direct kinematics can be reduced to the solution of a univariate polynomial of degree 4 or less. So, hence in most cases mobility is determined numerically using search ok. In a 4-bar mechanism fortunately mobility can be obtained in closed form and again this is the reason why mobility a 4-bar mechanism is used to demonstrate many of the concepts in parallel manipulators and closed loop mechanisms.

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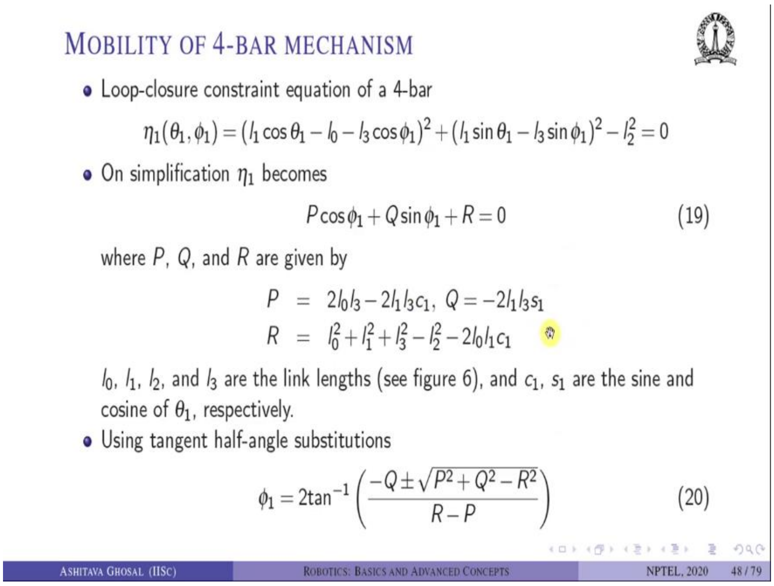

So, the loop closure equation of a 4-bar mechanism was derived in the last lecture. So, this equation $\eta_1 = (\theta_1, \phi_1)$ is basically the Freudenstein equation ok. It relates given θ_1 which is the input crank angle, what is the ϕ_1 which is the output crank angle in terms of the link lengths l_1, l_2, l_3 and l_0 ok.

So, on simplification this η_1, this equation becomes something like $Pcos(\phi_1) + Qsin(\phi_1) + R = 0$ where P Q and R are functions of the link lengths and also the angle θ_1 ok. You can see $cos(\theta_1)$ appearing here, $sin(\theta_1)$ appearing here and so on ok. So, if you substitute θ_1 ok $\tan\frac{\theta_1}{2}$ as x and then substitute this tangent half angle substitution.

So, basically x is $\tan\frac{\theta_1}{2}$, then we find out what is $\cos(\theta_1)$ and $\sin(\theta_1)$ remember $\cos(\theta_1)$ will be 1 plus x square divided by 1. So, $\frac{1-x^2}{1+x^2}$ and $\sin(\theta_1)$ will be $\frac{2x}{1+x^2}$, we will get a quadratic equation in x. So, this transcendental equation $Pcos(\phi_1) + Qsin(\phi_1) + R$ will be converted into a quadratic equation just like before we have seen many times and then we can solve for x as a quadratic equation ok.

We will have two roots and then we can find out the tan inverse of that and multiply by 2 and we will get phi 1. So, it turns out that $\phi_1 = 2\tan^{-1}(\frac{-Q \pm \sqrt{P^2+Q^2-R^2}}{R-P})$ ok.

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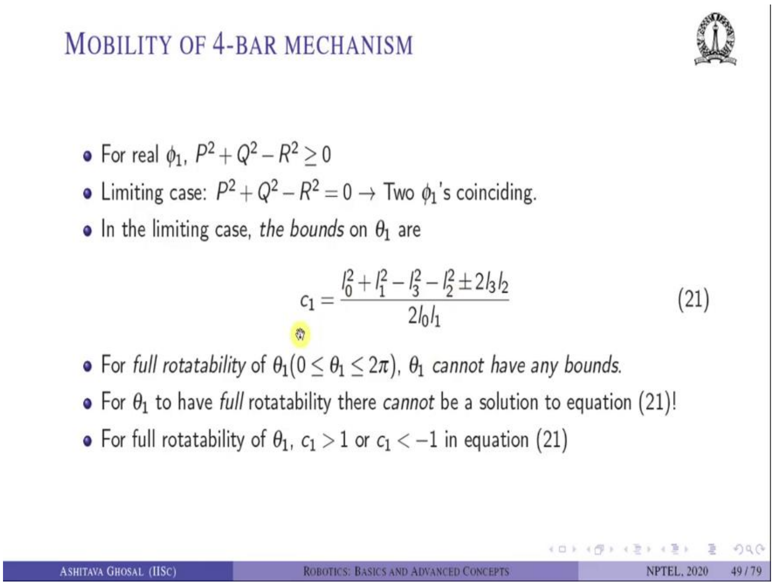

So, for real ϕ_1 $P^2 + Q^2 - R^2$ must be greater than or equal to 0. The limiting case when you get 2 equal real values of ϕ_1 is when $P^2 + Q^2 - R^2$ is equal to 0. So, we have 2 phi's which are coinciding ok. In the limiting case, the bounds on θ_1 are $\cos(\theta_1) = \frac{l_0^2+l_1^2-l_3^2-l_2^2 \pm 2l_3l_2}{2l_0l_1}$ ok. Where did I get this from?

So, we can see here that P Q R are functions of sine and cosine theta. So, when this term under the square root of is equal to 0, we can go back and find out what is cosine theta in that limiting case ok. This is what the expression we get. So, for full rotatability of θ_1 basically that the 4-bar mechanism can be assembled for any given value of θ_1 in the range 0 to 2 pi, θ_1 cannot have bounds ok; this is the important argument.

So, if I get a value of θ_1 from this expression ok, then for that coincident ϕ ok you will get a $\theta_1$value ok, but I want θ_1 to rotate fully else still ϕ be real. So, then we cannot have any bounds on θ_1 so, which basically means that there cannot be a solution to this equation.

So, let us look at the argument once more. So, for real ϕ_1, we get case addition which is $P^2 + Q^2 - R^2$ greater than 0. The two real values will coincide when $P^2 + Q^2 - R^2$is equal to 0. For this limiting case I can find the value of θ_1, $\cos(\theta_1)$ok. Now, if this $\theta_1$value were to exist ok, then we have a ϕ_1 value where it is coinciding, but we do not want any value of θ_1 because I want the crank to rotate fully ok.

So, θ_1 cannot have any bounds. So, there cannot be a solution to this equation ok. So, basically for full rotatability of θ_1, $\cos(\theta_1)$ must be greater than 1. So, if this number is greater than 1, we cannot have a solution to θ_1 or it should be less than minus 1 ok.

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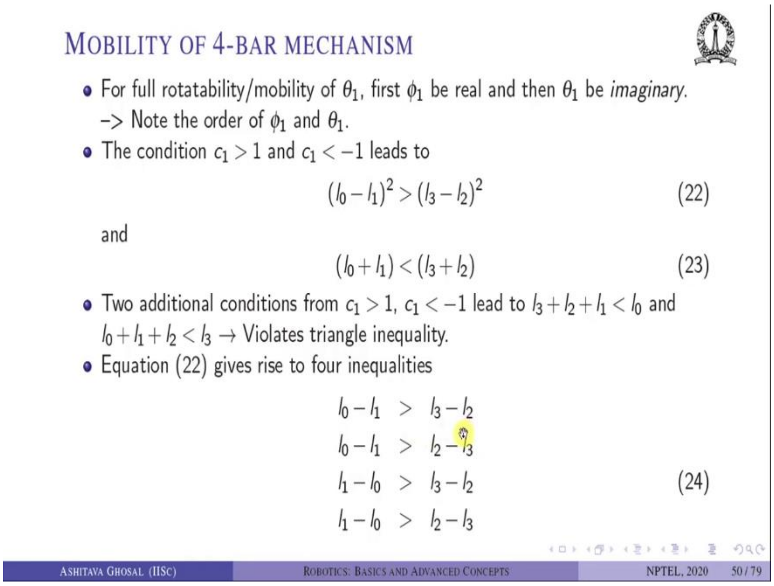

So, for full rotatability mobility to repeat of θ_1; first ϕ_1 must be real and then θ_1 be imaginary; note the order of ϕ_1 and θ_1. So, the condition c_1 greater θ_1 and c_1 less than - 1 leads to $(l_0 - l_1)^2 > (l_3 - l_2)^2$.

So, this you can derive from putting c_1 greater than 1 here. So, you will have some equation which is l_0 square plus l_1 square greater than or equal to something ok. And we also get $(l_0 - l_1) < (l_3 + l_2)$. We can also get 2 additional conditions from c_1 greater than 1 and c_1 less than 1 which leads to $l_3 + l_2 + l_1 < l_0$ and $l_0 + l_1 + l_2 < l_3$.

These two violate triangular triangle inequality triangle inequality means the sum of two sides must be greater than the third side for any triangle ok. So, finally, with this equation 22 gives rise to four inequalities because it is a square. So, we can have $(l_0 - l_1) > (l_3 - l_2)$ ok. We can also have $(l_1 - l_0) < (l_3 - l_2)$ if you take the minus sign and likewise for the other two.

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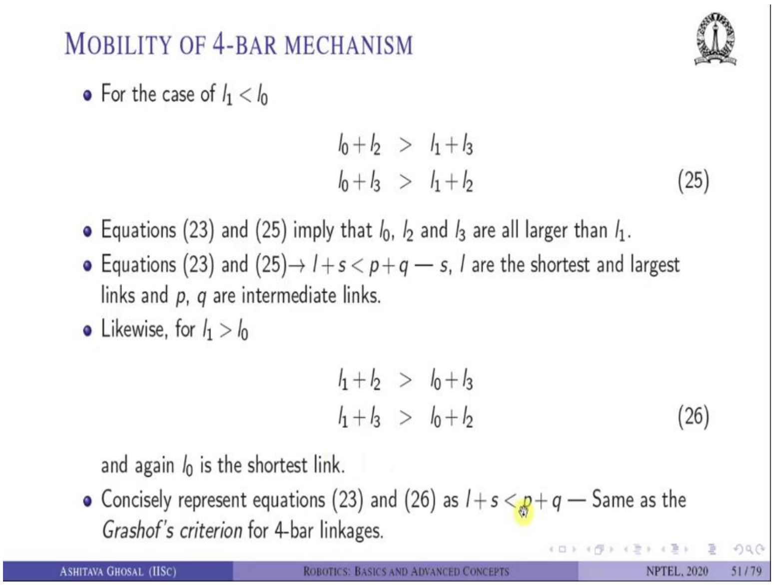

So, for the case $l_1 < l_0$; if I assume that the first link, the crank is less than the base link. We have $(l_0 + l_2) > (l_1 + l_3)$ and $(l_0 + l_3) > (l_1 + l_2)$ok. Remember l_0 is the base link; l_1 is the crank l_2 is the coupler and l_3 is the output link.

So, equations 23 and 25 imply that l_0 l_2 and l_3 are all larger than l_1 ok. Equation 23 and 25 also implies in a short form that $l + s < p + q$ where l, s and l are the shortest and the largest link and p and q are the intermediate links ok. So, likewise for $l_1 > l_0$; the previous one was when $l_1 < l_0$ if $l_1 > l_0$, we will have $(l_1 + l_2) > (l_0 + l_3)$ and $(l_1 + l_3) > (l_0 + l_2)$.

So, all these conditions basically coming from the fact that we want the crank the angle, θ_1 to rotate fully ok. So, all of these four conditions in 25; 2 conditions in 25 and 2 in 26 can be concisely written $l + s < p + q$ ok.

So, if somebody who has done a course in 4-bar mechanisms analysis or synthesis, we know that this is the Grashof's criteria which says that the length of the largest and the shortest link should be less than the two intermediate links for the crank to rotate fully and the crank is adjacent to the fixed link.

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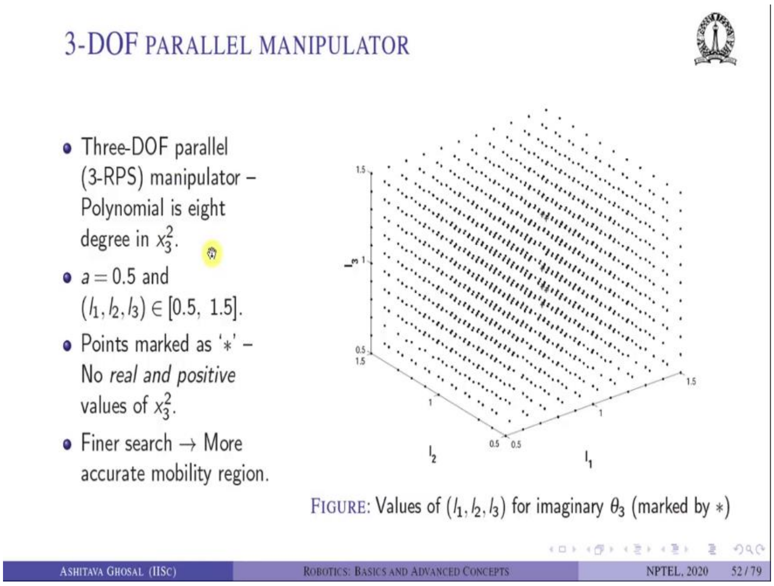

In the case of a 3 degree of freedom manipulator ok, we cannot obtain such nice results of when the θ_1 or the input actuator actuation can be of the full range. In this case, we saw that the polynomial in x_2^3 was 8th degree ok. For values of a 0.5 and (l_1, l_2, l_3) between 0.5 and 1 5, we can scan this space and solve the polynomials polynomial; 8 degree polynomial ok.

So, it turns out that the point marked star. So, for example, these two points here, this point here, at this point here, this point here for (l_1, l_2, l_3) in this range 0.5 to 1.5 in all 3 actuated variables, there are no real values. So, there are no real values for some values of (l_1, l_2, l_3) and hence for these values of (l_1, l_2, l_3); we do not or we cannot assemble this 3 RPS robot ok.s

So, the mobility of this 3 RPS robot let us go back to the definition. The values of actuated joint variables for which there are real values of the passive joint variables and if there are no real passive joint variables, then the mechanism cannot be assembled and the mechanism does not have mobility at those configurations.

So, it turns out that there are few points where the there are no real values $\tan\frac{\theta_3}{2}$ ok. Remember this is the numerical technique because we cannot obtain analytical conditions for the 8 degree polynomial when is the 8 degree polynomial giving real values.

So, like any numerical technique on any numerical search; if you refine the region; if you take smaller and smaller steps, the mobility region may look different. Nevertheless this is the way to determine the mobility of this 3 RPS manipulator.

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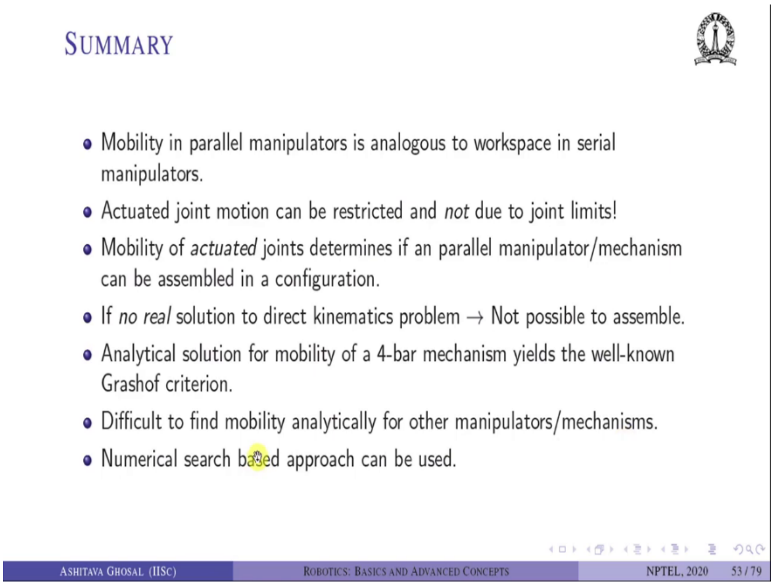

So, in summary the mobility in parallel robots is analogous to the workspace in serial manipulator ok. The actuated joints can be restricted and not due to joint limits; this is important. So, given a range of actuated joint; if I do not have real values of the passive joint variables, I cannot achieve those actuated joint variables and this has nothing to do with the joint limits which are there in serial robots.

So, the mobility of actuated joint determines if a parallel manipulator or a mechanism can be assembled in a configuration. So, if I tell you the passive and active joint variables, I can assemble the parallel manipulator. But if there are no real passive joint variables, we cannot assemble the parallel robot ok. So, just to repeat if no real solutions to the direct kinematics problem, it is not possible to assemble the parallel robot.

The analytical solutions for mobility of a 4-bar mechanism yields the well known Grashof criteria now $l + s < p + q$ for θ_1 to be fully rotatable ok; so, there is no problem of assemblability in a 4-bar mechanism when the crank rotates fully.

It is difficult to find mobility analytically for other parallel manipulators and mechanisms; simply because the direct kinematics problem cannot be solved in closed form ok. And hence for such parallel robots we have to do basically numerical search ok. We scan the actuated joint variable space and find out where the passive joints are imaginary.

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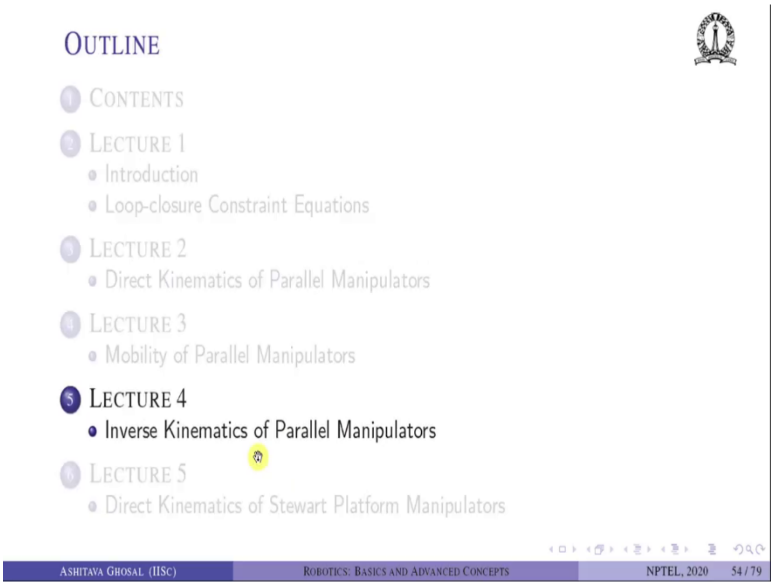

So, with this will stop ok. Clearly this is a big topic, lots of research has been done for different kinds of parallel robots for its mobility, but these are the ones which can be done quite easily or at least discussed quite easily. In the next lecture, we will look at inverse kinematics of parallel robots.