Robotics: Basics and Selected Advanced Concepts
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Lecture - 17
Application of Parallel Robots

Welcome to this NPTEL course on Robotics: Basics and Advanced Concepts. In this week we are looking at Application of Parallel Robots.

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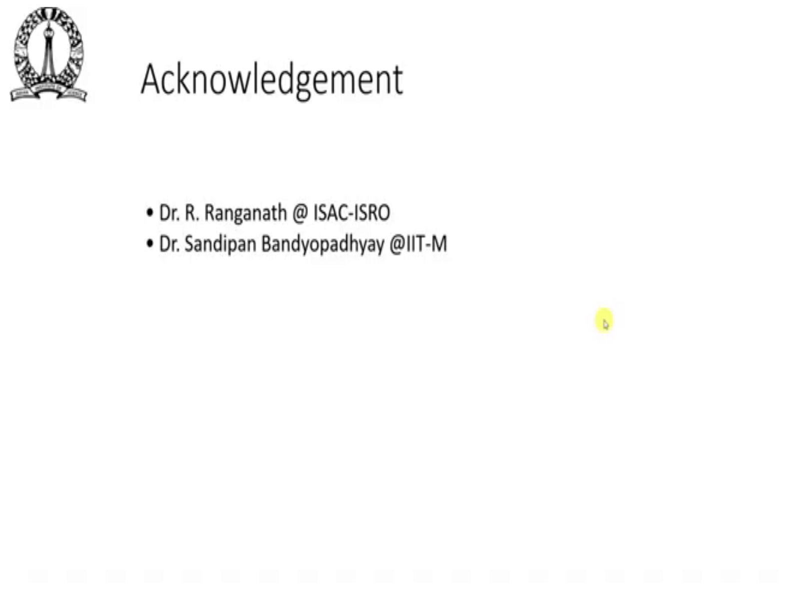

This is the second lecture in this week and in this lecture, we will look at Stewart platform manipulator as a sensor ok. So, quick acknowledgement this work was done by Dr. R. Ranganath who was a student from ISRO and Dr. Sandipan Bandyopadhyay is a faculty member at IIT Madras.

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So, the contents of this lecture are the following we will first introduce the Stewart platform again, when we look at the statics of a 6-6 Stewart-Gough platform then we will figure out how to obtain the singular configurations in a 6-6 Stewart platform, then we will look at the design of a 6 component force torque sensor based on a Stewart platform in a near singular configuration, then I will show you some experimental results and conclude this lecture.

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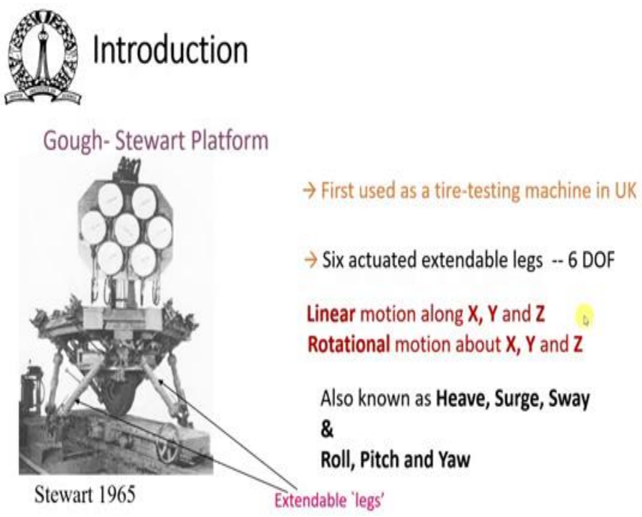

So, the Stewart Gough platform or the Gough Stewart platform was first used as a tire testing machine in UK ok. It was invented by Stewart in 1965 and later on it was found that Gough had invented this much earlier. So, the modern usage is Gough Stewart platform. So, in a Stewart platform there are 6 legs which can be extended ok. So, each of these legs are actuated.

So, it turns out that this Stewart platform has 6 degrees of freedom. So, this is the original picture of the tire testing machine by Stewart. So, basically, we have a top platform and there is a tire which is mounted here and then there are these 6 legs which can extend ok.

So, as you extend these 6 legs, this top platform can tilt, do roll pitch yaw they can also be linear motion along X, Y, Z axis and hence if this tire can be tested by rubbing by pressing by doing various things with some equipment at the floor. So, this linear motion along X, Y and Z and the rotational motion along X, Y and Z are also sometimes called Heave, Surge and Sway and Roll, Pitch and Yaw ok.

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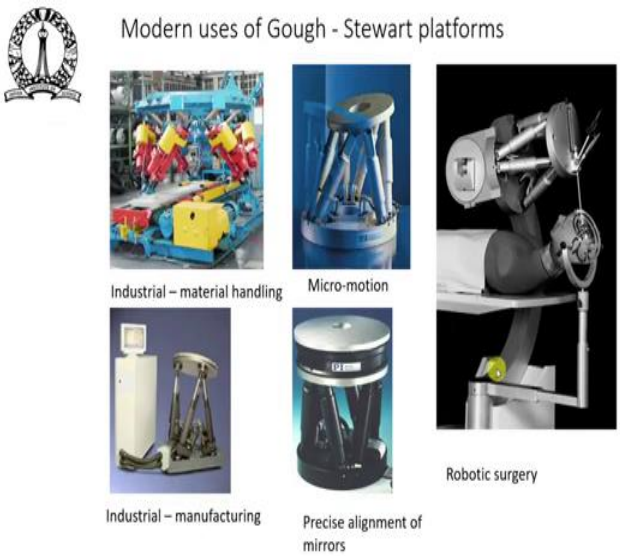

There are many many modern uses Stewart platforms now. So, initially the tire testing machine is now used for material handling, Stewart platform can also perform very very fine or micro motions ok. Stewart platform is a parallel device hence the errors are expected and shown to be smaller than a serial manipulator with the same degrees of freedom ok.

In a 6 degree of freedom Stewart platform the error is the maximum of the errors in each of these legs the errors could not add up as in the serial robot. It is also used in many industrial manufacturing setups, it is used for precise alignment of mirrors and it has also been proposed this is of course, a schematic of a Stewart platform being used for surgery.

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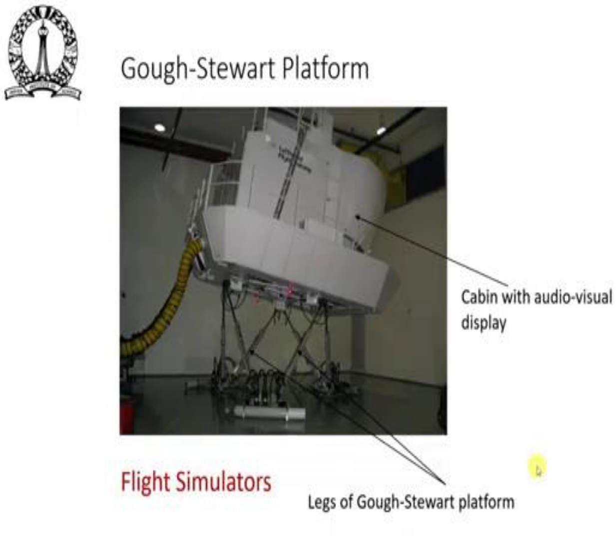

The most popular and famous use of a Stewart platform is the flight simulator. So, basically a steward platform can support a platform at 6 points, hence it can carry a large amount of load ok because the whole weight of this top platform is supported at 6 places.

The top platform is like a cabin with audio visual display and a top pilot or a trainee will be sitting inside this cabin with lots of audio visual displays around it and then when you moves the joystick or the controls of an aeroplane, this top platform can move in the 6 degrees of freedom it can do X, Y, Z translation and roll, pitch, yaw.

So, hence depending on what is projected on those screens, you can be trained to fly an aircraft. So, this is one of the most common uses of a Gough Stewart platform as a flight simulator.

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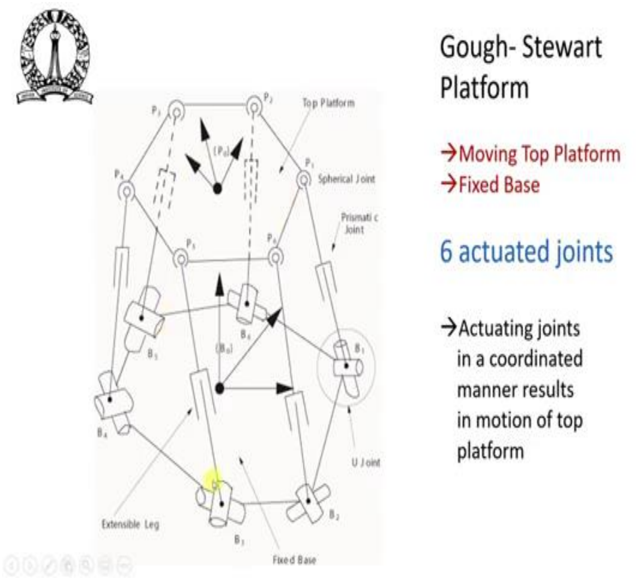

The kinematic model of a Stewart platform consists of a top platform which is this with a coordinate system $\{P_0\}$ attached to the top platform typically at the centre, most of the time this top platform is like a hexagon equal sided and at the top platform there are this spherical joint P_1 through P_6 there is also a bottom platform which have U joints or hook joints which are B_1 through B_6 ok.

So, the connection between the top and bottom platforms are through legs ok. So, for example, here the leg is connected like $B_2 - P_6$ ok where the leg consists of a prismatic joint which can change its length ok. There is a fixed coordinate system $\{B_0\}$ and as you actuate these legs the top platform can do this 6 degrees of freedom ok.

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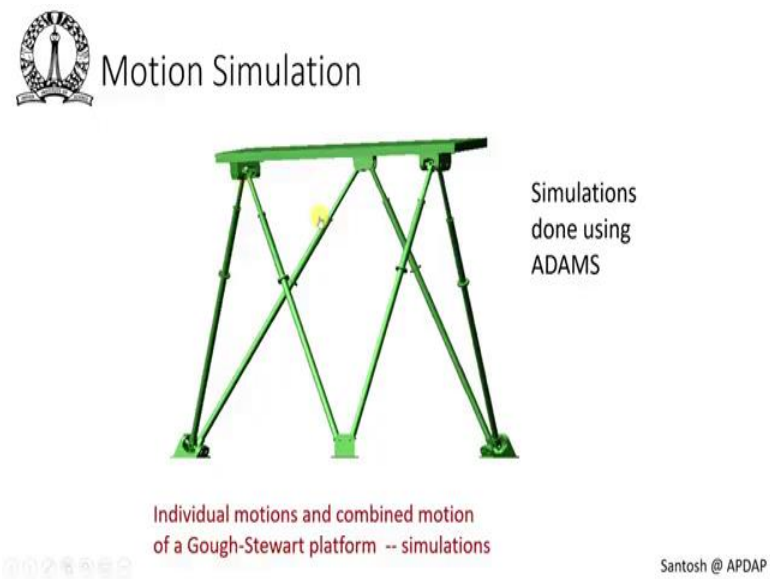

So, here is a video of Stewart platform simulation which was done using the software called ADAMS. So, as you can see that this leg lengths are changing and then because the way you are changing the leg length, you can move this to Stewart platform in various ways. So, this is some kind of motion where some legs have been moved. So, these are basically individual motions of along X, Y and Z and the combined motion.

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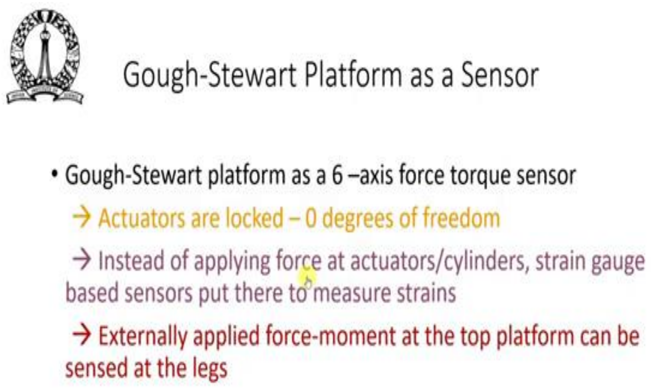

So, the Gough-Stewart platform can also be used as a sensor ok. So, basically when it is used as a sensor, it can measure the 6 components of the forces and torques which are acting on the top platform. So, if there is a force with components (F_x, F_y, F_z) and moment or torque which is (M_x, M_y, M_z) it can be rigged up to sense or measured these forces.

So, when it is used as a sensor these actuators are locked. So, when you lock the legs of the actuator, the whole thing becomes a structure and instead of applying force at the actuators or cylinders, strain gauge based sensors are mounted there. So, they can measure the strains due to the load which is acting on the top platform ok. So, essentially the external applied force moment at the top platform can be sensed at the legs and we will show a little bit of the mathematics.

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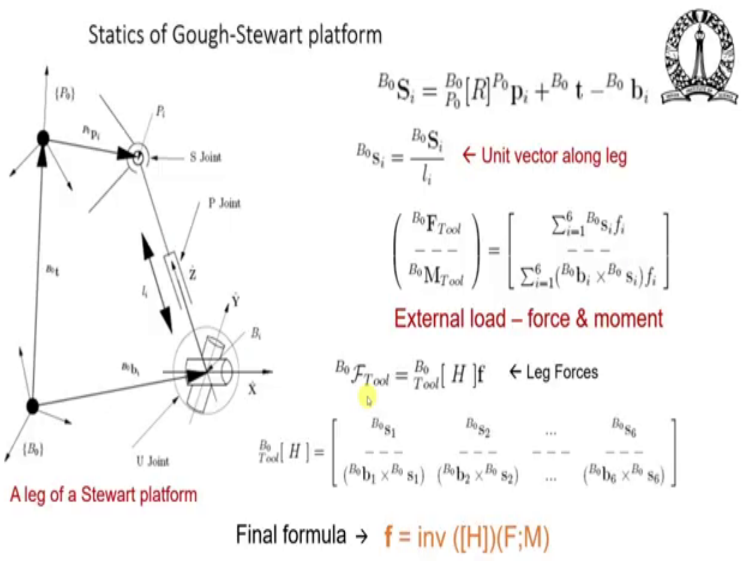

So, consider a single leg of a Stewart platform. So, basically, we have a fixed coordinate system $\{B_0\}$ top platform moving coordinate system $\{P_0\}$ each of these S joints can be located by a vector ${}^{P_0}\boldsymbol{p}_i$ ok. So, P_i is this point of the spherical joint, the U joint can be located by a vector ${}^{B_0}\boldsymbol{b}_i$, B_i with respect to B_0 this is a fixed coordinate system and then each leg can change its length by l_i.

So, the Z axis is along the direction of the joint like any other joint and then we have these two rotary joints which represent a U joint. So, the U joint is nothing, but rotation about X and Y ok. So, the position vector of any S joint ok can be written as ${}^{B_0}\boldsymbol{S}_i$.

So, with respect to the fixed coordinate system as some vector ${}^{P_0}\boldsymbol{p}_i$, pre multiplied by a rotation matrix ${}^{B_0}_{P_0}[R]$, because this vector is in the moving top platform coordinate system because this translation from the fixed origin to the top platform origin minus the location of the U joint ok.

So, this is the basic equation for the Stewart platform for each leg which locates a spherical joint. The length of the motion ok l_i is along the prismatic joint and we denote a vector which is ${}^{B_0}\boldsymbol{s}_i$ this is the unit vector along this prismatic joint in this one.

So, this is $\left(\frac{{}^{B_0}\boldsymbol{S}_i}{l_i}\right)$ and you can show that the force and the moment which are acting in the top platform denoted by ${}^{B_0}\boldsymbol{F}_{Tool}$ and ${}^{B_0}\boldsymbol{M}_{Tool}$ with respect to the $\{B_0\}$ coordinate system must be equal to the force which is acting along this unit vector along each length and the moment of this unit vector the force which is acting along the $({}^{B_0}\boldsymbol{b}_i \times {}^{B_0}\boldsymbol{s}_i)$ ok because the Stewart platform is in equilibrium all the joints are locked.

So, we can rewrite this equation as ${}^{B_0}\mathcal{F}_{Tool}$. ${}^{B_0}\mathcal{F}_{Tool}$ is the force and the moment acting on the end effecter as some matrix ${}^{B_0}_{Tool}[H]\boldsymbol{f}$. So, $\boldsymbol{f}$ are 6 of them. So, there are the 6 legs each of them are applying force along its length. So, we can rewrite it this way and the elements of this ${}^{B_0}_{Tool}[H]$ matrix are nothing, but the unit vectors along this leg and the moment of this unit vector with respect to the $\{B_0\}$ fixed coordinate system ok.

So, the first column is ${}^{B_0}\boldsymbol{s}_i$ then $({}^{B_0}\boldsymbol{b}_1 \times {}^{B_0}\boldsymbol{s}_1)$ and so on. So, eventually what we can write is that this ${}^{B_0}\mathcal{F}_{Tool} = {}^{B_0}_{Tool}[H]\boldsymbol{f}$. We can also invert this ${}^{B_0}_{Tool}[H]$ matrix and write $f = \left({}^{B_0}_{Tool}[H]\right)^{-1}(\boldsymbol{F};\boldsymbol{M})$. So, this is the starting point of a Stewart platform base 6 component force torque sensor.

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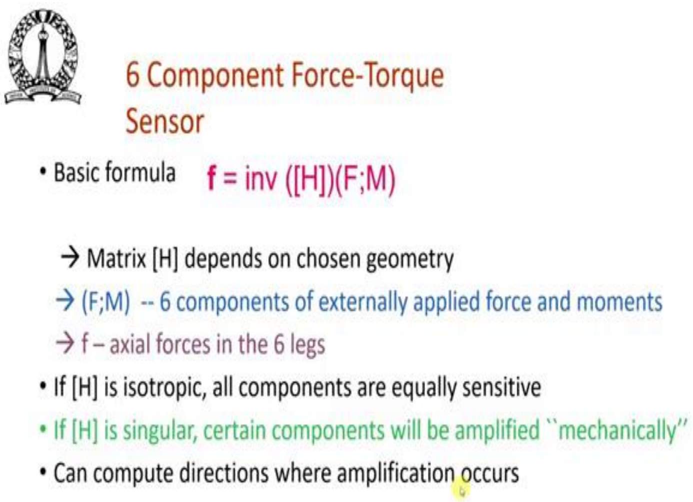

So, from this basic formula we can see that this matrix ${}^{B_0}_{Tool}[H]$ depends on the chosen geometry. What is the connection point between the top and the bottom platform ok? This quantity $(\boldsymbol{F};\boldsymbol{M})$ is actually not really a vector just like in any parallel robot ok just like in when you are discussing force and moment together, it is a combination of 3 components of force and 3 components of moments and this $\boldsymbol{f}$ on the left hand side are the axial forces in the 6 legs.

So, if ${}^{B_0}_{Tool}[H]$ were isotropic meaning if the eigenvalues of ${}^{B_0}_{Tool}[H]$ are all equal ok all components of $(\boldsymbol{F};\boldsymbol{M})$ will be equally sensitive ok, we can measure this $\boldsymbol{f}$ and we can see that all the values of this small $\boldsymbol{f}$ vector will be equally same. If ${}^{B_0}_{Tool}[H]$ is singular certain components this $(\boldsymbol{F};\boldsymbol{M})$ will give rise to larger small $\boldsymbol{f}$ ok and this is what we mean by a Stewart platform in a singular direction ok.

So, if it is in a singular configuration then some $\boldsymbol{F}$ and some $\boldsymbol{M}$ will give rise to large or infinite $\boldsymbol{f}$ if it is a near singular configuration then it will give you large $\boldsymbol{f}$ ok and we can compute these directions and in this direction some amplification will be happening as compared to the other directions.

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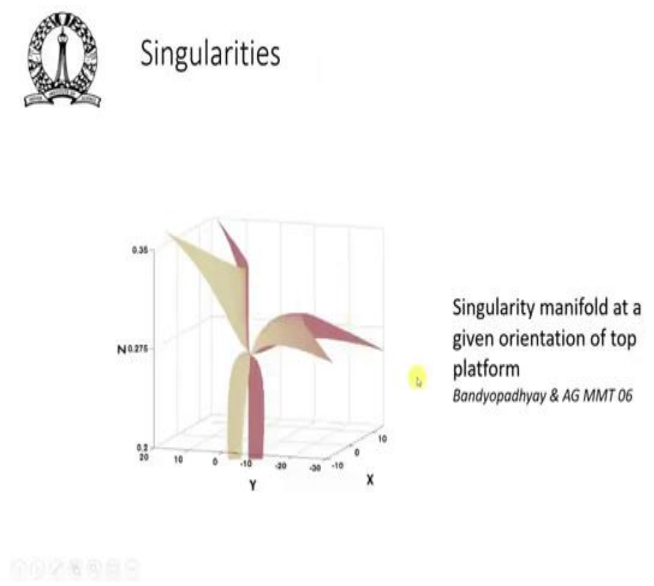

So, there was a work by Professor Bandyopadhyay he obtained the singularity manifold of a 6 6 Stewart platform ok in a semi regular configuration and he showed that the Stewart platform singularities lie on this curve which are basically quadratic in the X Y plane ok and cubic in the Z direction.

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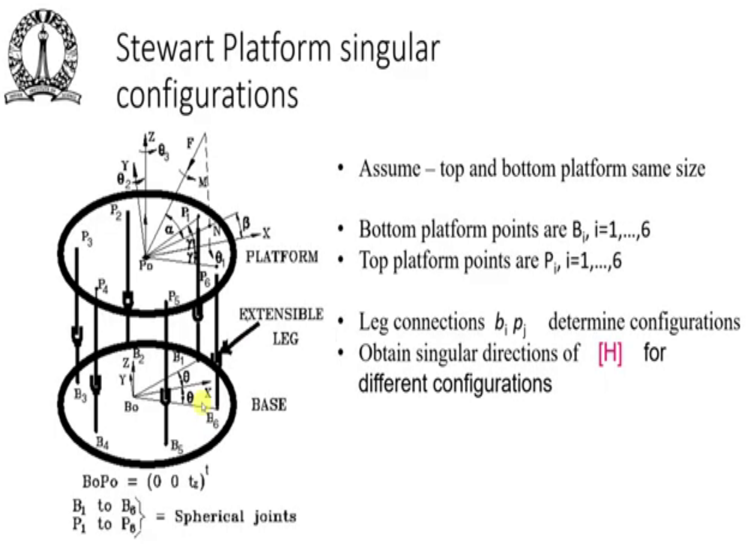

Let us look at the singular configuration of a Stewart platform in a little bit more detail. So, we will start with the Stewart platform where the top and the bottom platform are of the same size ok. So, $B_1 - P_1$, $B_2 - P_2$, $B_6 - P_6$ and all these things. So, the connection points are on a hexagon, the bottom points are also on a hexagon and the sizes of the hexagons are same ok.

So, the bottom platform points are denoted by B_i the top platform points are denoted by P_i and the leg connection can be denoted by a combination $b_i p_j$. So, as we combine them we can find the vector along the leg and the moment of the vectors along the leg and then we can form this $[H]$ matrix for any different combinations or any different configuration which we can think of.

(Refer Slide Time: 14:01)

Singular directions of Stewart Platform configurations

Sl. No	Leg connections						'0' Eig.Val of [H]	Singular Directions
	Leg 1	Leg 2	Leg 3	Leg 4	Leg 5	Leg 6		
1	b_1-p_1	b_2-p_2	b_3-p_3	b_4-p_4	b_5-p_5	b_6-p_6	3	F_x, F_y, M_z
2	b_1-p_2	b_2-p_1	b_3-p_3	b_4-p_4	b_5-p_5	b_6-p_6	2	F_x, M_z
3	b_1-p_2	b_2-p_1	b_3-p_4	b_4-p_3	b_5-p_5	b_6-p_6	1	M_z
4	b_1-p_2	b_2-p_1	b_3-p_4	b_4-p_3	b_5-p_6	b_6-p_5	0	None
5	b_1-p_3	b_2-p_2	b_3-p_1	b_4-p_4	b_5-p_5	b_6-p_6	2	$[1/3,1,0]^1$, M_z
6	b_1-p_4	b_2-p_2	b_3-p_3	b_4-p_1	b_5-p_5	b_6-p_6	2	F_y, M_z
7	b_1-p_1	b_2-p_3	b_3-p_2	b_4-p_4	b_5-p_6	b_6-p_5	1	F_y
8	b_1-p_2	b_2-p_3	b_3-p_4	b_4-p_5	b_5-p_6	b_6-p_1	2	M_x M_y M_z

Note: Superscript '1' denotes singular force direction in a plane.

14

Then we find the eigenvalues of this matrix. So, for example, if $b_1 - p_1$, $b_2 - p_2$, $b_3 - p_3$ and so on. What does this mean? That each of these legs are exactly vertical why vertical? Because the top platform and the bottom platform are exactly of the same size ok, they are both hexagons. So, if all these legs are exactly vertical you can show that there are 3 - 0 eigenvalues and these 3 - 0 eigenvalues of $[H]$ corresponds to singular direction corresponds to $\boldsymbol{F}_x$, $\boldsymbol{F}_y$ and $\boldsymbol{M}_z$ ok.

So, what does it mean? That if I apply a force along $\boldsymbol{F}_x$, $\boldsymbol{F}_y$ or $\boldsymbol{M}_z$ or moment along $\boldsymbol{M}_z$ it cannot be resisted ok. Remember the $[H]$ transformation matrix we had discussed in parallel robots, it shows you the directions in which the force cannot be registered. So, if you apply a very small force it gives rise to a very very large $\boldsymbol{f}$ in this when the forces are in the singular directions ok.

So, why large? Why not infinity? Because you can never make these legs exactly straight. So, the eigenvalues can never be exactly 0 or the determinant of $[H]$ can never be exactly 0. So, if it is small then it is near singular configurations ok. This is another configuration which you can see it is $b_1 - p_2$, $b_2 - p_1$.

So, basically the legs are crossed two adjacent legs are crosses. So, $b_3 - p_3$, $b_4 - p_4$ and then all the other legs are straight. So, first two legs connections cross and all other straight. So, in this case you can see that the eigenvalues of $[H]$ there are two of them which are 0 and they correspond to singular directions which $\boldsymbol{F}_x$ and $\boldsymbol{M}_z$ ok.

If you cross all the legs $b_1 - p_2$, $b_2 - p_1$, $b_3 - p_4$, $b_4 - p_3$, $b_5 - p_6$, $b_6 - p_5$ then none of the eigenvalues are 0 and hence there are no singular directions. We can also find if you do from 1 to 2, 2 to 3, 3 to 4, 4 to 5, 5 to 6 and 6 to 1. So, basically each of the legs are shifted by 1, then you can see that there are singular directions which are $\boldsymbol{M}_x$, $\boldsymbol{M}_y$ and $\boldsymbol{M}_z$.

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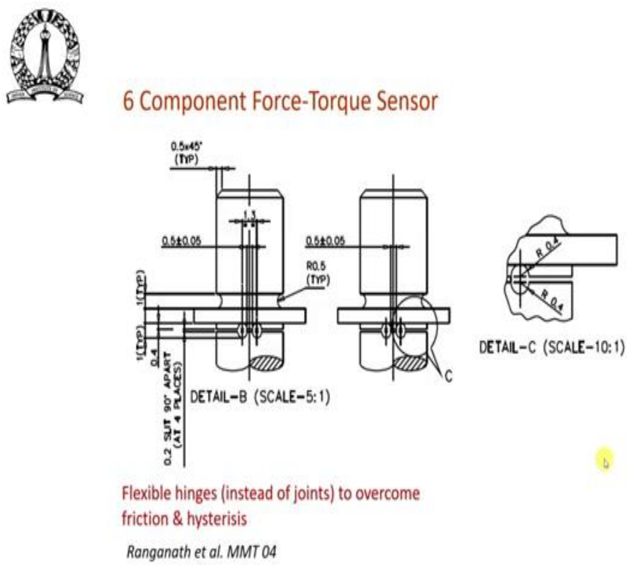

So, one next important thing for fabrication of a force torque Stewart platform is this how we can get rid of this spherical joint and the Hook joint. Why do we need to get rid of the spherical and the Hook joint? Because in an actual kinematic joint there will be friction ok. So, not only will the leg force be amplified at a near singular configuration, but even the friction of the joints will be amplified.

So, we need to somehow get rid of these forces in the joints at the Hook joint and at the spherical joint. So, what we do is, we make what are called as flexural hinges. So, flexural hinge is nothing, but you take a rod ok and then you make at some place you cut it such that the connection is like two cones ok.

So, it is like two cones meeting at a point. So, the top part of the leg can move with respect to the bottom part of the leg because the section is very very thin layer. So, its basically deflects. So, it is not really a physical joint, but it will deflect it will deform.

But as a sensor, we do not need too much motion at the joints anyway ok. So, sensor is normally a rigid or semi rigid structure which do not deform a lot. So, in this way we can make sure that the top platform will move with respect to the bottom platform without the effect of the friction and other problems associated with the spherical joint ok

One of the problems in joints is there is something called hysteresis ok. Here also there will be some hysteresis. So, depending on how it is moving you will slowly see that the for a given force while loading and while unloading the deformations are slightly different that is a property of a material. However, by careful choice of material and heat treatment and other things we can make sure that this hysteresis is very small ok.

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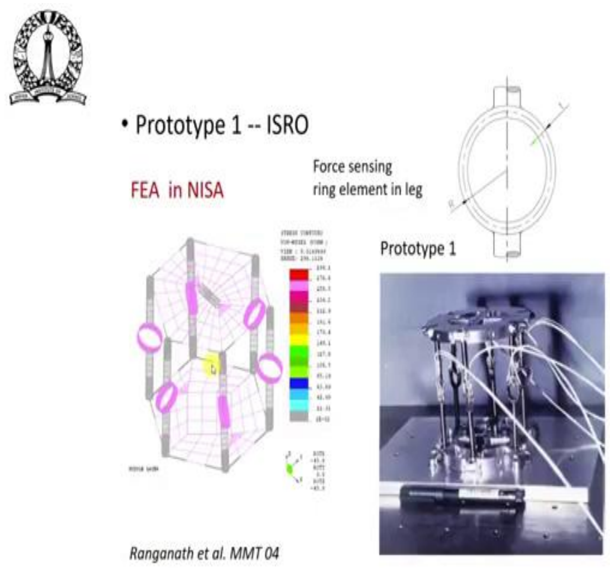

We also made a CAD model of this sensor where all the legs are exactly straight. So, $b_i - p_1$, $b_2 - p_2$ and so on and the reason is we have this small flexural hinges and we need to make sure that they do not break or they do not fail when certain loads are applied. So, this is the model of the 6-6 Stewart platform which is made in NISA and finite element analysis was done in NISA.

So, basically these are the top platforms, this is the bottom platform and then there are this flexural joints connecting each leg to the top and bottom platform and as I said we do not have sliding joint we do not have actuators there. So, instead of an actuator we put a ring which can measure the strains or the deformation in each leg.

So, basically the ring looks like this. So, this is the top part of the leg, this is the bottom part of the leg. So, if I apply a load along this leg ok at the mid plane you will get the largest strain at the mid plane you will also get the largest stress ok. This the deformation it is here is more, but the stress is more at the mid plane.

So, we will put some strain gauges four of them one outside surface one inside surface of this ring on both sides and then we use this strain gauge in a full bridge configuration to measure the strains when some axial load along the leg is applied ok. So, we checked all these calculations, we did this finite element analysis and we showed that the von-mises strain on these legs and the different parts of this Stewart platform were well within the tolerance are well within the acceptable limits.

So, this Stewart platform was built as you can see it is not very big, it is similar to the size of a pen with 6 legs and then there are these wires which are connected to the strain gauge and then we take this wires to a strain indicator.

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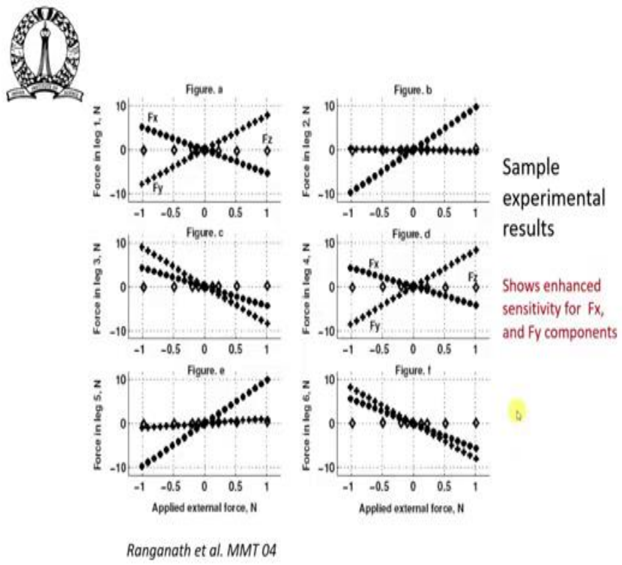

So, then we apply some known loads on the top platform ok. So, if you apply a known force on the top platform ok, which is this applied external force and then you measure the strains in each of these legs and from the strains we find out what is the force in each of the legs ok. How can we do that? We do some calibration we apply a known force of so, many Newton's find out what is the strain in each leg and then we can have some sort of a table which tells you this force corresponds to this $\boldsymbol{F}_i$.

So, what you can see here is, when you apply $\boldsymbol{F_x}$ and $\boldsymbol{F_y}$, the output in the each of the legs. So, this is leg 1, 2, 3, 4, 5 and 6 this is the force measured in each of the legs, it is much larger than when you apply only $\boldsymbol{F_z}$ ok. Does that make sense?

Yes, because the configuration is very close to singular configuration where the sensitive directions are $\boldsymbol{F_x}$, $\boldsymbol{F_y}$ and $\boldsymbol{M_z}$ ok. So, a small $\boldsymbol{F}$ will give rise to a large leg force ok whereas, the $\boldsymbol{F_z}$ is there is not much amplification ok the z force the vertical force is also supported at 6 places.

So, hence it is very small ok. So, we get an amplification of the order of 6 ok. So, one Newton force applied along the x or the y direction will give rise to some force which is 6 times if you had applied the force along the z direction.

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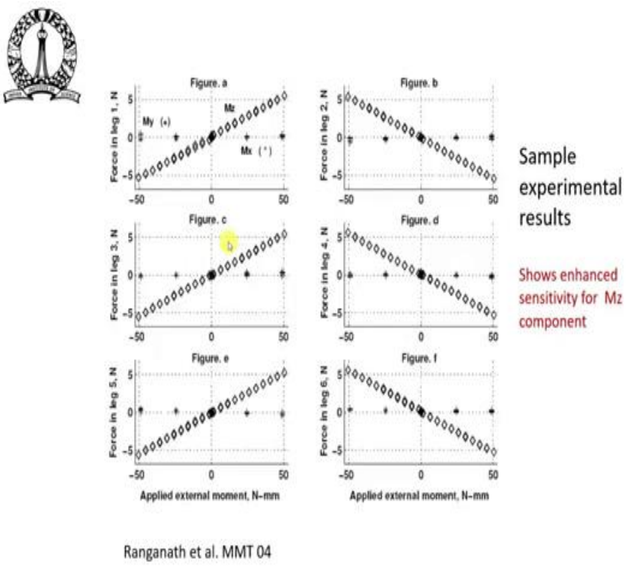

We also applied some moments on the top platform and again you can see that this $\boldsymbol{M_z}$ direction is more sensitive as expected. So, we know the Stewart platform with all the legs exactly vertical is sensitive to $\boldsymbol{F_x}$, $\boldsymbol{F_y}$ and $\boldsymbol{M_z}$ and these tests show or these results show that is indeed true ok. So, you can see that the slope of the $\boldsymbol{M_z}$ which is applied by leg is much larger than $\boldsymbol{M_x}$ and $\boldsymbol{M_y}$. $\boldsymbol{M_x}$ and $\boldsymbol{M_y}$ are both very close to the horizontal line ok little bit more, but not much.

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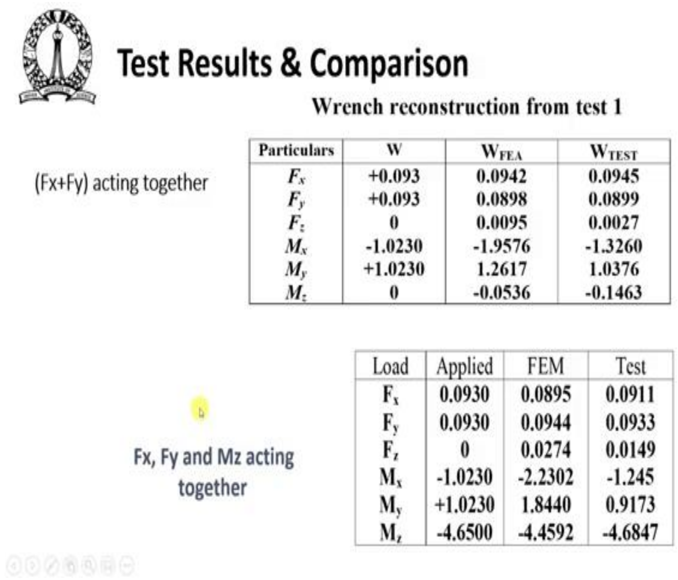

Particulars	W	W_{FEA}	W_{TEST}
F_x	+0.093	0.0942	0.0945
F_y	+0.093	0.0898	0.0899
F_z	0	0.0095	0.0027
M_x	-1.0230	-1.9576	-1.3260
M_y	+1.0230	1.2617	1.0376
M_z	0	-0.0536	-0.1463

Load	Applied	FEM	Test
F_x	0.0930	0.0895	0.0911
F_y	0.0930	0.0944	0.0933
F_z	0	0.0274	0.0149
M_x	-1.0230	-2.2302	-1.245
M_y	+1.0230	1.8440	0.9173
M_z	-4.6500	-4.4592	-4.6847

Once we have this proof of concept that we can indeed measure $\boldsymbol{F_x}$, $\boldsymbol{F_y}$ and $\boldsymbol{M_z}$ more sensitively, then we form that $[H]$ matrix again using some calibration then we find the inverse of the $[H]$ matrix ok and then we do some real tests. So, for example, if you apply $\boldsymbol{F_x}$ and $\boldsymbol{F_y}$ of this value 0.093, 0.093 and so on.

And similarly some $\boldsymbol{M_x}$ and $\boldsymbol{M_y}$ which of these two values we can do a finite element analysis and find out what should be the leg forces ok which are these and then sorry we can find out the leg forces and from the leg forces we can compute what should be the forces and moments which the device is measuring.

So, the finite element analysis is telling you that we can get we apply 0.093 and analysis is telling you 0.0942 0.093 and this is 0.898 instead of 0, we are getting 0.0095 and so on ok. Likewise, we can also perform this actual test and you can compute what the force it is measuring, and it turns out it is 0.945 0.0899 and so on.

So, as you can see it is reasonably close, we know this is what we have applied, and this is what we have measured, and this is what we have computed using the finite element software. All are sort of close ok this $\boldsymbol{M_z}$ is not really close because this is minus 0.14 whereas, is supposed to be 0 ok but nevertheless it is not too bad.

If you apply $\boldsymbol{F_x}$, $\boldsymbol{F_y}$ and $\boldsymbol{M_z}$ together. So, we are applying 0.093 and some $\boldsymbol{M_x}$, $\boldsymbol{M_y}$ and $\boldsymbol{M_z}$ what we can see the finite element is telling you analysis is that these are the values which you should get from computation the test is telling you these are the values which you should get test means you measure the leg forces and you multiply by the $[H]$ matrix which you now know ok and then you get the measured forces. So, as you can see here also its not too bad it is quite reasonable ok.

So, this shows that the 6 component force torque sensor that we have fabricated can indeed measure external forces and moments ok and more importantly it can measure very small values of $\boldsymbol{F}_x$, $\boldsymbol{F}_y$ and $\boldsymbol{M}_z$ it is more sensitive to $\boldsymbol{F}_x$, $\boldsymbol{F}_y$ and $\boldsymbol{M}_z$.

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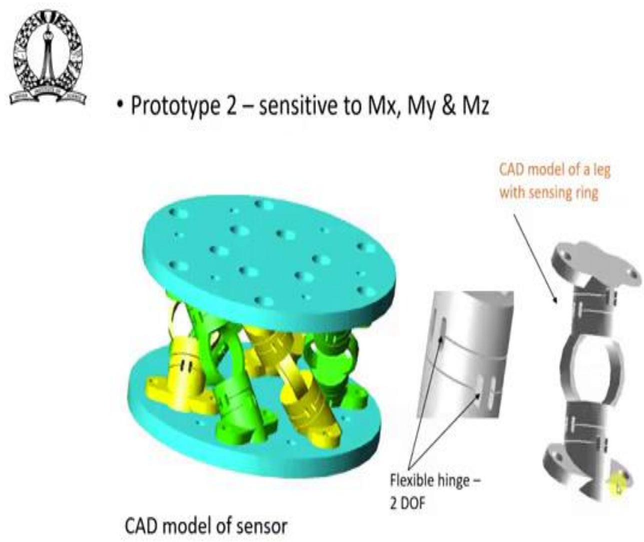

We also built another prototype where the legs was shifted 1 ok remember 1 goes to 2, 2 goes to 3, 3 goes to 4 like that and for that configuration I showed you that the Stewart platform based force torque sensor would be sensitive to $\boldsymbol{M}_x$, $\boldsymbol{M}_y$, $\boldsymbol{M}_z$ ok. Here also we made this hinges flexible hinges this time it is slightly different it is not like a cone two cones touching each other; however, here it is like 2 one degree of freedom cuts ok, but this is 90 degrees to each other.

So, there is one actually at the connection between the top and the bottom platform there is a thin plate two thin plates which are perpendicular to each other ok. So, just the connection point is not like a cone ok like two cones meeting and this helps because this can carry much more load.

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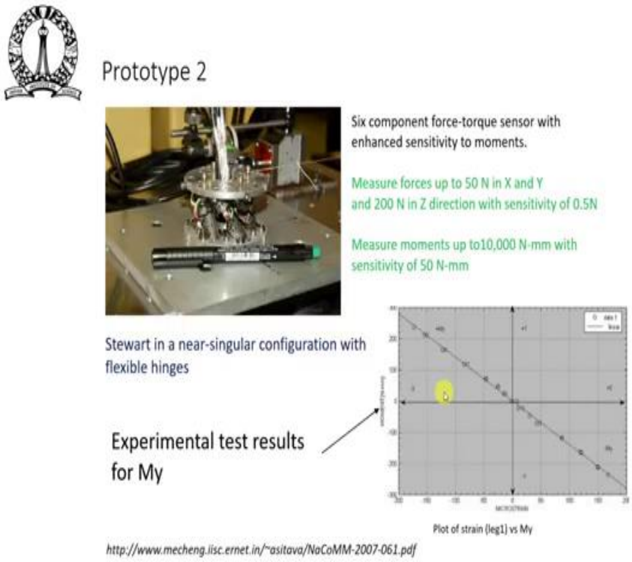

So, this Stewart platform was also built ok it could measure forces up to 50 Newtons in the X, Y and 200 Newtons in Z direction with the sensitivity of 0.5 Newtons. It could measure moments up to 10,000 Newton-millimetre with a sensitivity of 50 Newton millimetre ok. This is a sample plot or a calibration plot of strain versus moment.

So, we can apply moment $\boldsymbol{M_y}$ and we can measure the strains, and this is what it looks like. So, it is fairly linear. So, this is also one of the requirement for a typical sensor that we would like the sensor to be linear. So, if you double the moment the strain should more or less double and likewise reduce if you reduce the external force or moment.

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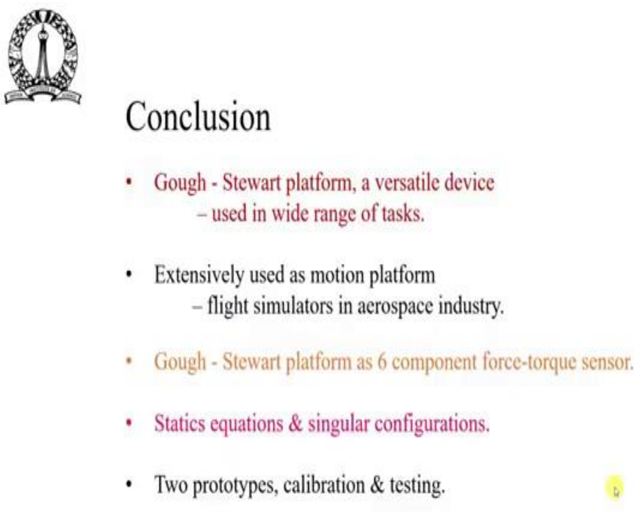

So, in conclusion the Gough-Stewart platform is a very versatile device ok. It can be used in a wide range of tasks. It can be used for tire testing originally, it can be used for orienting mirror, it can be used for various things in industry for manufacturing. One of the most common uses of a Stewart platform is for flight simulators in Aerospace industry.

In this talk or in this lecture I have showed you that the Stewart platform can also be used as a 6 component force-torque sensor. Moreover, we have used this Stewart platform in a near singular configuration such that we get very high sensitivity along the singular directions and I showed you two prototypes which are built calibrated and tested.