Robotics: Basics and Selected Advanced Concepts
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Lecture - 25
Redundancy resolution in human arm

Welcome to this NPTEL course on Robotics: Basics and Advanced Concepts. We are looking in these lectures on Redundancy in robots and hyper redundant robots and its Resolution. In the previous week I had told you how to use redundancy to avoid obstacles and various techniques to resolve the redundancy.

So, we looked at pseudo inverse of the Jacobian matrix, modal solution and also the tractrix base solution ok. In this week we will look at how the redundancy is resolved in human arms ok.

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Quick acknowledgement, this work has been done by Puneet Singh,a student in the robotics line and the Centre for Neuroscience in IISC, the funding was by Robert Bosch Centre for Cyber Physical System.

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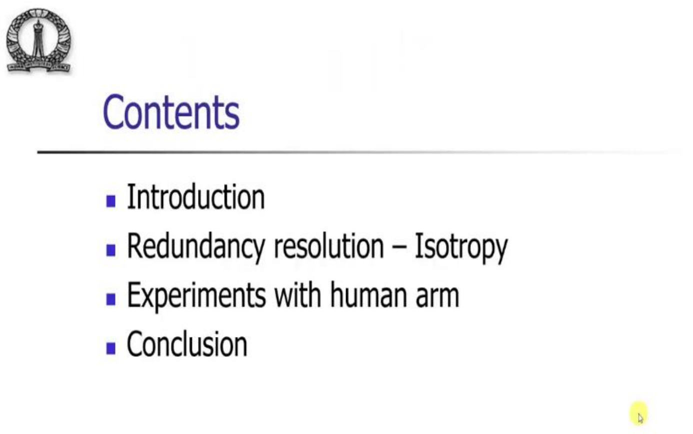

In this lecture we have the following contents we will first introduce again redundancy and also look at the redundancy in human arm. We will also look at redundancy resolution by another technique, where we use the redundancy to make the velocity distribution isotropic. And then I will show you experiments with human arm and conclusion.

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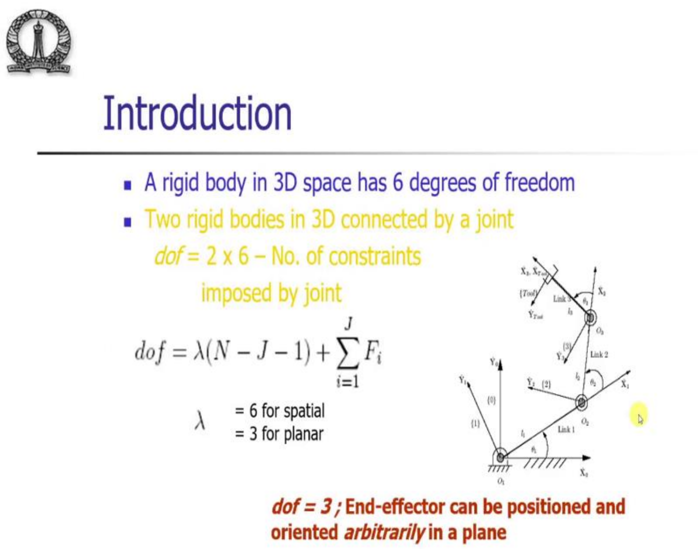

So, again very quick introduction to redundancy a rigid body in 3D space has 6 degrees of freedoms, two rigid bodies in 3D space connected by a joint will have 2 times 6 minus the number of constraints imposed by the joints. The degree of freedom is given by this well-known Grubler's formula ok.

So, for example, in this 3 R robot it has 4 links N is 4 J is 3 and $\sum_i F_i = 1 + 1 + 1$, and λ is 3 which will give you 3 degrees of freedom ok. What it means is we can position and orient the end-effector arbitrarily in a plane.

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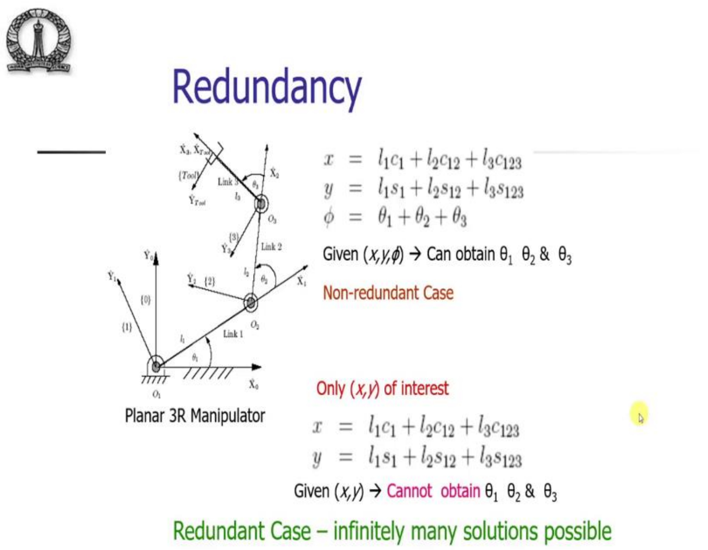

And again as I have discussed several times now by now. If you are not interested in the orientation of the last link, if you are only interested in the position of the last link then we have 2 equations in 3 unknowns and then given x and y we cannot obtain θ_1, θ_2, θ_3 as there are infinitely many solutions ok.

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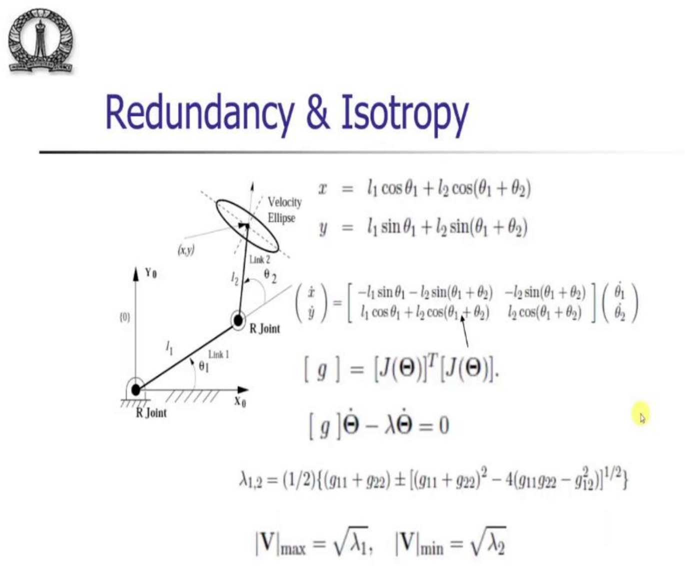

So, one way to resolve this redundancy specially for this 2 degree of freedom case is we can look at the velocity at the tip of this robot at the end-effector ok. So, for example, the position vector x and y can be related to the θ_1 and θ_2 for this 2R robot as $x = l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2)$ and $y = l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2)$.

The velocity can be obtained by taking the derivative of these two equations and reorganizing. So, $x = (-l_1 \sin\theta_1 - l_2 \sin(\theta_1 + \theta_2))\dot{\theta}_1 + (-l_2 \sin(\theta_1 + \theta_2))\dot{\theta}_2$ and similarly $\dot{y}$ will contain now $\cos\theta_1$ and $\cos(\theta_1 + \theta_2)$.

So, this matrix is the Jacobian matrix which we have seen in the past. So, we can derive this $[g]$ matrix which is $[J(\mathbf{\Theta})]^T[J(\mathbf{\Theta})]$ and this also we have seen in the past and then this is the 2×2 matrix and we can find the eigenvalues of this matrix. So, this eigenvalues are given by $[g]\dot{\mathbf{\Theta}} - \lambda\dot{\mathbf{\Theta}} = 0$.

So, in this case this $[g]$ is a 2×2 matrix and $\lambda = \left(\frac{1}{2}\right)\left((g_{11} + g_{22}) \pm \sqrt{(g_{11} + g_{22})^2 - 4(g_{11}g_{22} - g_{12}^2)}\right)$. So, this again was shown earlier. So, what these eigenvalues tell me is that the maximum and minimum velocity possible at this point at the tip at any point in the workspace ok is given by $\sqrt{\lambda_1}$ and $\sqrt{\lambda_2}$ and the tip of the velocity vector traces an ellipse ok.

So, recall this tip of the velocity vector traces an ellipse when there is a constraint on $\dot{\theta}_1$ and $\dot{\theta}_2$. So, we have used $\dot{\theta}_1^{\ 2} + \dot{\theta}_2^{\ 2} = 1$. If $\dot{\theta}_1^{\ 2} + \dot{\theta}_2^{\ 2} = k^2$ then the size of the ellipse is scaled by k, but the shape of the ellipse is same ok. So, this was done earlier.

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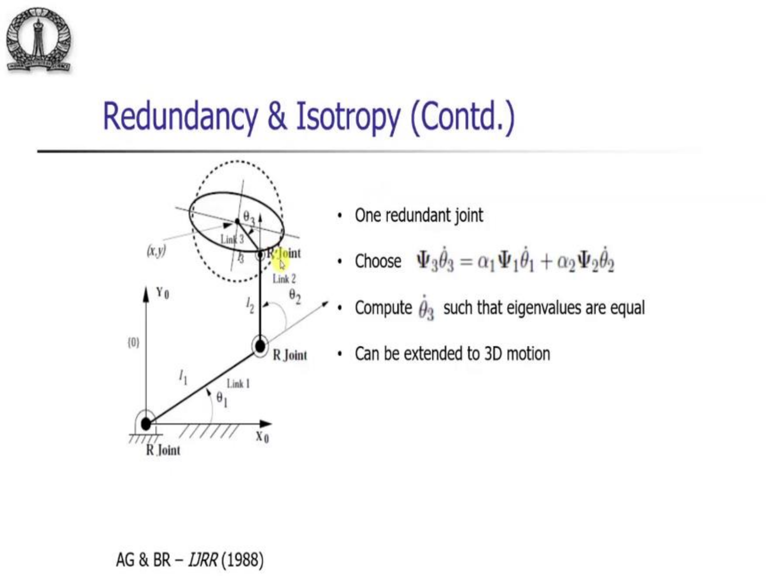

So, now if I want to make this ellipse a circle ok. So, and suppose I have 3 joints ok. So, I have now an example where I have 3 links θ_1, θ_2 and θ_3. So, now, the velocity vector at the tip will be a function of $\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3$. So, at this point. Now we have one redundant joint because I am only interested in x and y we are not interested in the orientation of the last link.

So, I can choose $\mathbf{\Psi}_3\dot{\theta}_3$, $\mathbf{\Psi}_3$ is a variable is a vector which comes from the derivative of the x and y with respect to θ_3. In terms of $\mathbf{\Psi}_1\dot{\theta}_1$ and $\mathbf{\Psi}_2\dot{\theta}_2$. So, $\mathbf{\Psi}_1$, $\mathbf{\Psi}_2$ and $\mathbf{\Psi}_3$ determine the Jacobian for this 3 joint case ok. So, we can compute $\dot{\theta}_3$ in this manner ok and we force that the Jacobian eigenvalues of this Jacobian are equal. So, that will give me a way to compute $\dot{\theta}_3$.

So, this is a very old paper which appeared in 1988 which showed how to compute $\dot{\theta}_3$ in the redundant system in this 1 degree of freedom redundancy 1 redundant joint instead of an ellipse we can make it into a circle. So, the tip of the velocity vector now lies on a circle ok. So, ellipse means certain directions are easier to go and certain directions are harder to go.

If you are lying on an ellipse that is what is happening. If the velocity vector lies on the circle then all directions are equal ok. So, this is also termed as isotropic configuration ok. So, we have done this isotropic configuration for planar 2R, but in this case what I am showing you is that if you have one extra joint and if you use $\dot{\theta}_3$ from that extra joint in this form satisfying this equation, we can make it look like a circle and the velocity distribution is isotropic ok.

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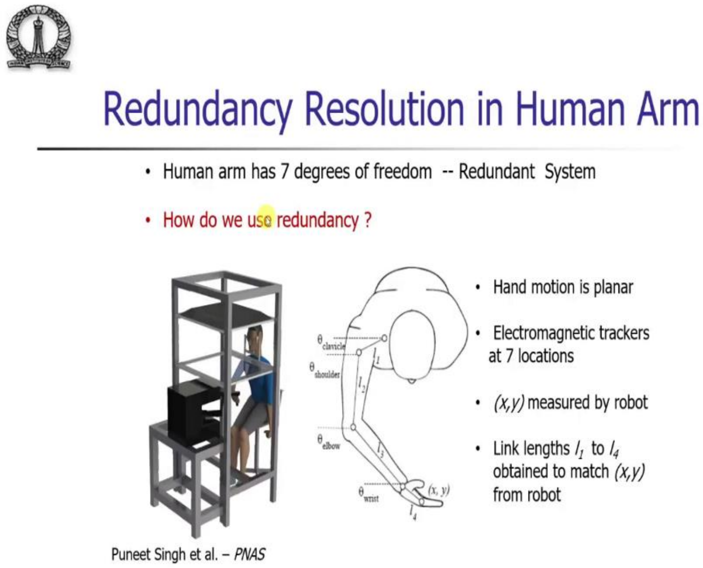

Now, let us see what happens in a human arm. So, the human arm has 7 degrees of freedom ok. So, if you count the number of joints there is a shoulder joint with a 3 degrees of freedom, there is an elbow joint which is 2 degrees of freedom and then there is a wrist joint with another 2 degrees of freedom. So, there are 7 degrees of freedom this is not counting any of the degrees of freedom at the fingers ok.

So, we do not need 7 degrees of freedom to position and orient an object, we know it requires 6 degrees of freedom ok a PUMA robot or an industrial robot has only 6 joints and it can position and orient the end-effector in 3D space. In a human arm there are 7 degrees of freedom 7 joints ok.

So, question is, how do we use this redundancy? So, this is a question which was asked and it was answered by Puneet one of the PhD student. So, what we do is we start doing some experiments. So, the experiment is following ok. So, there is a human person sitting here he is moving a robot a planar robot ok. So, the trajectory of the hand is a planar robot ok. So, he cannot see his hand.

The motion of the hand is shown as a cursor on this screen ok which is using some optical device it is projected on this TV screen and then from the TV screen it is shown here ok. So, the idea is he cannot see his hand, but when he moves his hand he can see where is the end point of his hand ok. The motion of the hand is planar because this robot is planar.

We also mount electromagnetic trackers at 7 locations on the hand. So, some in the shoulder, some in the elbow, some in the wrist and so on. Since its a planar motion we can only measure x and y of this point. So, we have x and y of this something with the fingers are grasping. So, I have some degrees of freedom in the shoulder, some degrees of freedom in the elbow and some in the wrist.

So, there are 4 joints which are active when you are moving this object which you are grasping which is this robot in a plane ok. So, this x and y is measured by the robot because the robot can measures the location of the end-effector of the robot and we have these link lengths l_1, l_2, l_3 and l_4.

So, this is the place where you are grasping the object and there are four angles which is called as clavicle, shoulder, elbow and wrist. So, these are four thetas which are in play here ok. So, we can obtain these link lengths l_1, l_2, l_3 and l_4 for different subjects such that the model I will show you the model very soon that the location of this point (x, y) based on l_1, l_2, l_3 and l_4 matches the position of this x and y of the robot.

So, we can play around with this link length for different subjects such that both the x, y match. So, that is first step.

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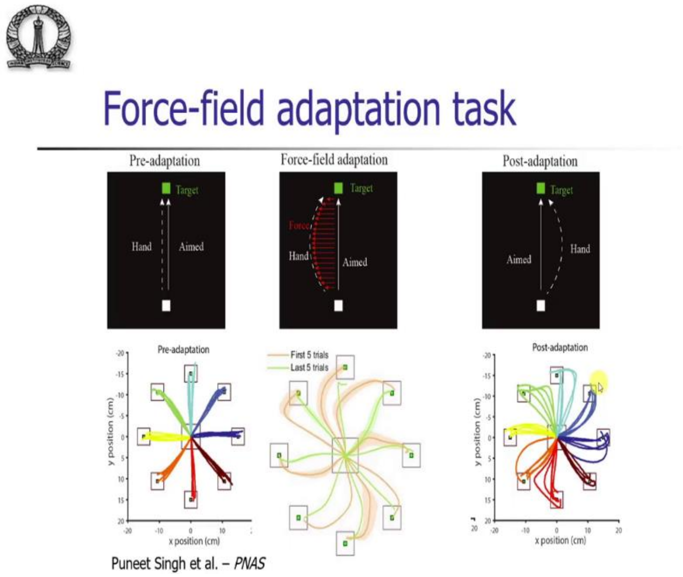

Then what we do is we do the experiment. The experiment consists of the following tasks. So, the subject sees a starting point which is this white point and then a green dot which is the target is projected ok and you are supposed to move the hand towards the green target ok and we do this target in 8 random different directions.

So, first target might appear which is at horizontally, then it can suddenly appear 90 degree at an angle 90 degrees and so on. So, these eight directions are 0, 45, 90, 135 and so on and 315. So, these targets are appearing the person starts from one central location and goes to these targets ok and while he is moving? We locate or measure the θ's we also measure the x and y and we can see what is the trajectory of the hand ok.

So, these are called kinematic reaching tasks. So, I want to go from one point to another point, I want to reach that place and then I am recording what is happening to this during this reaching tasks ok. So, there is a phase which is called pre-adaptation. So, basically we suddenly show this target and then record and what you can see is that reaching task is not really exactly straight, but it is more or less following a straight path.

So, we want to go straight directly to the target. So, there is some small deviation this is basically based on the subject ok. How he reaches to the task and you can see that there are a bar or a circle or a square around this target and if you reach anywhere inside this region we consider the task as a success ok the person has reached that target. So, we can see that there are some small overshoots, but it is not exactly straight it follows some sort of a slight curve ok.

So, these are the one subject doing this reaching tasks when a target is shown. The next task after some time after about some 10 trials approximately 10 trials in each direction we switch on the force field in the robot. So, what is the force field? Basically as you are moving towards the target the robot will apply a perpendicular force ok and the force is proportional to the velocity ok.

So, as you can see he is trying to move go from white to this green there is a force which is acting initially the velocity is small. So, the force is very small as you go towards the middle of the you know task your typical velocities are larger and when you come to the end you stop or you slow down. So, hence the force is also smaller ok.

So, this is what is happening and this force is applied by a robot. So, now, again we record what is happening to the trajectory of the hand. So, what you can see is as soon as the force is switched on which is this yellow curves, you can see a lot of variation. So, this bar or the shaded region around is what is happening in the first five trials this yellow trajectories are what is happening in the first five trials. So, there is some variation.

As you keep on doing this task you learn ok and then you become straighter and this green trajectory shows what happens when you are towards the end of the trials. So, as you can see that the error is decreasing and the trajectory is towards the direction of the force ok that is important.

So, we are trying to adapt to this externally applied force, we learned that there is an externally applied force and then our trajectories become straighter and straighter which is never exactly straight, but it is becoming straighter. Then after about 200 such reaching tasks ok we switch off this force.

So, when you switch up the force the hand tries to over compensate, it thinks that there is still a force which is acting and it will go in the opposite direction ok. So, this is what is shown in this last picture here. So, now, there is no force, but you think there is a force. So, you over compensate and you go in the opposite direction.

So, as you can see when the force is initially applied that curve is like this and when it is switched off the curve is in the opposite direction ok and in all of these we are recording the x, y point we are recording the rotations at the four joints ok and then comes the analysis part ok.

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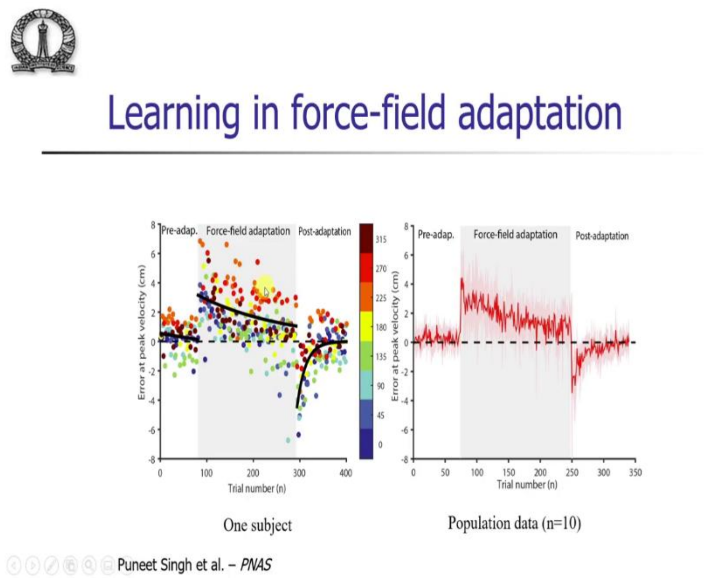

So, first thing is we plot what is the largest error ok. So, what is the largest error? The largest error is when it has the maximum velocity or at some place where the distance from this horizontal straight line from the target to the initial point this distance is largest.

So, that is the largest error. So, we are going to plot this largest error across trials. So, the initial portion there is no force this is called the pre adaptation period. As soon as you apply force the errors become very large. So, this orange line is along some angle 225, these blues are when you are moving in the 0 direction. So, these are 0, 45 or they went in 315 ok.

So, as you can see the errors are very large initially and then slowly the errors go down and we can set an exponential curve we set an exponential curve because it is known in neuroscience that the learning is a first order process. So, if you have some error thus next time the error will be smaller, it decays exponentially ok and in the post adaptation when the force is switched off again the error is initially very large, but it is in the opposite direction and then it again comes to 0.

So, these various dots are the maximum error by a subject in different directions between 80 and approximately 200 something trials and then the last 100 trials are again when the force is switched off. So, this shaded area is called the force field adaptation, this is post adaptation and this is pre adaptation ok. So, this is an exponential curve. So, we can plot it as $e^{-\beta n}$ the here also there is an exponent which is fitted and here also there is an exponent which is fitted ok.

So, now we see what happens for 10 subjects. So, we do this same experiments with ten subjects and this dark line shows the average error for all the 10 subjects and this here shows the average error for all these 10 subjects in the force field adaptation and this is the average error in the post adaptation, this light colored lines are the variation across subjects ok.

So, some subjects the error is much larger in some place and so on ok. So, this is the data which we have calculated from the actual measurements of the maximum error.

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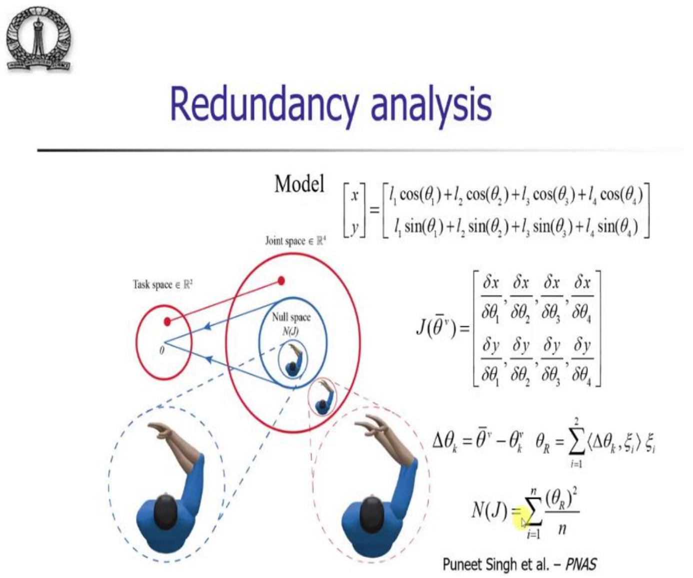

So, now let us look at a model we want to analyze this data. So, the first thing we analyze we model is this is x and y can be written like a planar robot with 4 joints ok. So, in this case we have $x = l_1 \cos\theta_1 + l_2 \cos\theta_2 + l_3 \cos\theta_3 + l_4 \cos\theta_4$ very similar to the planar 2R case except now this is not $(\theta_1 + \theta_2)$ why because the actual measurements of the joint are absolute angles.

We can also measure y which is now in terms of sin exactly very similar to the planar 2 R case ok, but with 4 joints. We can calculate the Jacobian matrix which contains $\frac{\delta x}{\delta \theta_1}$, $\frac{\delta x}{\delta \theta_2}$ and similarly $\frac{\delta y}{\delta \theta_1}$, $\frac{\delta y}{\delta \theta_4}$.

So, it is a 2×4 matrix. So, there are 2 rows which is x and y corresponding to the velocity along x direction and y direction and 4 columns because there are 4 joints ok. So, we can find out this Jacobian at which place where you have the maximum error. We can also find what is the average error or average θ across subjects ok. So, average θ and then we can find the difference between the θ between the average and the maximum ok. So, why do we need this average?

So, basically if I want to go from point A to point B what is the actual trajectory? So, we assume that when you take the average of all this θ that is what the brain is commanding that is what the desired θ should be, but then there is variation across from this average ok. So, we find out the Jacobian matrix at the average maximum velocity, then we find the variation θ across these subjects between the average and the maximum and let us call this $\Delta\theta$.

We can also find the null space of this Jacobian matrix ok. So, the null space of this Jacobian matrix will be 2 dimensional why? Because it is a 2×4 matrix the null space will be 2 dimensional and the null space is what represents the redundancy of this system ok. So, let us look at why? So, if you look at the joint space which is 4 dimensional. So, any point in these joint space maps into some x and y ok.

So, this is given right hand side I can find out x and y and this is what it looks like. So, I can go to x and y, but I can go to x and y in different ways ok. So, the null space tells you that we can go to this place, but the tip is at the same place x and y is at the same place, but there is internal motion of the hand.

So, I can go and touch my nose, but then my elbow can be at different places even though and shoulder rotation can be at different places even though my finger is still touching the nose. So, I can reach some point x and y; however, the θ variables can be different. If you are in the red region which is not in the null space if I rotate this θ joints the tip will move ok.

So, the null space is where motions of the joint do not cause motions of the tip. So, and that is basically another way of saying that those are the redundant degrees of freedom ok. So, if you think about it what we are saying is I can reach using many many different ways and one way to estimate what are these many ways is to compute the null space of the Jacobian matrix ok. So, the null space is 2 dimensional we find the variation of θ from the mean and we project onto the null space.

So, this is $\Delta\theta$ dot product with the null space dimension and times ξ_i will give me ξ_1 plus $\Delta\theta_2 \cdot \xi_2$ along ξ_2 will give me the θ_R which is the redundant theta ok. So, that is the θa which does not cause any motion of the tip, but there are internal motions ok. So, let us repeat once more, I can compute the Jacobian matrix, I can compute the null space of the Jacobian matrix ok.

This is a simple 2×4 system you can easily do it numerically also we find what is the θ average when you are going from one place to another place and the θ average at the maximum velocity or at the maximum error and then we see what is the variation of θ for different trajectories about the average. So, this average minus theta k^{th} trial is $\Delta\theta_k$ and we project this $\Delta\theta_k$ onto the null space of the Jacobian matrix and this is a proxy or this is a measure of the redundancy in your arm.

And then we square this number because we want a single number to plot later on across all the trials and then this is called as the $N(J)$ which is the null space of the Jacobian matrix ok. So, I hope this is clear.

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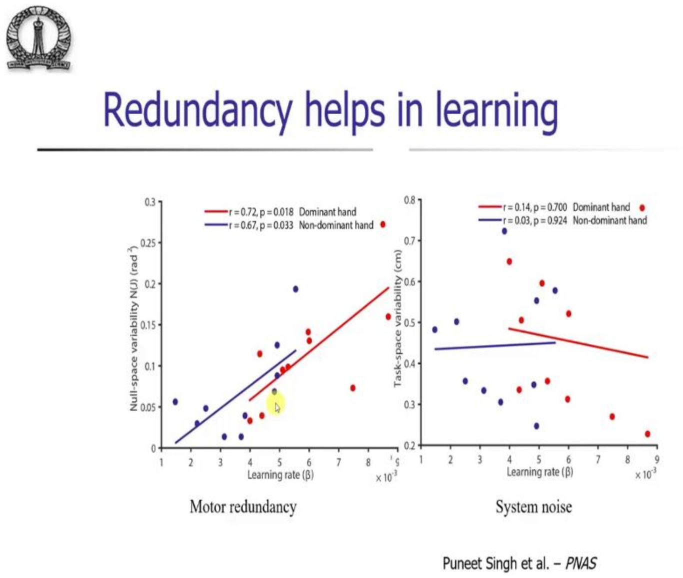

So, once we find what is $N(J)$ we can plot $N(J)$ versus the learning rate ok. Remember the learning rate was $e^{-\beta n}$. So, β was the exponent for the exponential fit ok. So, β is a parameter which tells you how fast you are learning. So, if β is large ok.

Then the exponential curve will drop faster and you are learning faster your errors are becoming smaller ok. So, let us plot this null space that enough computed versus this learning rate for different subjects ok. So, these dots are for one subjects this is for another subject this is for. So, all the blue dots 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 these are the different subjects we compute their learning rate in that force adaptation period and we also compute the $N(J)$ which is the null space variability ok. Likewise, we can also do these experiments for non-dominant hand ok. So, non-dominant hand for some people is the left hand, but for left handed people it is the right hand. So, for these 10 subjects we figured out which one was the dominant hand and which was non dominant hand.

And we asked them to do the same experiment with both dominant and non-dominant hand and again we collected the data and we find out what is the β or the learning rate for the dominant hand and the non-dominant hand. So, the red dots are the data of learning rate and null space variability with the non-dominant hand. So, for example, for one subject the β was let us say bit around 7.5 into 10 minus 3 and null space was let us say 0.7.

Now, we can plot all these points and then we can fit a straight line ok we find the best fit straight line for the non-dominant hand which is the blue line and the dominant hand which is the red line ok. So, what can we conclude? What we can conclude we can easily see is that the dominant hand the learning rates are larger ok the red curve red straight line is further to the right the learning rate is larger ok which is which makes sense right.

We learn faster when we are using a dominant hand. More importantly we can also see that the learning rate is correlated with the null space variability.

So, for those subjects whose β is larger their $N(J)$ is also larger and this is true for both the dominant and non-dominant hand both show a positive correlation with the learn of learning and $N(J)$ and the mathematically or quantitavely quantitatively the this is shown by the r value. So, the r value is 0.72 for the dominant hand and the p value is 0.018 for the dominant hand.

Whereas, the r value is 0.67, but the p value is a little larger for the non-dominant hand nevertheless it shows that there is a positive and significant correlation between the learning rate and the null space of the person. However, we can also find out the learning rate and the task space variability. So, this was joint variability ok we can also compute what is the variability from the mean x, y of the end point when it reaches and what you can see is that the end point and the learning rate are not very well correlated.

In fact, the r values are very poor and not only that, but the p values does not make sense ok. p means what is the probability that the null hypothesis is correct that if you have a p which is less than 0.05; that means, the result is not by chance whereas, if you have a p value much larger than 0.05 that it is random it is a noise ok which is by chance. So, what it is showing you is that subjects who learn faster they also have a larger null space or they make use of the null space more effectively ok.

So, whereas, if you want to reach a task then the variability which is at the end is not correlated to the learning rate ok. So, this is a very important concept or very important finding and this was published in this Proceedings of the National Academy of Sciences in USA.

It shows that those who use redundancy they learn faster to adapt to external forces. In this case the external force was applied when you are trying to reach a certain task or when you want to do a certain task. Let us continue.

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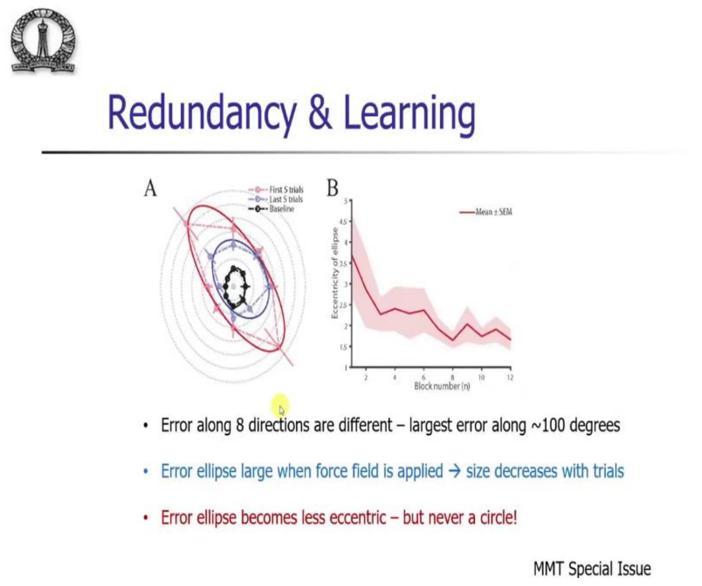

Now, we can also plot what is the error distribution in the first 5 trials the last 5 trials and in the baseline when there was no force ok. So, in the 8 different directions we can plot the error. So, the dark spot is the mean error the bar shows what is the variation of the error ok and you can see in the first 5 trials after the force is applied the errors are much larger ok size of the ellipse is much larger whereas, as you learned in the last 5 trials the size of the ellipse is becoming smaller.

It is also becoming somewhat more circular ok. So, it was shown that the eccentricity of the ellipse eccentricity is the ratio of the major to the minor axis ok the eccentricity of the ellipse is decreasing as you do trials ok. So, it is very very eccentric initially and then slowly it is going towards a circle, it never goes to a circle, one would be a circle ok, but it is decreasing the largest error is roughly along 100 degrees ok.

So, if you think of it your hand is like this, you are going straight that is 90 degrees you are going left or right that is 0 or 180 degrees. So, the error this largest when you are sort of going towards 100 degrees ok not exactly away from you and not exactly horizontal somewhere in between ok.

So, as I have said the error ellipse is large when the force is applied and the size decreases with trials. So, basically you are learning to adapt to the force more importantly the error ellipse becomes less eccentric ok, but never a circle ok. So, remember one of the thing which I showed you that if I have an extra degree of freedom, I can make the velocity distribution at a point in the workspace isotropic, circular.

So, we were hoping that our human hand is also making the velocity ellipse into a circle. So, velocity is related to error. So, whenever you have large velocity you have larger errors. So, we hoped that the velocity ellipse or the error ellipse will become more circular, but that is not true it is becoming less eccentric, but not exactly circular.

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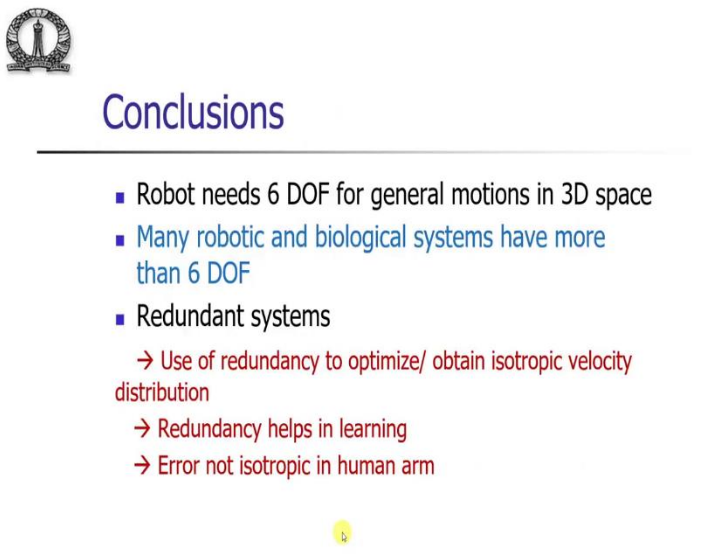

So, in conclusion, we know that a robot needs 6 degree of freedom for general motion in 3D space we have seen this many times now many robotic and biological systems have more than 6 degrees of freedom. So, our human arm has 7 degrees of freedom. So, these are redundant systems ok.

So, in mechanical system you can use this redundancy to optimize some joint variable or some function or we can make the velocity distribution isotropic ok. In human hand it is not becoming isotropic, but what is the redundancy being used for? It is clearly seen that the redundancy helps in learning ok those subjects who use redundancy more, they learn faster the error is not isotropic in the human arm, but it is decreasing ok with learning.