Robotics: Basics and Selected Advanced Concepts
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Lecture - 36
Force Control of Manipulators

Welcome to this NPTEL course on Robotics: Basic and Advanced Concepts. In the last lecture we looked at control of constrained and parallel manipulators and also Cartesian control of serial manipulators. So, in this lecture we will look at Force Control of Manipulators and we will use these ideas of Cartesian control of serial manipulators that we developed last lecture.

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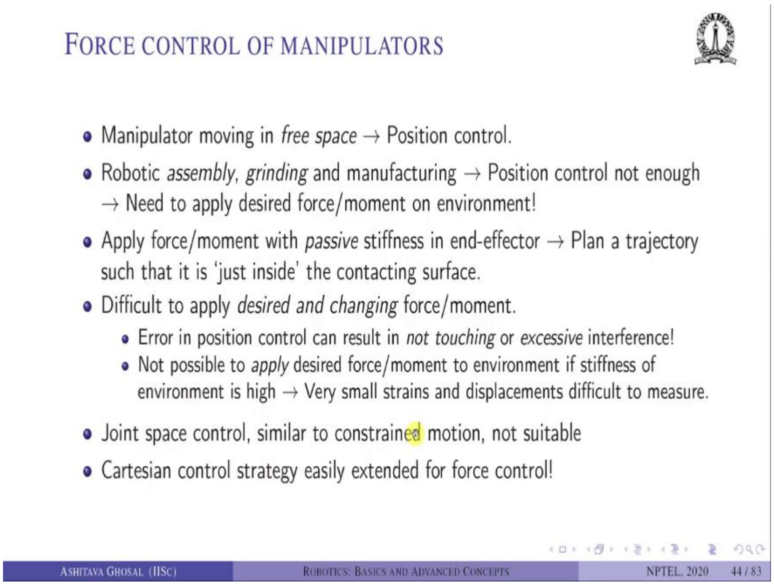

So, let us continue if the manipulator is moving in free space we need to do position control, basically we need to track the desired trajectory. If the robot is assembling, grinding, and manufacturing ok, so, there is some contact with the environment position control is not enough ok. So, we in all these three cases of assembly, grinding, and manufacturing we need to apply desired force and or moment on the environment ok.

So, one possibility is that we apply force and moment with passive stiffness in the end effector ok. So, most end effectors or in fact, for that matter most joints will have some passive stiffness ok, say it is compliant in some sense it is not completely rigid.

So, if you have such joints or with some stiffness, we plan a trajectory such that it is just inside the contacting surface, so think of it this way we want to grind at surface, we make sure that the trajectory is just a little bit inside the contacting surface ok.

As a result when position control is being tried, it will try to reach to that point just below that surface and as a result it will always be in contact with the surface ok. And it will also apply some force, if you know how much is the stiffness which is at the end effector. This is not so easy to do ok, for the following reason ok. The main problem is there is always some error in position control ok, and we cannot be exactly following the trajectory which is just inside the surface.

So, let us say a fraction of a millimeter inside the surface ok. So, due to the error in position control, it is possible that the grinding tool is not touching the surface or it is going too much inside the surface. So, hence we have excessive interference ok. The other problem in this approach of using passive stiffness is that it is not possible to apply desired force to the environment ok.

Most of the time the environment stiffness is very high, there will be very very small strains and the displacement due to the passive stiffness for a very you know stiff environment is difficult to measure, and it is possible that the stiffness also changes or the environment also changes. So, a joint space control similar to constraint motion are what we saw that the tip is tracing a curve in the workspace is not suitable as I had mentioned earlier ok,

We have 6 degree of freedom manipulator it is not so easy to convert that curve into joint variables, because inverse kinematics is may not be analytical, there are other reasons also which we had discussed. We can however use Cartesian control strategy very easily ok, and this can be extended to the control of force which the end effector is applying to the environment.

So, let us start with some very simple basic problem, we have this mass m, which is in contact with this environment, the environmental stiffness is K_e, it is applying some force f to the environment.

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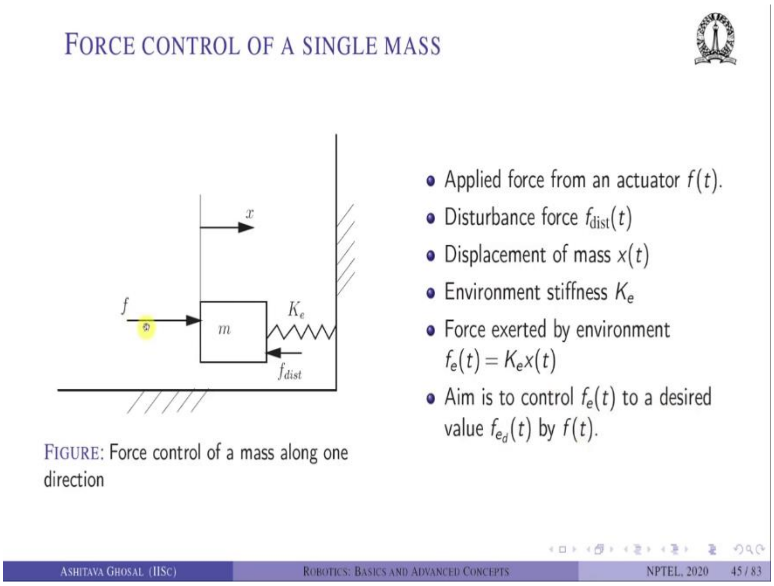

The displacement of this mass is $x(t)$ and we also assumed due to uncertainties that there is a disturbance force which is acting on this mass ok, and we are looking at only one direction, force being applied in one direction and control of that force in only one direction ok.

So, the applied force from an actuator is $f(t)$ the disturbance force is $f_{dist}(t)$, the displacement of the mass is $x(t)$, the environment stiffness is K_e as I have mentioned. So, hence the force exerted by the environment is $f_e(t) = K_e x(t)$ like spring constant times the displacement, and the aim is to control $f_e(t)$ to a desired value $f_{e_d}(t)$ by applying $f(t)$ this external force from an actuator ok.

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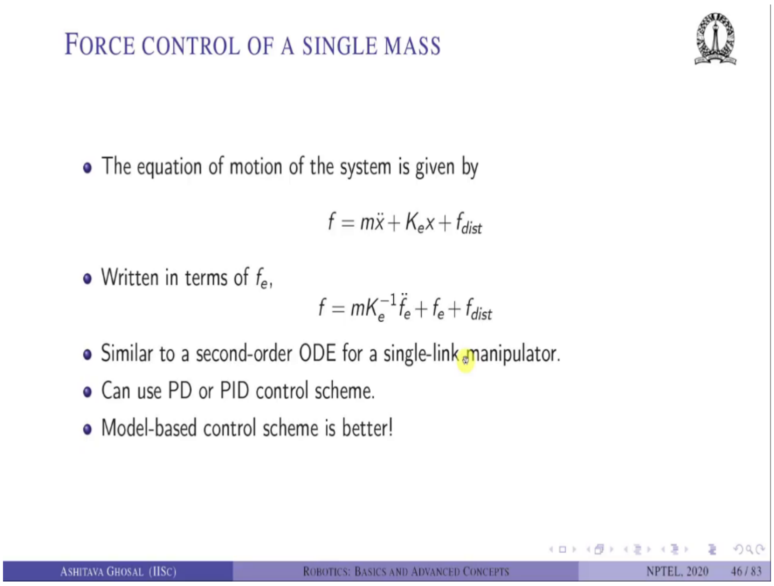

So, the equation of motion of this system can be very simply written as $f = m\ddot{x} + K_e x + f_{dist}$, standard spring mass system with an external force, we know that $K_e x = f_e$ ok. So, we can rewrite this equation in terms of f_e. So, we can write $f = mK_e^{-1}\ddot{f}_e + f_e + f_{dist}$. So, this is very very similar to a second order ODE which we discussed for a joint rotation for a single link robot ok.

In this case, you have a single mass which is applying a force in that single link manipulator we had a torque acting at the joint and the joint was rotating ok. So, we can use PD or PID control to make sure that this f which you are applying gives the desired f_e which we want ok.

However, we are now very familiar with model based control ok ,we see that there are lots of advantages of model based control schemes and hence, let us try and see if we can develop a model based controller for this task of applying a desired force onto the environment.

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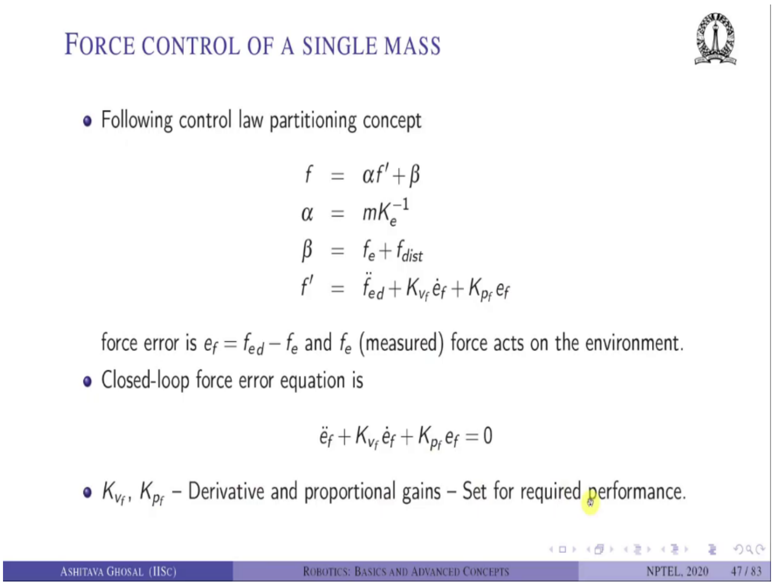

So, just following the control law of partitioning concept, we will say $f = \alpha f' + \beta$ for the moment let us assume $\alpha = mK_e^{-1}$, $\beta = f_e + f_{dist}$ and hence, we will be left with $f' = \ddot{f}_{ed} + K_{v_f}\dot{e}_f + K_{p_f}e_f$. So, e_f is nothing but the error or the difference between the desired force and the measured force.

So, we must have some sensor which will measure the force also. So, if you go back and substitute all this and simplify, we will get a second order force error equation ok. This is the closed loop force error equation; it is second order linear ok, with constant coefficients. So, K_{v_f} and K_{p_f} are the derivative and proportional gains and we can set them to some value for required performance.

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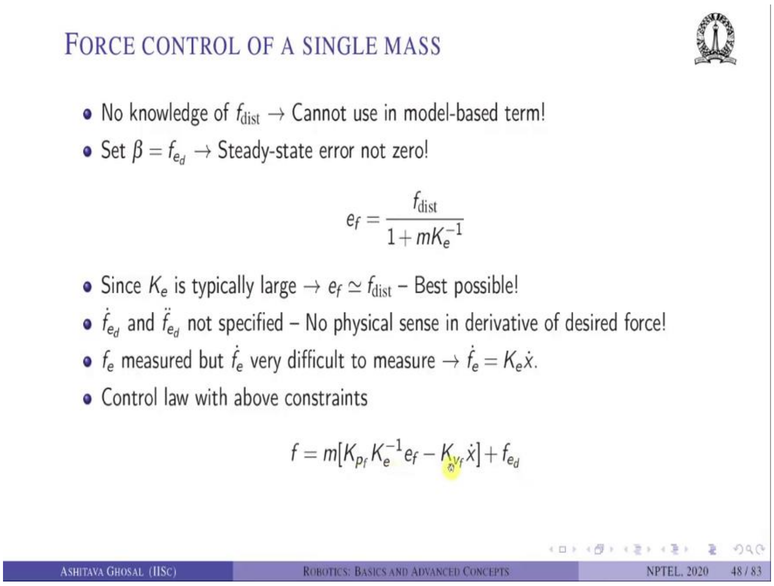

Now, there are some issues ok, first is we do not have any knowledge of f_{dist}. So, hence, we cannot use it in the model based term. So, remember we have said $\beta = f_e + f_{dist}$, but in principle the whole idea of f_{dist} is we do not know the disturbing force. So, hence we cannot compute β, because this is $f_e + f_{dist}$.

So, if you set $\beta = f_{e_d}$, not $f_e + f_{dist}$ we can see that the steady state error will be $e_f = \frac{f_{dist}}{1+mK_e^{-1}}$, it is not zero; however, it is small why because K_e is typically very large ok. So, K_e of say a steel will be very very large number and this K_e^{-1} will be m divided by some large number so, it is very small.

So, the error is of the order of f_{dist}. So, this is the best that we can do. The second issue is we can measure f_{e_d}, but we cannot normally measure the rate of change of the force applied to the environment ok. So, there is also no physical sense in the use of derivative of a desired force. So, if $\dot{f}_{e_d}$ and $\ddot{f}_{e_d}$ cannot be specified ok then, we might as well drop it.

Likewise, f_e is measured, but $\dot{f}_e$ is very very difficult to measure we can measure that force exerted by the environment by strain gauges or some sensor, but the rate of change of the force applied by the environment or applied to the environment is impossible, but we can estimate $\dot{f}_e$ by $K_e\dot{x}$. So, if you can measure how much is the velocity of that mass which is $\dot{x}$, then knowing or getting an idea of K_e we can find $\dot{f}_e$.

So, hence dropping $\dot{f}_{e_d}$, $\ddot{f}_{e_d}$ and also $\beta = f_{e_d}$ and $\dot{f}_e = K_e\dot{x}$ if you make all these substitutions the control law now, becomes $f = m\left(K_{p_f}K_e^{-1}e_f - K_{v_f}\dot{x}\right) + f_{e_d}$.

So, you can see, it is proportional to the error ok, it is derivative gain times $\dot{x}$. So, there is no $\dot{f}_{e_d}$ and there is a bias term f_{e_d}. So, this is the best that we can do, if you are going to use a model based scheme to control the force which this mass is applying to the environment ok. So, how do we choose K_{p_f}, K_e and K_{v_f}?

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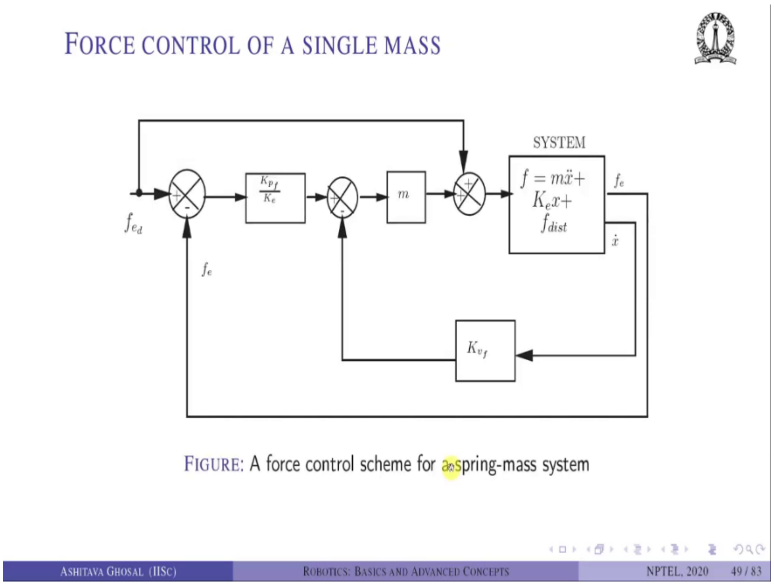

So, ok before that let us quickly go through the block diagram, we have this spring mass single mass connected to a spring applying a force and also experiencing a disturbing force. So, the input is f we measure f_e and $\dot{x}$ with $\dot{x}K_{v_f}$ we subtract ok and then with f_e and f_{e_d}, we find the error in the force we multiply by $\frac{K_{p_f}}{K_e}$, and then we subtract this $K_{v_f}\dot{x}$ and that multiplied by mass will give the force and then we add this f_{e_d} directly to obtain the force ok.

So, this is the block diagram of a force control of a spring mass system ok, and if we have set the gains K_{p_f} and K_{v_f} properly then this f_{e_d} will approach the whatever we want ok, the measured f_e which is the force measured by the sensor and f_{e_d} will go to zero the difference between those two will go to zero.

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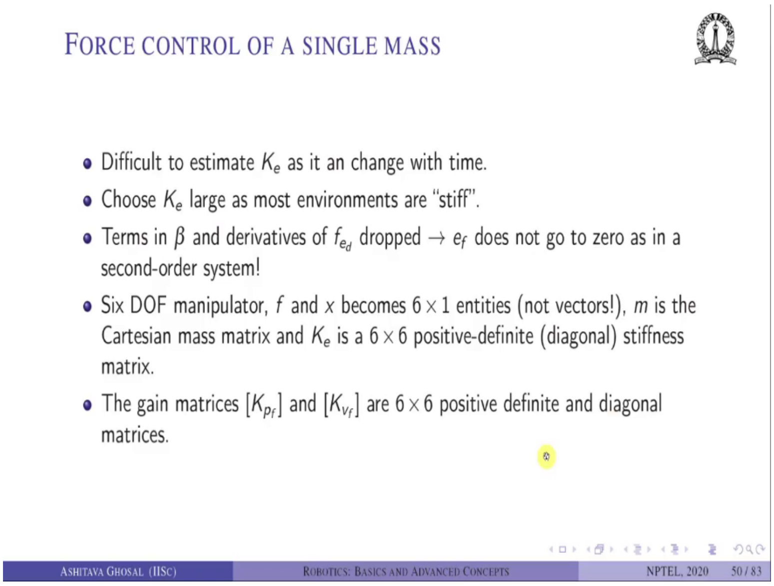

So, as I said it is difficult to estimate K_e as it can change with time ok, the stiffness you know initially you are grinding a steel then you can be grinding something else aluminum. So, the stiffness can change with time. So, what is the basic idea, we can choose K_e large as most environments are "stiff".

So, what is large we have to choose the large number and then play around with it, we have to do some simulations to see what is happening; terms in β and the dropped derivatives of f_{e_d} are not there ok. So, some terms like f_{dist} was dropped $\dot{f}_{e_d}$ and $\ddot{f}_{e_d}$ also were dropped, as a result you can show that the error between the desired force and the measured force does not go to zero as in a second order system ok.

It becomes a small number, it is of the order of the disturbance force, but it is not zero, we can extend this idea to 6 degree of freedom manipulator. So, in this case f and x becomes 6×1 entities remember not vectors so, f becomes force and moment which is acting at the end effector m is the Cartesian mass matrix and K_e is a 6×6 positive definite diagonal stiffness matrix ok, x contains x, y, z and the orientation, a representation of orientation of the end effector.

So, this gain matrices K_{p_f} and K_{v_f} are 6×6 positive definite and diagonal matrices ok. So, as I said by choosing this K_{p_f} and K_{v_f} for each one of these axis, we can hope to ensure that we can apply the desired force ok.

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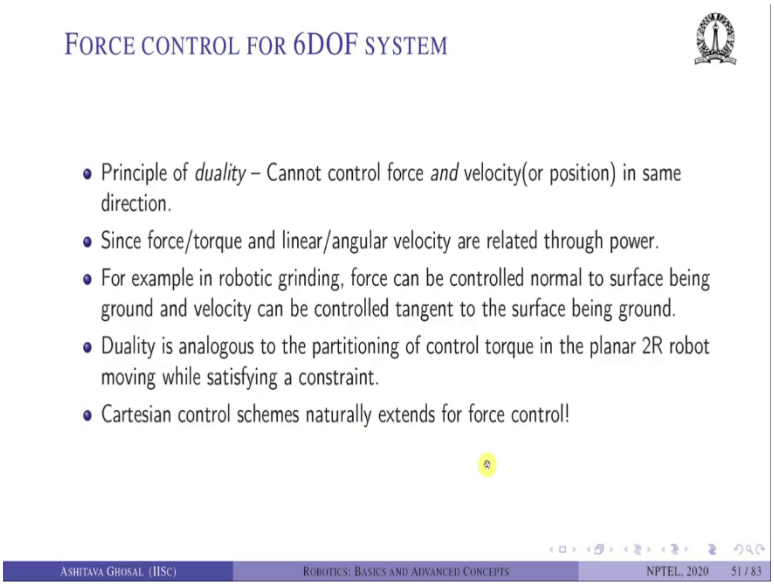

Now, let us continue. So, when we are dealing with forces ok, there is this very important notion or principle of duality. So, what it says is we cannot control force and velocity in the same direction ok.

Why is that? because if you say that let us consider a simple mass, we know $F = ma$. So, if you say that the force is specified the acceleration is automatically determined I cannot choose arbitrary force and arbitrary acceleration ok. Likewise, I cannot choose arbitrary torque and arbitrary rotation of the end effector, they are related and in fact, the force torque and the linear and angular velocity are related through power through Newton's law and Euler's equation.

So, for example in robotic grinding, force can be controlled normal to the surface being ground and the velocity can be controlled tangent to the surface being ground ok, we cannot control the force in the tangent to the surface and velocity in the other direction ok. Because, it does not make sense ok, or we cannot control both the force and the velocity tangent to the surface.

So, this duality is analogous to partitioning of control torque in planar 2R robot while satisfying a constraint. Remember, when we saw that the tip of the 2R robot was tracing a curve $f(x, y) = 0$, I showed you there was a τ_ϕ which was tangent the surface and a τ_n which was normal to the surface.

So, these two are different ok, we cannot have both velocity in the normal direction and force in the normal direction, only force in the normal direction and velocity in the tangent direction ok. So, this Cartesian control scheme naturally extends for force control.

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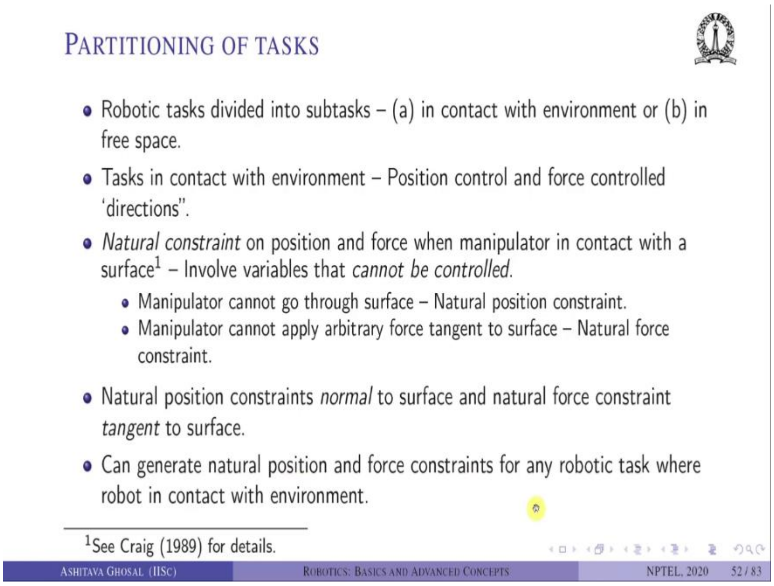

So, what do we do, in any robotic task we divide into sub tasks first in contact with the environment or in free space ok.

So, if the robot is moving and then goes and touches a surface, till it is moving in free space it is moving in free space ok, and when it gets in contact with the environment we have to use a different kind of control scheme. So, tasks in contact with the environment position controlled and force controlled directions need to be found out ok.

So, if you are in contact with the environment certain directions you can do position control, certain directions you can do force control, ok. How about when it is in free space, in all directions or for all components x, y, z and orientation we do position control, there is no force which is acting on the end effector.

So, to divide the task into position and force control directions, there was this notion of natural constraint and artificial constraints which were developed in the late eighties ok, you can see this book by Craig for details. So, a natural constraint on position and orientation occurs when the manipulator is in contact with the surface, it involves variables that cannot be controlled ok.

A manipulator cannot go through a surface. So, the natural position constraint exists when it is in contact with the surface. A manipulator cannot apply arbitrarily force tangent to the surface ok, so, there is a natural force constraint in the tangent direction.

So, whenever it is in contact with the environment, we can find certain variables which are natural position constraints position means both x, y and z orientation and certain force constraint which are f_x, f_y, f_z and moments m_x, m_y, m_z which cannot be controlled ok.

So, if you think about it, the natural position constraints are normal to the surface and the natural force constraints are tangent to the surface ok. So, depending on what kind of task you are doing we can generate the natural position and force constraints for any robotic tasks ok, whenever it is in contact with the environment and I will show you some examples.

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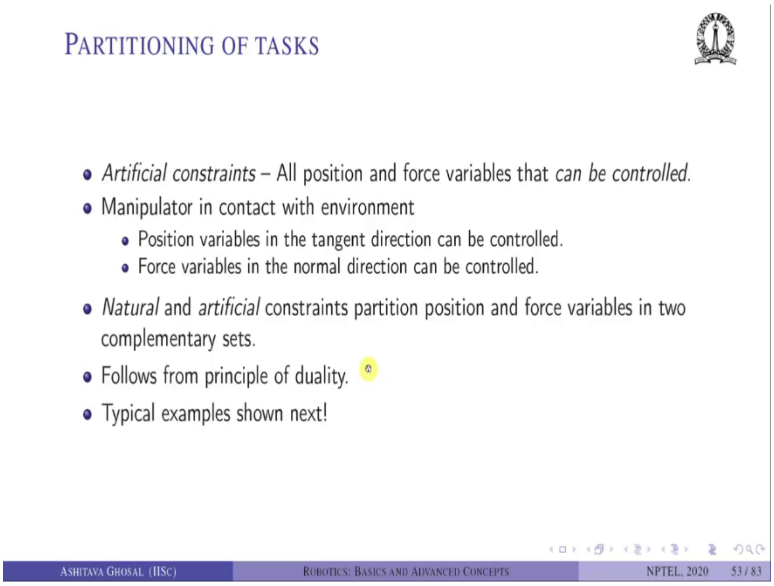

On the other hand, there are some constraints which are called artificial constraints and these are all position and force variables, position means x, y, z, and orientation, force means f_x, f_y, f_z and moments that can be controlled ok. So, as an example, let us say that the manipulator is in contact with the environment, the position variables in the tangent direction can be controlled.

So, I can trace a part in the tangent directions, I can go faster or slower, I can choose a desired trajectory in the tangent direction of the position variables. I can also control the force in the normal direction ok; I can say apply one Newton force at all times in the normal direction. So, this natural and artificial constraints, partitions the position and force variables into two complimentary sets ok, and where what do these complimentary sets really mean basically, they follow from the principle of duality.

So, if there is a force constraint in the normal direction ok, there cannot be a position constraint in the normal direction. I can control the force in the normal direction, but not the position ok, this is again from the principle of duality. So, next we show some examples.

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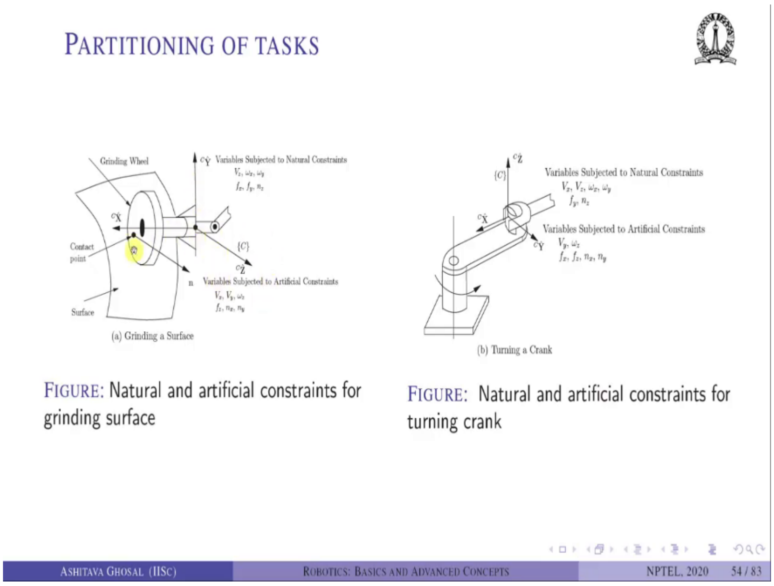

So, for example, consider this grinding example which we have been talking about all the time.

So, we have this grinding wheel and there is a surface this wheel is grinding a surface. So, first thing to do is we need some coordinate system ok, which in that coordinate system we will write all these position and force variables and say which ones can be controlled and which ones cannot be controlled. So, typically we set a coordinate system $^c\hat{X}$, $^c\hat{Y}$, $^c\hat{Z}$ with this curly bracket $\{C\}$, which is the constraint coordinates.

So, now let us look at the six position variables meaning, V_x, V_y, V_z, ω_x, ω_y, ω_z and six force variables, which is f_x, f_y, f_z, n_x, n_y, n_z; n_x, n_y, n_z are the moments which are acting on this constraint coordinate system. So, as you can see this V_z. So, in this example Z direction is normal to the surface this is along the normal and X and Y, X is along the grinding wheel in some sense along the axis of the grinding wheel and Y is perpendicular to X and Z.

So, as you can see V_z is a natural constraint why? because, you cannot go into the surface or you cannot leave the surface then it is no longer in contact ok. Likewise, f_x and f_y the force variable along X and Y direction ok, cannot be controlled ok and likewise n_z which is a rotation above this Z axis these are natural constraints. So, if you can think of some friction ok so there is a force, but the normal force and the friction force are related, we cannot have arbitrary friction force f_x and f_y.

So, variables which are subjected to artificial constraints meaning the variables which can be controlled I have this V_x and V_y. So, they are the variables in the tangent space at that contact point ok. So, I can say that I want to go in the X direction with such and such velocity and such and such velocity in the Y direction. So, V_x and V_y can be controlled. Likewise f_z can be controlled; I can say that we want to apply a certain force along the Z direction.

How about ω_x and ω_y? you can see that ω_x cannot be controlled, because if you have some ω_x, if you rotate about the X axis ok then it will leave contact ok.

Similarly ω_y, if I rotate about the Y axis the grinding wheel will lose contact ok. So, V_x, V_z, ω_x, ω_y, f_x, f_y, n_z these are the natural constraints and the variables which are artificial constraints or variables which can be controlled are V_x, V_y, f_z and think a little bit about it which is also n_x, n_y and ω_z.

So, as we can see these two are complimentary, if there is a V_z here there is a f_z here, V_z and f_z cannot be in the same set, f_x and V_x cannot be in the same set, because of the principle of duality. Let us take one more example, which is I have a crank I have a wrench which is turning a crank ok. So, again we set up a constraint frame which is ${}^{c}\hat{Z}$, which is the vertical direction ${}^{c}\hat{X}$ is along the direction of the crank and ${}^{c}\hat{Y}$ is the direction in which you are rotating ok.

So, the variables which are artificial constraints meaning which can be controlled are this velocity along the Y direction, rotation speed about the Z axis f_x, f_z, n_x and n_y. So, you can see that these two are different, here we had V_x, V_y, ω_z here we have V_y, ω_z. There is no V_x here likewise, the variables which you cannot control are V_x you cannot have a velocity in this direction then, you know you are sort of bumping against the rest of the body of the crank.

Likewise, we cannot go up because then you will lose contact ok and f_y and n_z. ω_x, ω_y, f_y and n_z are natural constraints. So, for every robotic task such as grinding or turning a crank and we will see later for an assembly operation, we can divide the six position variables and the six force variables into variables which can be controlled and which cannot be controlled ok.

So, this is the hard part, this the programmer needs to think about it and partition these variables, but he has some tools to do it basically, we have to use this principle of duality and also what exactly is the task doing ok. We know in the normal direction forces can be controlled in the tangential direction positions can be controlled and so on.

So, some ideas are there, we use those ideas to partition this twelve variables, six position and six force variables ok.

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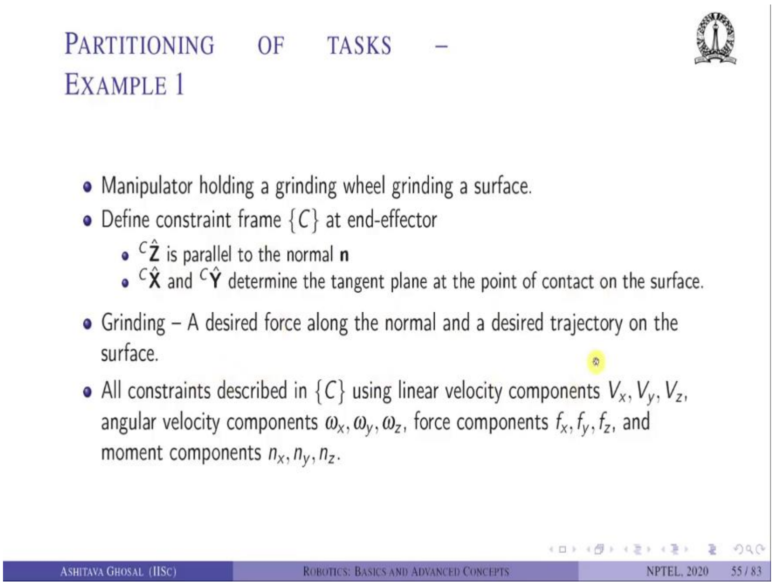

So, whatever I said is summarized here. So, the manipulator is holding a grinding wheel on a surface we define a constraint frame $\{C\}$, ${}^c\hat{Z}$ is along the normal ${}^c\hat{X}$ and ${}^c\hat{Y}$

determine the tangent plane at the point of contact, for grinding at desired force along the normal and a desired trajectory on the surface is required ok.

So, all constraints described in $\{C\}$ using linear velocity components V_x, V_y, V_z, angular velocity components ω_x, ω_y, ω_z, force components f_x, f_y, f_z ,and n_x, n_y, n_z, ok.

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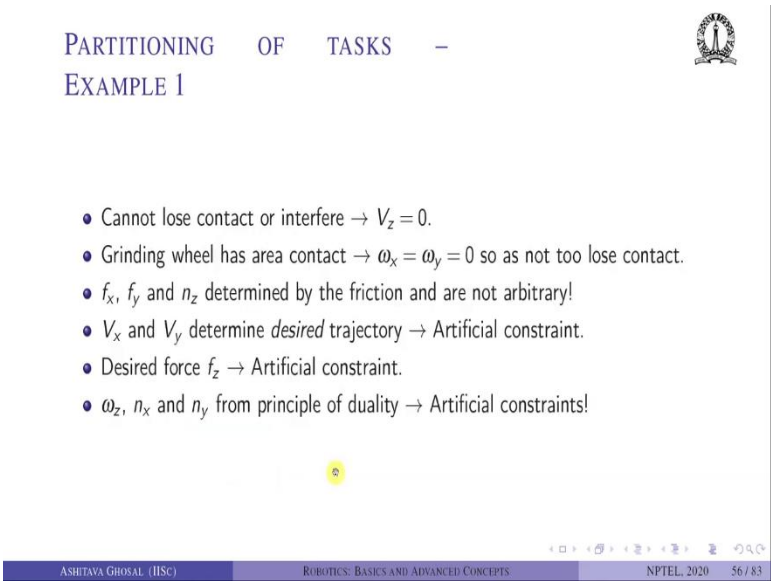

So, we cannot lose contact ok or interfere so, hence $V_z = 0$. Grinding wheel has area contact so $\omega_x = \omega_y = 0$, just a little bit more detail.

So, as not to lose contact, f_x, f_y, n_z determined by friction and are not arbitrary ok, we cannot set whatever we want. V_x and V_y determine the desired trajectory how we are moving this grinding wheel on the surface. So, hence they are artificial constraint, we also need a desired force along the Z direction hence this is an artificial constraint and likewise ω_z, n_x and n_y from the principle of duality are artificial constraints ok.

So, we can partition all these 6 + 6 variables, force and position variables, into something which are cannot be controlled and something which can be controlled.

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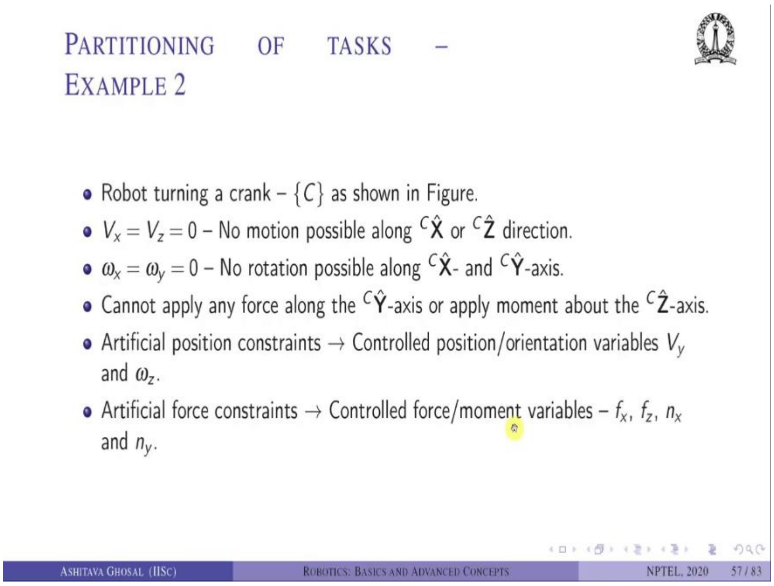

In the robot turning a crank again $\{C\}$ is as shown in the figure, X axis is along the direction of the crank Y is the direction, where you are rotating, and Z is the vertical direction ok.

So, again $V_x = V_z = 0$. So, no motion possible along X and Z ok because, otherwise it will interfere or lose contact, $\omega_x = \omega_y = 0$, so no rotation possible along ${}^c\hat{X}$ and ${}^c\hat{Y}$ axis, we cannot apply any force along the Y axis or apply a moment about the Z axis. So, the artificial constraints are V_y and ω_z, artificial force constraints are f_x, f_z, n_x, and n_y ok. So, you can think about it and see that what is mentioned here looks sensible and looks correct.

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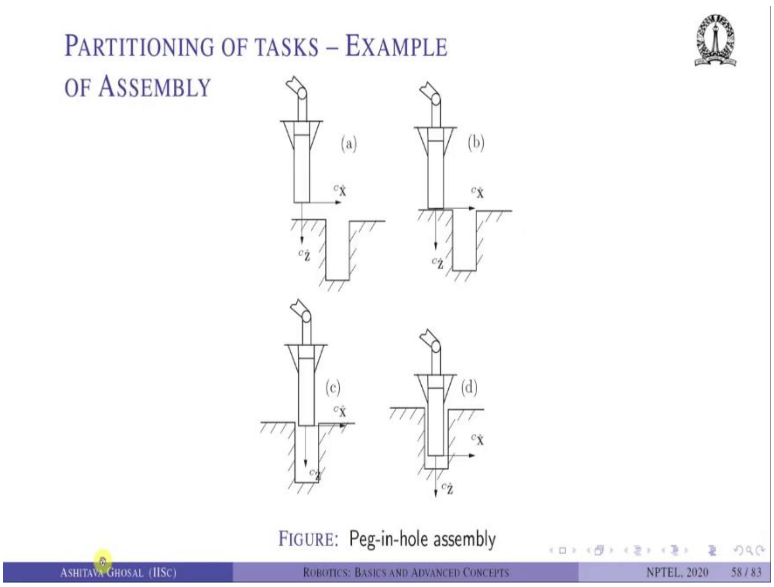

Let us look at one more task, this is a very important robotic task which is basically assembly of two components and here we are going to look at a very simplistic case of assembling a peg in a hole ok. So, this is the peg, this is the hole on the surface and we want to assemble this peg into this hole ok, moreover first as far as simplification is concerned we will assume it is a 3D task ok. So, it is in the X-Z plane the Y is in the normal to this plane of this hole.

So, first case here (a) is showing when the robot is moving this peg in space ok. So, it is not in contact with the environment. So, the constraint frame is X is along this direction Z is along the vertical direction. After a while this peg comes in contact with the surface ok, this is the second kind of sub task which this peg in hole requires that you come and touch the surface.

Then, you move along the surface such that the peg comes directly on top of the hole, you may not come directly on top of the hole at the first instance. So, you have to move a little bit here and there, then once it is directly on the top of the hole you push it down. So, the full peg in hole assembly is divided into moving in free space, touching the surface and moving on the surface, then finding the hole and moving into the hole and then at the end of the motion nothing is moving ok, it is all assembled ok.

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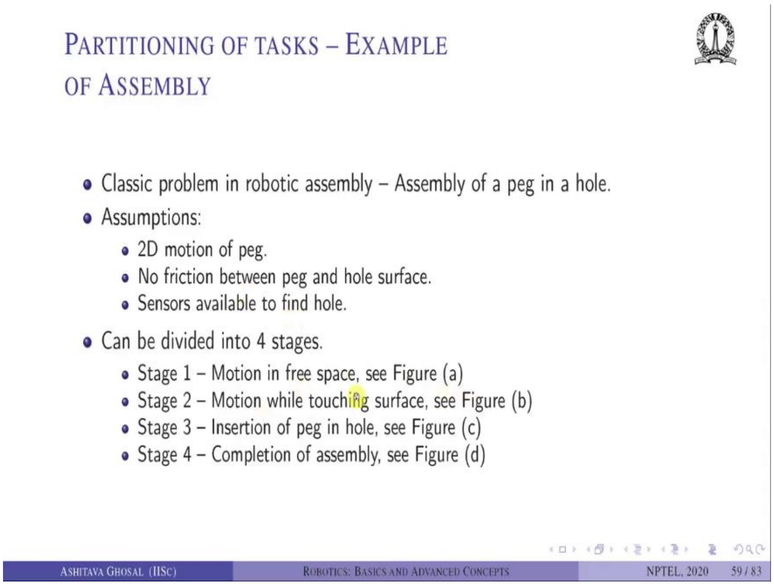

So, this is the classic problem in robotic assembly of a peg in hole. So, as I said the assumption is 2D motion of the peg, no friction between peg and hole surface, and sensor is available to find the hole ok, as I said you move along till you find the hole there must be a sensor to find the hole. So, as I said it is divided into 4 stages.1st is motion in free space ,which is figure (a), motion while touching the surface which is figure (b), stage 3 is insertion of the peg in the hole, figure (c) and stage 4 completion of the assembly ok.

So, let us go back and see these pictures. So, this is stage 1 in free space, stage 2 moving on the surface, stage 3 insertion into the hole, and stage 4 and everything is done, it is completed ok.

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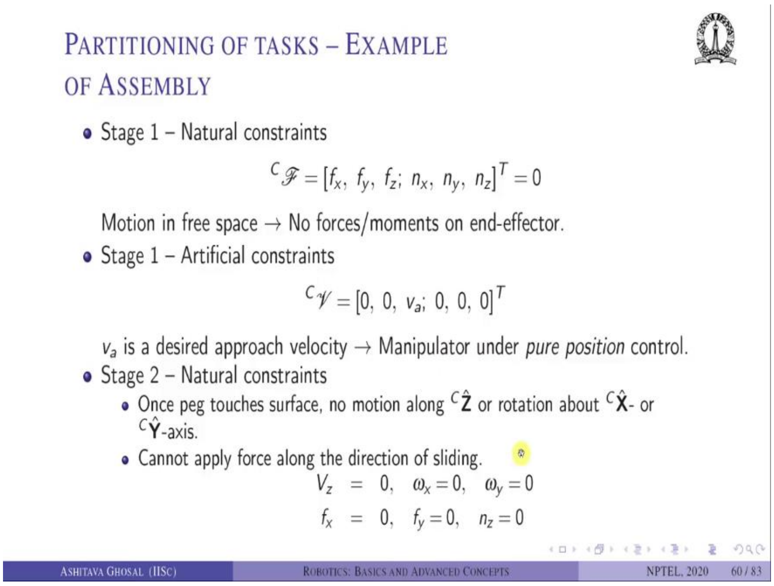

So, let us try and find the natural and artificial constraints for each one of these stages. So, the 1st stage is very simple ok, what is the natural constraint and what is the artificial constraint.

The artificial constraints are the position x, y, z and the orientation are completely controlled ok. So, we have V_x, V_y, V_z, ω_x, ω_y, ω_z. So, V_x and these are 0 and we want to approach the plate along the Z direction. So, this is some desired Z velocity we also do not want to rotate this peg. So, all the desired angular velocities or angular orientations are zero.

So, corresponding to this $f_x, f_y, f_z, n_x, n_y, n_z$ these are natural constraints ok. The variables which cannot be controlled when it is moving in free space are nothing but the forces which are acting on the end effector, there are no forces which are acting on the end effector ok. So, in stage 1 the manipulator is under pure position control, in stage 2 once the peg touches the surface, no motion along the Z axis or rotation about the X or Y axis is possible ok.

So, you cannot penetrate the surface or you cannot rotate at one place ok. So, we also cannot apply force along the direction of sliding, we can control the position along the direction of sliding, but we cannot control the force ok. So, $V_z = 0$, $\omega_x = 0$, $\omega_y = 0$, $f_x = 0$, $f_y = 0$ and $n_z = 0$, these are the natural constraints ok. If you think about it, I cannot penetrate the solid which is V_z and so on and $f_x = f_y = 0$.

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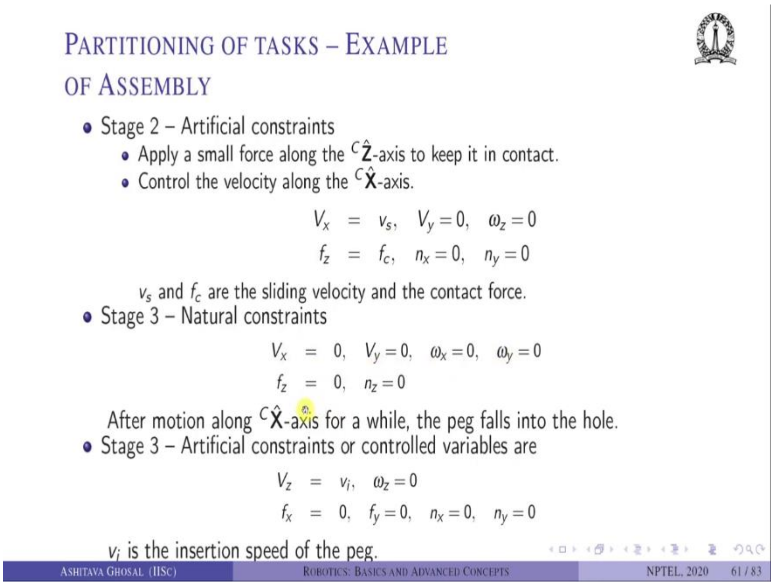

So, what are the artificial constraints? we would like to apply a small force along the Z axis to ensure that it is always in contact, we control the velocity along the plane along the X and Y axis. So, V_x we have $V_x = V_s$ is some V sliding velocity $V_y = 0$ and $f_z = f_c$ and this $n_x = n_y = \omega_z = 0$, this follows from the principle of duality. So, we would like to control the sliding velocity and the force along the Z direction.

In stage 3 the peg is going into the hole ok. So, what are the natural constraints think about it we see that V_x, V_y, ω_x, ω_y, f_z and n_z are 0. Why is $f_z = 0$? $f_z = 0$ because it is not in contact.

We assume that this peg is sort of smoothly going into the hole ok. What are the artificial constraints, what are the variables you would like to control, we would like to control the

Z motion we want to go slowly ok so, $V_z = v_i$. We also want to control that there is no force coming from the sides you do not want to touch the sides let us say.

So, $f_x = f_y = n_x = n_y = \omega_z = 0$. So, at every stage we can find out what are the variables which you want to control and which we cannot control ok.

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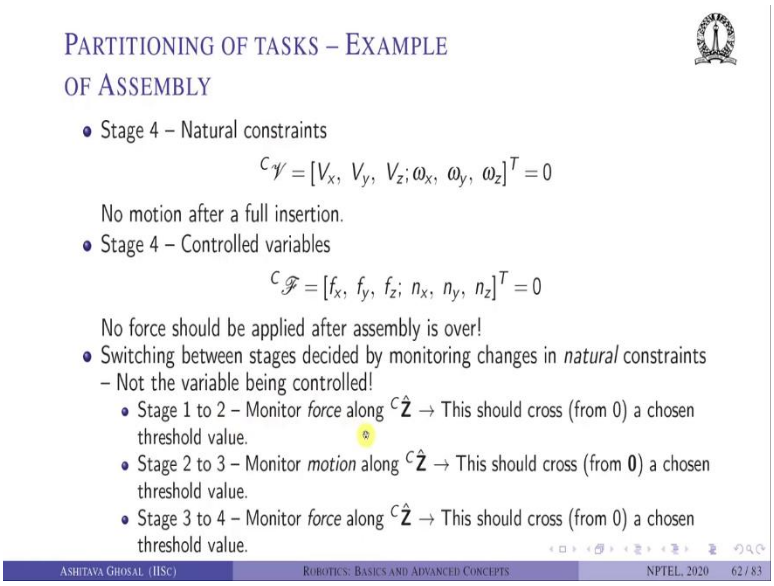

In the last stage, when it is completely inside then what are the natural constraints? I cannot move. So, all the V_x, V_y, V_z, ω_x, ω_y, ω_z, they are all cannot be controlled, they are basically zero ok it is completely inside and it is assembled ok.

So, no motion after full insertion so, what are the variables that we control when it is under full insertion, we need to make sure that it is you are not applying too much force ok. So, there is some desired force along the x, y and z and the desired moment along the x, y and z so, most of the time it will be 0 or some small number, which is ok.

So, I am assuming that there has to be no force applied after the assembling is over ok. So, we have four stages and we know the natural and artificial constraint in all the four stages, the interesting thing is that we can find out how to switch from one stage to another stage just by monitoring the natural constraints ok.

So, there is a use for the artificial constraints those are the variables which we can control, we can design a controller which will control those variables, but how do we switch from stage 1 to stage 2? we monitor the natural constraints. So for example, we in stage 1 to stage 2 you monitor the force along the Z direction.

So, when it is moving in free space f_z cannot be controlled, $f_z = 0$. As soon as it touches, if you are monitoring the force along the Z direction you will see some number the sensor will see some force in the Z direction. So, as soon as its sees some force in the Z direction we should switch from stage one to stage two ok.

So, this force should cross from some 0 to a chosen threshold value. Likewise, from stage 2 to stage 3 what is stage 2? Stage 2 is moving on the surface while in contact, but at stage 3 is when is just on top of the hole and supposed to go in. So, again we should monitor the ${}^{c}\hat{Z}$. So, you monitor the motion along the Z axis. So, in stage two there cannot be any motion along the Z axis ok.

But as soon as it is in top of the hole it will start to go down so, there will be some Z motion. So, this should also cross from 0 to some chosen threshold value and finally, from stage three to stage four you monitor the force along the Z direction again. So, at the end of the assembly while assembling we are assuming it is smoothly going into the hole the peg, at the end when the assembly is over the peg goes and touches the bottom of the hole. So, again you will see some force in the Z direction.

So, as soon as you see some change in the value of f_z ok along ${}^{c}\hat{Z}$, you switch from stage 3 to stage 4. So, it is a very nice way of looking at this assembly that we can monitor the natural constraints to check when we should switch from one stage to another stage, and in each stage, we control the artificial constraints ok.

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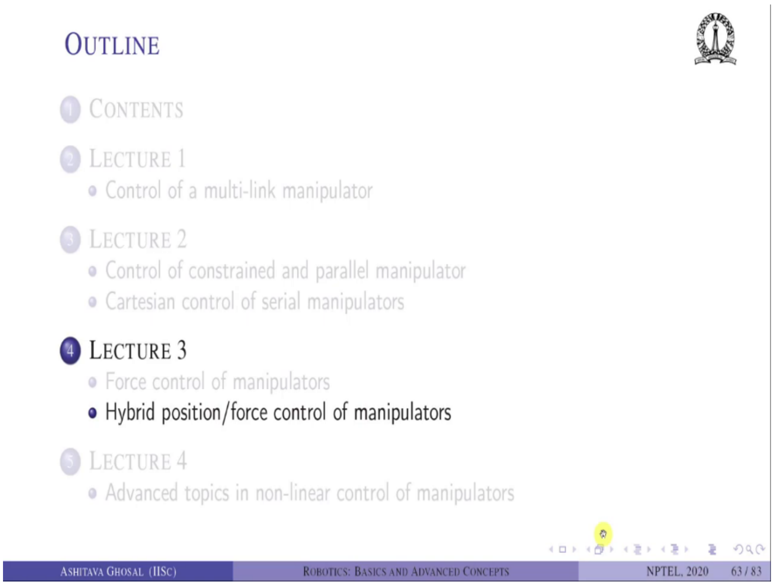

Now, let us look at this very important controller which is the hybrid position and force control of robot manipulators.

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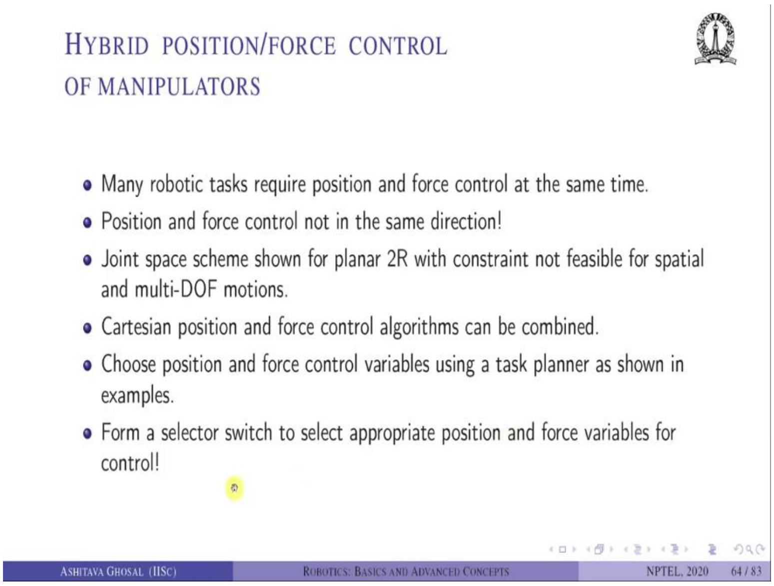

So, till now we have seen that we need to control position along certain directions and force along some other directions ok. So, can we have a controller which can control the desired force in the desired direction, achieve the desired force in the desired direction and achieve the desired motion in the motion directions, which can be controlled ok that is called as hybrid position force control of robots ok.

So, a little bit of introduction, many robotic tasks require position and force control at the same time. So, as in the case of grinding or assembly I showed you certain components of force need to be controlled and certain components of position or velocity needs to be controlled. And these position and forces cannot be in the same direction ok, because of principle of duality.

The joint space schemes shown much earlier where we had a planar 2R with constraint is not feasible for spatial and multi degree of freedom motion simply because, we cannot do the inverse kinematics analytically, we have very complicated way of looking at the force in the normal direction and position control in the tangent direction, in terms of the joint variables.

The Cartesian position and force control algorithm can be very easily combined I will show you. So, I showed you that you have the single mass applying a force to the environment so, that is the force control part you copy that six times and get it for the end effector and likewise I showed you how to obtain the Cartesian position control using $F_{\mathcal{X}}(q, \dot{q})$, $[M_{\mathcal{X}}(q)]$, $C_{\mathcal{X}}(q, \dot{q})$ and so on ok.

We write the equations of motion in terms of Cartesian end effector coordinates ok. We can choose the position and force control variable using a tasks planner ok, as shown in the example. So, using this idea of which variables can be controlled in force domain and which variables can be controlled in the position or velocity domain we can decide ok.

So, that is called as a task for planning and then, basically what we will do is we will have a selector switch ok, which selects the appropriate position and force variables for control ok. So, we have six components of position x, y, z and orientation and six components of force and moment f_x, f_y, f_z, n_x, n_y, n_z. We have a switch, which will send let us say X from the position part, but not f_x from the force part ok similar.

So, it will be one from the position control part and zero from the force control part, so it, does not violate the principle of duality ok.

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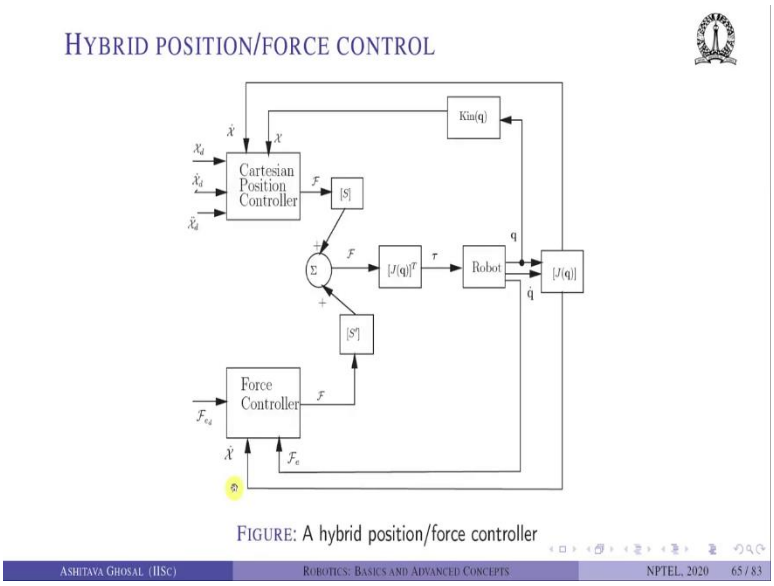

So, that is exactly what is shown in this figure. So, we have a robot the input to build to the robot is always the torque ok and we measure q and $\dot{q}$ now, using q, we can find the Jacobean matrix and we can find $\mathcal{X}$ and $\dot{\mathcal{X}}$.

So, we can do direct kinematics and find $\mathcal{X}$ and we can do Jacobean and find $\dot{\mathcal{X}}$. In this Cartesian position controller we have $\mathcal{X}_d, \dot{\mathcal{X}}_d, \ddot{\mathcal{X}}_d$. So, what is this $\mathcal{X}$? this is x, y, z and the orientation. So, it is a six dimensional quantity and what is the output of the Cartesian controller? it has $f_x, f_y, f_z, n_x, n_y, n_z$.

We had discussed this by using the equations of motion in the Cartesian coordinates ok. Using position and orientation of the end effector we can derive the equation of motion in terms of the Cartesian coordinates. Likewise, let us look at the bottom part we are going to measure the force coming from the environment, we also measure the velocity of the end effector ok this $\dot{\mathcal{X}}$ and this $\dot{\mathcal{X}}$ is more or less same ok.

So, you go to Jacobean and you find $\dot{\mathcal{X}}$ and the input to this force controller is the desired force which this end effector must apply to the environment ok. So, $\mathcal{F}_{e_d}$ and this is $\dot{\mathcal{X}}$ and $\mathcal{F}_e$ so, remember for one axis I had shown you that it will be some $m\left(K_{p_f}K_e^{-1}e_f - K_{v_f}\dot{x}\right) + f_{e_d}$.

So, I have derived this force control law for one direction for a single mass spring which is applying a force on the environment. So, the output of the force controller is also a force Cartesian force means f_x, f_y, f_z and moments n_x, n_y, n_z. So, now we send it to these two selector switches S and S', S' means it is compliment of S.

So as I said, if I want to control let us say the X coordinate ok, V_x then I cannot take f_x from this part also. So, if it is V_x here which is 1 in the selector switch, f_x will be zero. So, likewise we have some ones and zeros in the diagonal and the complimentary ones so, where wherever there is a one in the S there will be a zero in the S'.

So, effectively what happens, we will get some forces coming from the position controller and some forces coming from the force controller and they are consistent with the principle of duality and you sum them over and you get the net force and then, if we post multiplied by $[J(q)]^T$ and we will get the torque.

So, if this torque equal's $[J(q)]^T\mathcal{F}$ is coming from statics, this is the very basic fundamental equation and why do we need to do this because, the robot itself can only apply joint torques it cannot apply external forces and moments in the end effector ok. So, let us go over this once more. So, I have one part which is the Cartesian position controller, which gives you the desired position orientation and its derivatives ok as input.

It also finds a measured position and velocity ok; from direct kinematics and Jacobian and then we run this through this controller and we get the Cartesian forces. Likewise, we measure the external force which the environment is applying and we measure what is the motion of the end effector, we also have an input what is the desired force which the robot should apply and send it through this force controller six component of the single axis force controller that we had discussed.

And we get the six components of the force vector f_x, f_y, f_z, n_x, n_y, n_z. And then we send it through these two switches, which selects the correct ones depending on the task ok and then we pre multiply by $[J(q)]^T$ and we get the torque ok.

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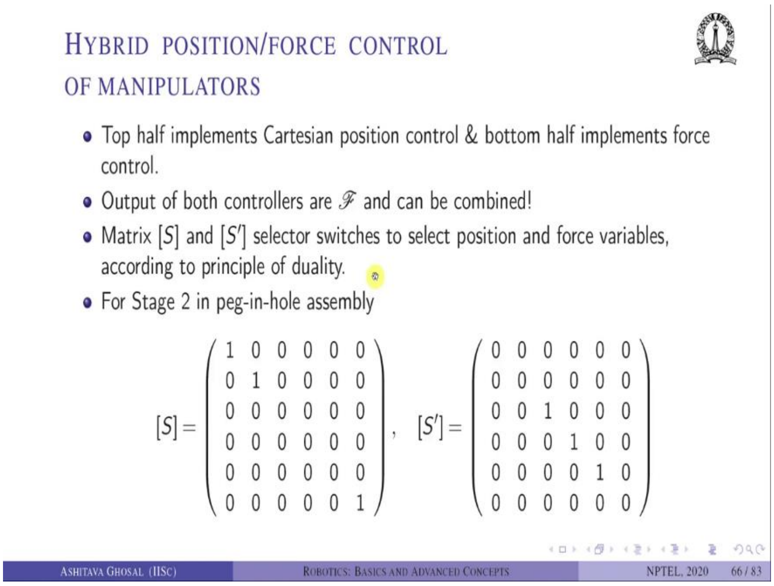

So, as an example, so as I said the top half implements Cartesian position control bottom half implements force control ok.

So, the output of both controllers are Cartesian forces and moments and hence, they can be combined. The matrix S and S' is our selector switches is to select the position and force variables, according to the principle of duality. So for example, in the stage 2 peg-in-hole assembly, what was stage two? The manipulator is now touching the peg on the surface and it is moving on the surface ok, till it goes into the hole.

So, while it is moving on the surface we would like to control V_x and V_y. So, S this is V_x, V_y, V_z, then ω_x, ω_y, ω_z. So, these are the force components coming from V_x, V_y, V_z, ω_x, ω_y, ω_z. So, we would like to control V_x and V_y and also ω_z. The S part we are trying to make sure that the force in the Z direction is controlled.

So, this is f_x, f_y, f_z, and this is n_x, n_y, n_z. So, we make sure that you are applying a contact force and likewise n_x and n_y are controlled ok. So, the controller sends through the force coming from V_x and V_y and ω_z and the force through f_z and n_x and n_y. So, together these two selector switches will now give all the six components of force and velocity ok, so six components of force which can be controlled.

And these six components come from suitable V_x, V_y, V_z, ω_x, ω_y, ω_z and the force along X, Y, Z directions and moment along X, Y, Z directions ok.

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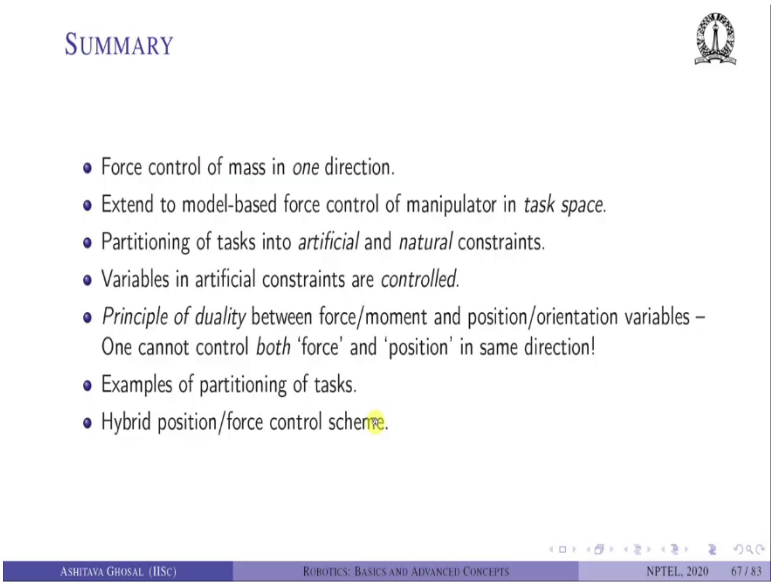

So, in summary I showed you how we can control the force exerted by mass in one direction ok. So, if you have a desired force which this mass needs to exert on the environment, we can design the control scheme which will do that ok. We extend to model based force control of manipulator in task space ok.

So, we used f_x, f_y that curly $\mathcal{F}$ and so on, we can partition a task into artificial and natural constraints, so, the variables in the artificial constraints are controlled and by principle of duality between force moment and position orientation variables, we cannot control both 'force' and 'position' in the same direction ok. So, quote unquote force means it could be f_x, f_y, f_z and n_x, n_y, n_z moments likewise, position could be position and or orientation.

And I gave you several examples of partitioning of tasks, grinding, turning a crank, and also peg in hole assembly and finally, I showed you how we can control position and force together ok, what is called as hybrid position force control scheme ok. So, this uses the Cartesian control of forces ok.

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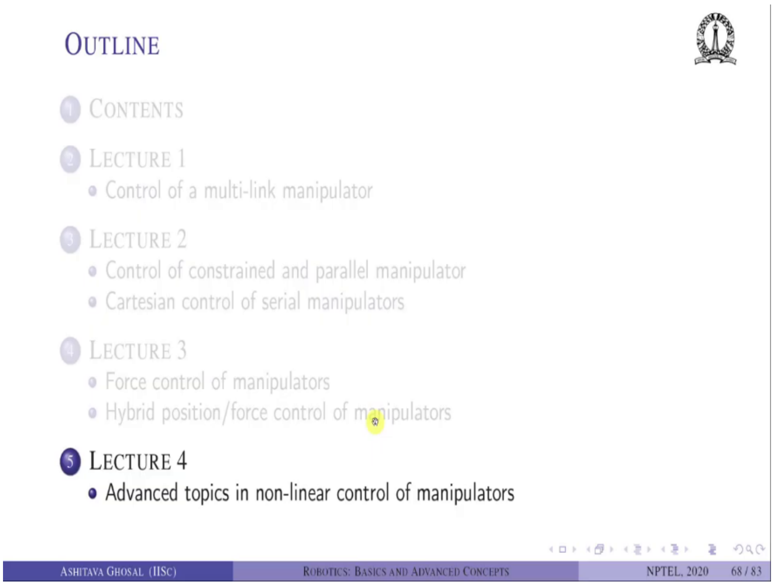

So, with this we come to an end to this lecture, in the next lecture we will look at some of the advanced topics in non-linear control of manipulators ok.