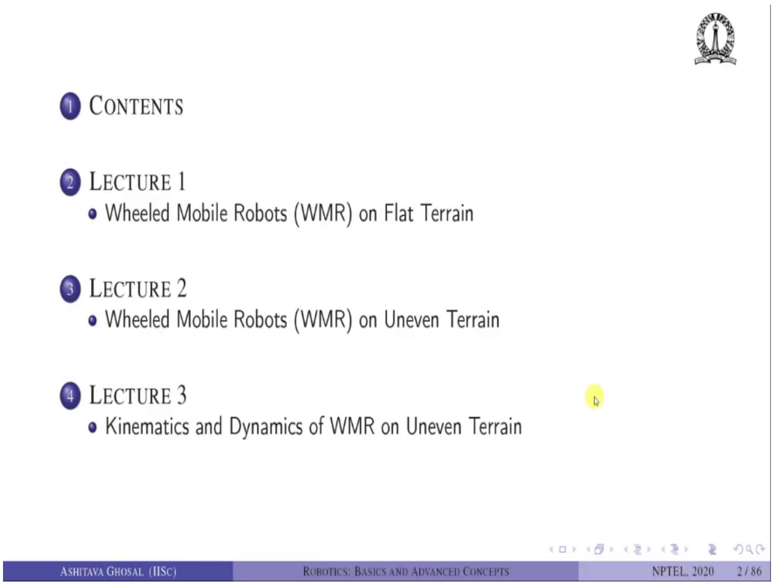
Robotics: Basics and Selected Advanced Concepts
Prof. Ashitava Ghosal
Department of Mechanical Engineering
Indian Institute of Science, Bangalore

Lecture - 38
Wheeled Mobile Robots

Welcome to these NPTEL lectures on Robotics Basics and Advanced Concepts. Till now we have looked at robots which have fixed base, they are either serial or parallel. In these lectures, we will now look at Wheeled Mobile Robots ok.

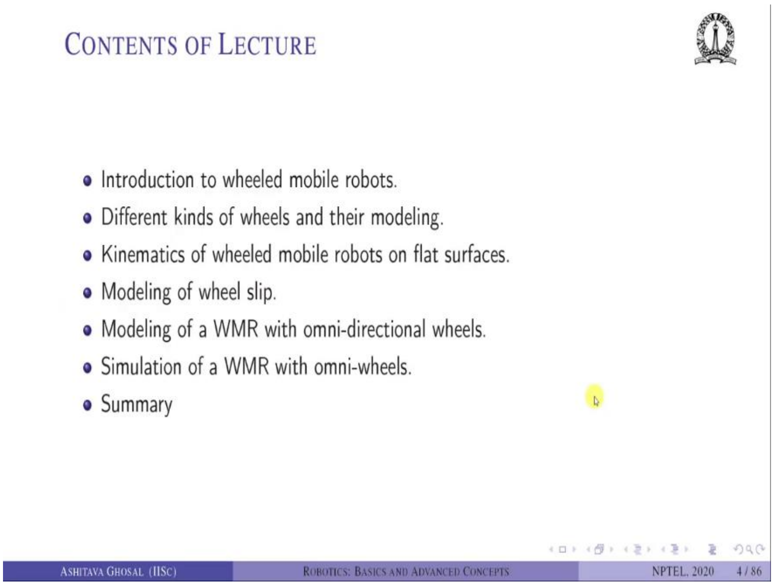
(Refer Slide Time: 00:37)



So, there are 3 lectures in this topic. The 1st topic is on Wheeled Mobile Robots on Flat Terrain. The 2nd topic is on Wheeled Mobile Robots on Uneven Terrain, and then we will look at Kinematics and Dynamics of Wheeled Mobile Robots on Uneven Terrain ok.

There are many, many aspects of wheeled mobile robots things like obstacle avoidance, sensing, intelligence, map making and so on. But in these lectures, we will concentrate on the kinematics dynamics, and basically modeling and simulation of the wheeled mobile robots. So, let us continue wheeled mobile robots on flat terrain.

(Refer Slide Time: 01:22)

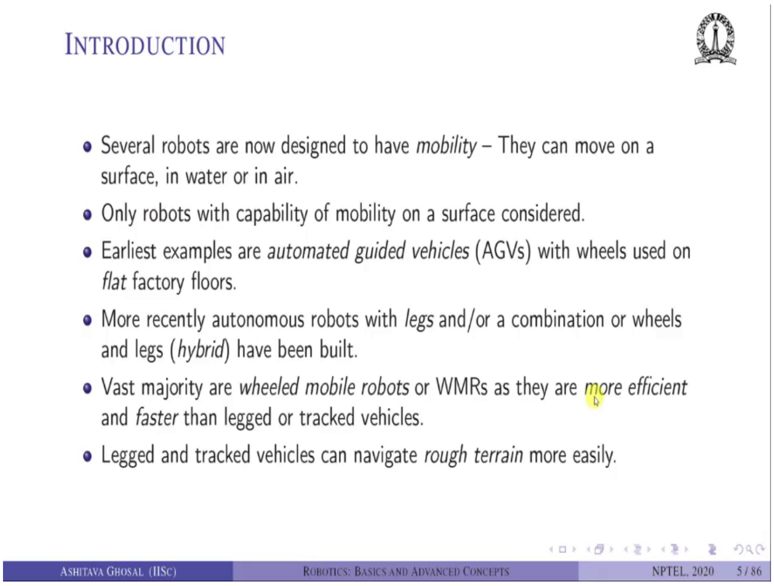


So, we will first look at some wheeled mobile robots. I will introduce you the topic of wheeled mobile robots. Then we will look at different kinds of wheels and their modelings ok. A wheeled mobile robot by definition should have a wheel. So, there are different kinds of wheels.

Then I will very briefly tell you the kinematics of wheeled mobile robots and flat surfaces. One of the important aspects of wheeled mobile robots moving on flat surfaces or for that matter on any surfaces is this notion of slip ok. The wheels of a wheeled mobile robot when it is moving on a surface will always slip; without slip actually we cannot even move. So, we will look at how to model this wheel slip. And we will next look at modeling of a wheeled mobile robot with omni-directional wheels.

So, this is a new kind of wheel which has 2 degrees of freedom ok. And we will look at how to model a wheeled mobile robot with omni-wheels. And we will look at some simulations of wheeled mobile robot with omni-wheels.

(Refer Slide Time: 02:32)

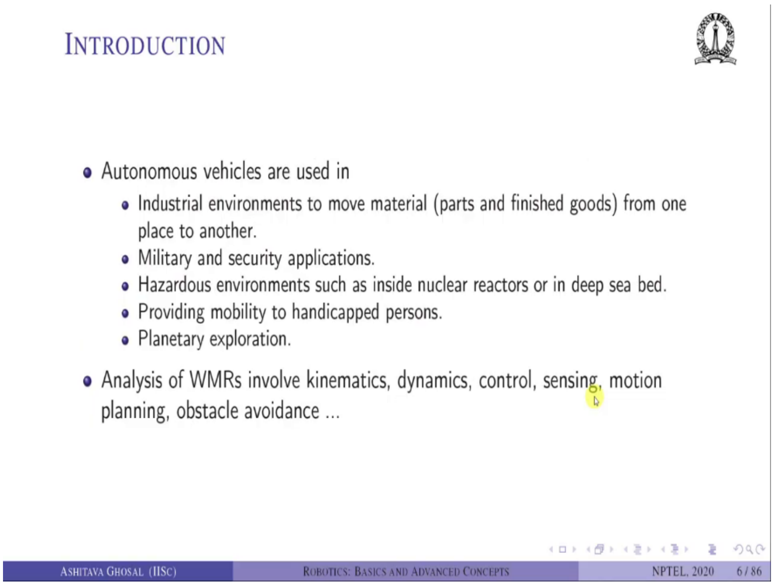


So, many, many robots have been designed to have mobility ok. So, for example, they can move on surface in water and in air. In these lectures, we will look at robots with capability of mobility on a surface ok. So, we will not worry about drones and robots which can move under water and so on. So, the earliest examples of wheeled mobile robots are automated guided vehicles also called as AGVs, used with wheels. And these were used on flat factory floors ok.

So, AGVs could take material components from one station to another station, and in an automated manner or sometimes they would be guided by some wire or some tracks which were put on the factory floor. More recently autonomous robots with legs and or combination of wheels and legs which are also called hybrid have been built ok. So, one of the robots which I had shown you in one of previous lectures was a quadruped. So, that is a mobile robot with four legs.

The vast majority of wheeled mobile robots also sometimes in these lectures I will say WMR, are wheeled mobile robots ok, because they are very efficient, they can move much faster than legged or tracked vehicles ok. There is an advantage of legged and tracked vehicles because they can navigate rough terrain. So, for example, tanks and bulldozers and something they have a track because they need to go over uneven terrain.

(Refer Slide Time: 04:17)



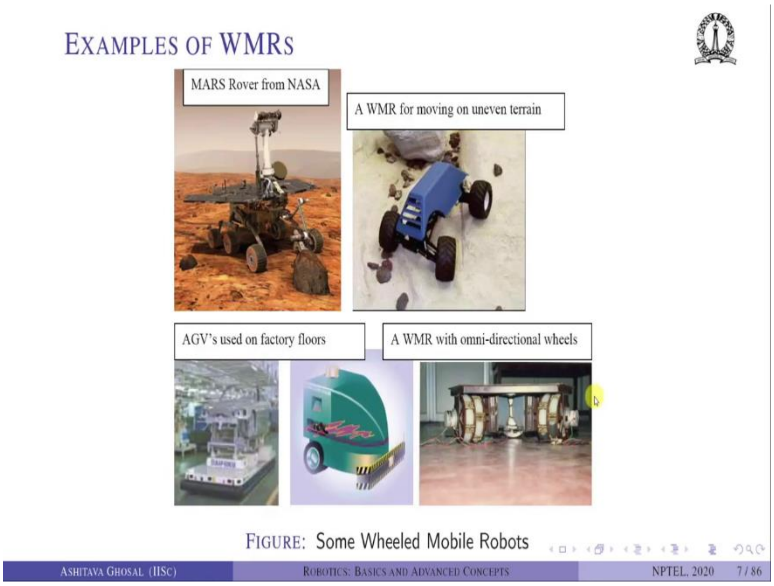
Autonomous vehicles are used in industrial environments to move material, parts and finish goods from one place to another. They are used in military and security applications ok like tanks, or even autonomous vehicles which can patrol a surrounding.

They are also used in hazardous environments such as inside nuclear reactors or in deep sea bed. Nowadays, these autonomous vehicles also provide mobility to handicap persons ok. So, we have autonomous wheelchairs on which a person can sit and then these wheelchairs can navigate on roads or through crowds and so on.

Autonomous vehicles are nowadays very popular because they are on Mars ok. So, there are this there is this wheeled mobile robot which is exploring surface of the Mars in an autonomous fashion. The analysis of WMRs involve kinematics, dynamics, control, sensing, motion planning, obstacle avoidance, many many things ok. So, there is something called map making, there is something called SLAM - Simultaneous Localization And Mapping.

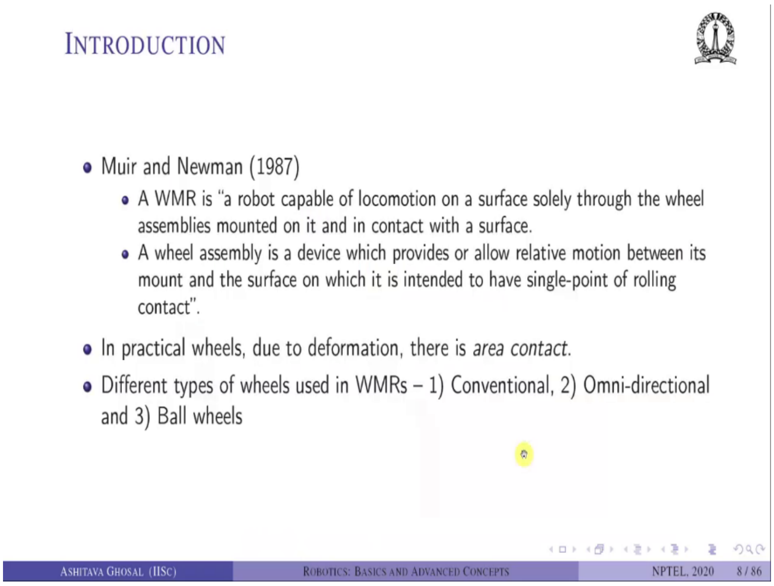
So, we need to worry about many aspects when we look at wheeled mobile robots. In this set of lectures, we will only look at kinematics and dynamics ok. We are not going to look at sensing, motion planning, obstacle avoidance and so on; a little bit of control will also be discussed.

(Refer Slide Time: 05:55)



So, a few examples of WMRs. So, on the top, we have this MARS rover from NASA. Then we can have some wheeled mobile robots this is from internet which is moving over uneven terrain. We also have AGVs, these were the first wheeled mobile robots these two. And then we had built a wheeled mobile robot with what are called as omni-wheels ok. We will discuss this in more detail. This robot was built in our lab.

(Refer Slide Time: 06:28)

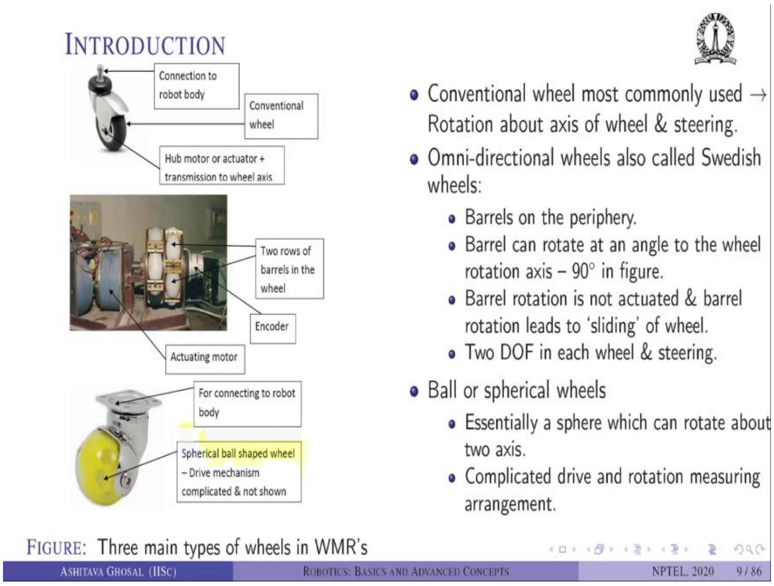


So, what is the definition of a wheeled mobile robot? According to this Muir and Newman in 1987, A wheeled mobile robot is a robot capable of locomotion on a surface, solely through wheel assemblies mounted on it and in contact with the surface ok. A wheel assembly is a device which provides or allow relative motion between its mount and the surface on which it is intended to have single-point of rolling contact ok. So, the important word is it is capable of locomotion on a surface.

The other important word is single point of rolling contact. Most of the time this is not really correct, the contact is never point contact ok. In practical wheels, due to the deformation of the wheel, there is area contact.

And there are different types of wheels used in wheeled mobile robots. First is conventional, then there is omni-directional wheels, and ball wheels. So, we will see later. So, this has 1 degree of freedom, this has 2 degrees of freedom, and this has more degrees of freedom ok.

(Refer Slide Time: 07:41)



So, what is a conventional wheel? It is basically something very similar to a castor wheel. So, this is the wheel there is an axle ok. And then there are there could be a hub motor or there could be a motor somewhere which will transmit the motion to this wheel. This is a normal standard wheel which is used in most wheeled mobile robots ok.

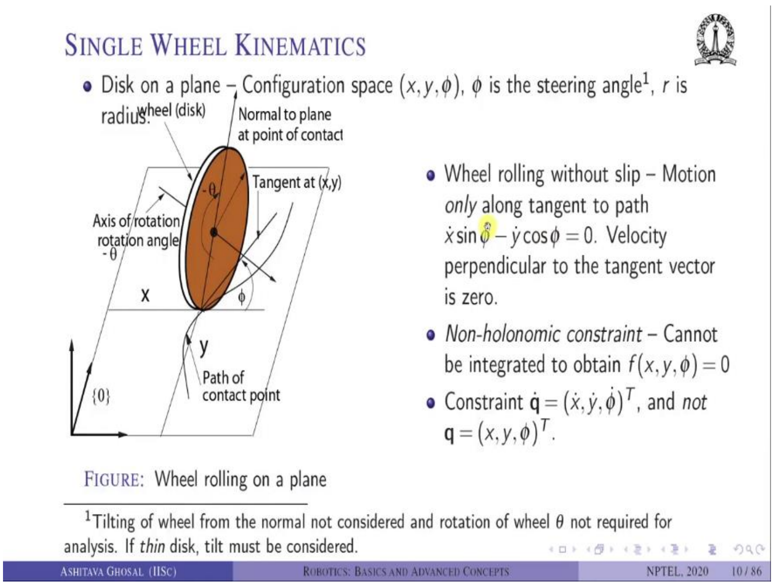
So, there is a connection point to the body. So, conventional wheels are most commonly use. So, they can rotate about this wheel axis ok, and it can be also steered. So, the steering is from the body. Then we have what are called us omni-directional wheels. So, basically this is a wheel but the wheel is made up of barrels which can roll in a perpendicular direction. So, the whole wheel can rotate about this axis. So, you can see there is a motor and we have mounted some encoder to measure the rotation.

But these balls themselves can rotate. And the shape of the ball is such that when it is on the surface ok, there is a point contact ok. So, there are barrels in the periphery. The barrels can rotate at an angle to the wheel, in this figure it is 90 degrees in the figure but it can be at any other angle. The barrel rotation is typically not actuated ok. So, the barrel rotation leads to sliding of the wheel. So, at one place if the barrels rotate, the wheels can slide, not in the driving direction but perpendicular to it.

And there are 2 degrees of freedom in each wheel. So, it can rotate and it can also slide ok. So, ball wheel is another kind of robot. So, it is basically nothing but a spherical ball ok which can rotate about 2 axis ok. These are called ball or spherical wheels. So, in this, this is of course a toy ball wheel ok. So, there is a drive mechanism which is very complicated but it is not shown here. And then there is on the top a way to connect to the body of the robot.

So, it requires a very complicated drive and rotation measuring arrangements. So, most of the time simple 1 degree of freedom wheels like this, conventional wheel is used, sometimes this omni-directional wheels are also used. The omnidirectional wheels are also called Swedish wheels ok in literature.

(Refer Slide Time: 10:21)



So, let us look at a single wheel kinematics. So, basically a single wheel can be modeled as a disk on a plane ok. So, this is the plane, this is the XY plane in some reference coordinate system 0. The axis of rotation of the wheel is this way, and the rotation angle is θ. So, let us assume that there is a contact point on this wheel which is at some (x, y), and then this wheel is tracing this path on the plane. There is also a tangent at the point of contact which is shown here ok.

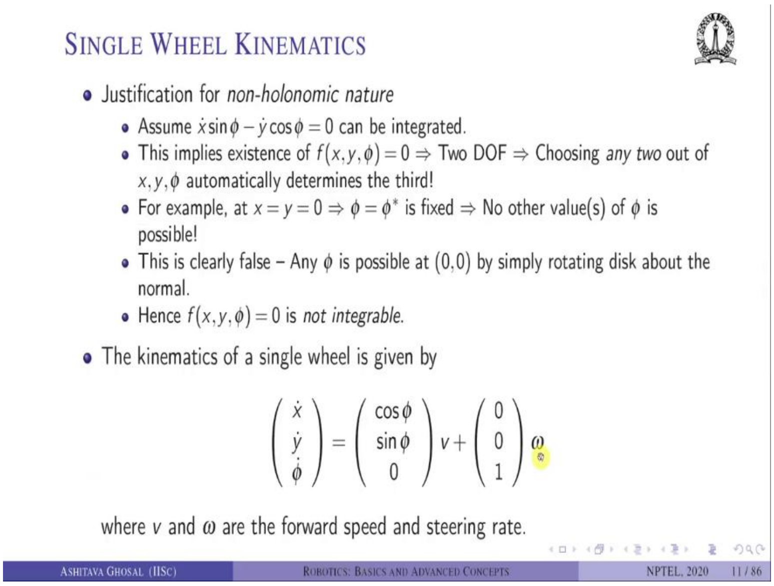
So, the configuration space of this wheel consist of the following parameters. One is this contact point which is x and y, and it is also the steering of the wheel or the direction in which the wheel is instantaneously moving along this tangent ok, so that is an angle ϕ. So, this (x, y, ϕ), where ϕ is the steering angle are enough to describe the motion of this wheel. If the wheel is very thin ok, then one more thing is required the wheel can also tilt about the vertical axis about this axis ok. So, then the tilt should also be consider.

The rotation of the wheel θ is does not appear in the kinematics. We will see later ok because we want to look at what is happening instantaneously. So, if the wheel is rolling without slip the motion is only along the tangent to that curve ok, and we can write the expression $\dot{x}\sin\phi - \dot{y}\cos\phi = 0$. Remember the wheel is not slipping, and it is moving along this tangent ok. The velocity perpendicular to this tangent vector is 0 ok.

This is what the basic equation of a single wheel or a disk which is rolling on a plane without slipping ok. This is a non-holonomic constraint ok. What do we mean by non-holonomic constraint? We cannot integrate this equation and obtain an expression like $f(x, y, \phi) = 0$.

If it were integrable, then it is just a normal holonomic constraint ok. The constraint is in terms of $\dot{q}$ which is in terms of $\dot{x}, \dot{y}, \dot{\phi}$; and not in terms of x, y, and ϕ. So, the constraint restricts the $\dot{q}$ variables by this equation; however, it does not restrict x, y, and ϕ.

(Refer Slide Time: 13:13)



So, how do we know it is non-holonomic? Ok, we assume that this $\dot{x}\sin\phi - \dot{y}\cos\phi = 0$ can be integrated. So, if it were holonomic, this constraint could be integrated. So, this means there is an existence of a function $f(x, y, \phi) = 0$. It is 2 degrees of freedom.

So, choosing any two out of x, y, and ϕ automatically determines the third because of this function ok. It is a single function is x, y, and ϕ. So, if I choose any two of them let us say x and y, ϕ will be automatically determined ok. So, ϕ will be equal to ϕ^* which is fixed.

As an example, if $x = y = 0$, then at that point if there were a function like this, ϕ will be ϕ^* and fixed. No other value of ϕ is possible ok. So, this is clearly not, this is clearly not true; it is false ok. Why, because think of the disk at this point at (0,0) ok.

I can easily rotate this disk about this direction which is normal to the plane at point of contact at (0,0). So, I can rotate the disk at a same (x, y), and I can achieve any ϕ, so that is what is mentioned here. So, this is clearly false because any ϕ is possible at (0,0) by simply rotating the disk about the normal ok.

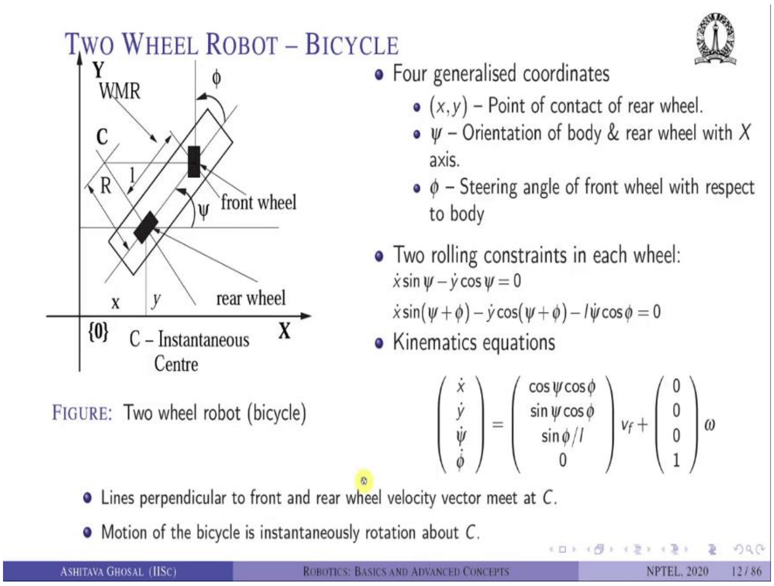
So, hence such a function does not exist, and hence this equation cannot be integrated. So, I gave you an example of $x = y = 0$, and ϕ is fixed, we can also do the same thing for if I choose x equal something and ϕ equal something, y will be fixed but then that is also not true, I can achieve any y.

So, the kinematics of a single wheel is given by

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{pmatrix} = \begin{pmatrix} \cos\phi \\ \sin\phi \\ 0 \end{pmatrix} v + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \omega$$

So, v is the forward speed, and ω is the steering rate. So, the wheel is rolling forward at some velocity v, and it is also being steered by some angular rate ok, so that is given by this.

(Refer Slide Time: 15:46)



Let us look at a 2 wheeled mobile robot ok. A 2 wheeled mobile robot is nothing but a bicycle ok. So, in a bicycle, we will have 4 generalized coordinates. One is some (x, y) which is the point of contact of let us say the rear wheel at this place; ψ which is the orientation of the body and rear wheel with respect to the X-axis.

So, this is the X-axis, this is Y-axis, and this is a fixed reference coordinate system $\{0\}$. So, at x and y, there is an angle ψ which represents this body and the orientation of the rear wheel. And then there is a steering angle of the front wheel ok. So, those are few who have driven a bike you know that the front wheel can be steered ok, and the back wheel can be represented by (x, y) and some orientation with respect to a fix coordinate system. So, there are 2 rolling constraints in each wheel.

So, we have $\dot{x}\sin\psi - \dot{y}\cos\psi = 0$. And we also have $\dot{x}\sin(\psi+\phi) - \dot{y}\cos(\psi+\phi) - l\dot{\psi}\cos\phi = 0$, where l is this length between the 2 wheels ok. The first one is very similar to one single wheel. And the second equation is for the front wheel.

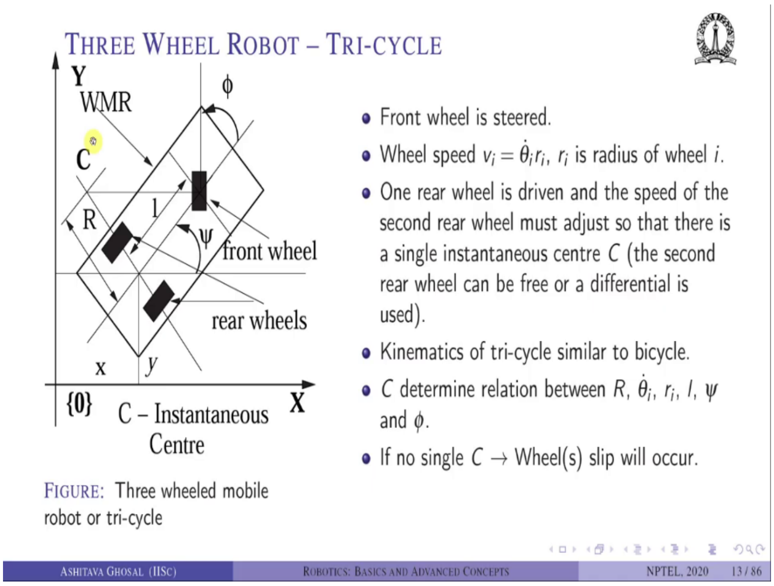
And we can write the kinematic equations of a two wheeled bicycle as

$$\begin{pmatrix}\dot{x}\\ \dot{y}\\ \dot{\psi}\\ \dot{\phi}\end{pmatrix} = \begin{pmatrix}\cos\psi\cos\phi\\ \sin\psi\cos\phi\\ \sin\phi/l\\ 0\end{pmatrix} v_f + \begin{pmatrix}0\\0\\0\\1\end{pmatrix}\omega$$

ω is the steering rate ok. So, the line, so this is the expression which gives you the four generalized coordinates how they change with time. The line perpendicular to the front and rear wheel velocity meet at a point ok. So, this is called as the instantaneous centre of rotation of this bicycle ok.

So, you can think of the motion of this bicycle as rotating instantaneously about a point in this X-Y plane ok. Later on, we will see that this instantaneous center plays a very important part when there is slipping ok.

(Refer Slide Time: 18:17)



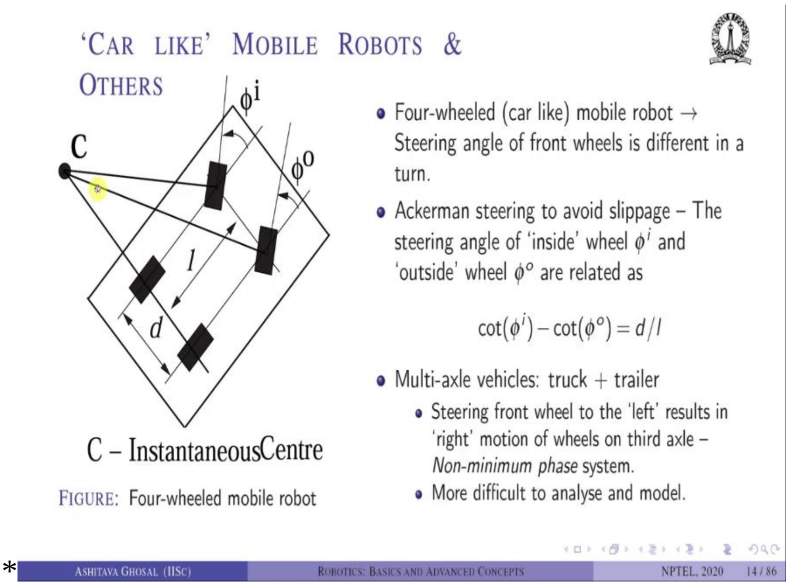
If you have a 3 wheeled robot ok mobile robot, so this is like a tricycle ok. In this picture, the front wheel is steered, the 2 rear wheels are at a distance from each other at some distance ok. And then the body of this tricycle makes an angle ψ with the X-axis. We will assume that the centre point between these 2 rear wheels is some x and y. The wheel speed for each wheel is $\dot{\theta}_i r_i$ where r_i is radius of the wheel i.

So, one rear wheel is give driven and the speed of the second rear wheel must adjust. So, that there is a single instantaneous centre of rotation. So, if this velocity is like this, and this velocity is different than when it is instantaneously meeting ok at this point, so then it is not possible to drive ok.

So, typically, in a tricycle one of the rear wheels is free, or in an automobile with 3 wheeled automobile there is a differential. So, as you take a turn, the speeds of these 2 rear wheels will adjust such that there is an instantaneous centre ok. The kinematics of a tricycle is similar to a bicycle we have the same set of variables. C determines the relationship between R, $\dot{\theta}_i$, r_i, l, ψ and ϕ. If no single C - wheel slip will occur ok. So, you can think of that this wheel is rotating faster.

So, it has some velocity vector this wheel has a velocity vector such that the instantaneous centre or these 3 wheels a velocity vector are such that C does not exist. There is not a single C, then one of the wheels must slip.

(Refer Slide Time: 20:24)

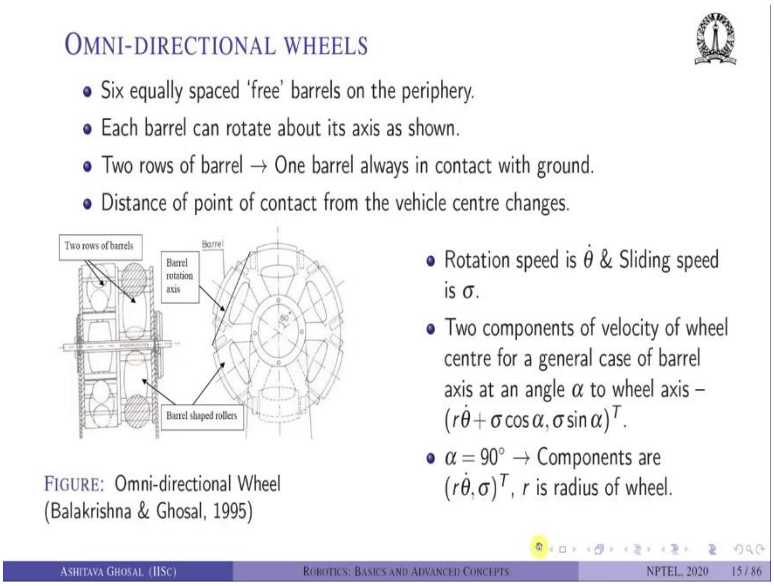


Finally, let us look at a car like mobile robot with four wheels ok. So, if you have a 4-wheeled car like mobile robot, the steering angle of the front wheel is different in a turn. So, these 2 wheels need to be steered differently, otherwise again the instantaneous centre is not there ok. So, there are sophisticated linkages which make sure that when you steer the car ok, the 2 wheels rotate differently ok. So, the inside wheel is ϕ^i and the outside wheel is ϕ^o.

And you need to make sure that this $\cot(\phi^o) - \cot(\phi^i) = d/l$. So, d is this, and l is this ok. If you have multi axis vehicles like a truck and trailer, the steering of the front wheel to the left results in the right motion of the wheels of the third axle ok. So, if you think about it, if you have a driver who is driving a truck with a trailer, if the front wheels are moved to the left, the rear wheel will move to the opposite direction ok.

So, these are called non-minimum phase systems ok. And they are very difficult to analyze or more difficult to analyze and model. So, we will not look at this four wheel and tractor trailer type of arrangements ok. So, most of our discussion will be based on a 3 wheeled mobile robot in this set of lectures.

(Refer Slide Time: 22:06)



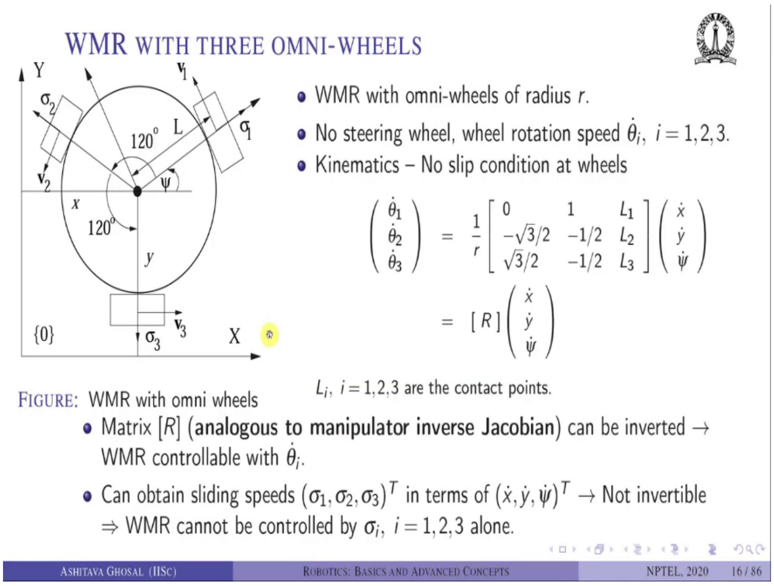
Let us look at omni-wheels. So, basically we have these 2 rows of barrels ok on a single axis. So, these are 6 equally spaced free barrels on the periphery, so 1, 2, 3, 4, 5 and 6 ok, 60 degrees apart. So, each barrel can rotate about this barrel rotation axis and the wheel itself can rotate about the wheel axis ok.

So, there are 2 rows of barrels which were used in our design such that one barrel is always in contact with the ground ok. If the barrels are at an angle not this axis is not 90 degrees, so then we can get away with one row of barrel ok. So, the distance between the point of contact from the wheel center changes this is what happen.

So, if the wheel is rotating such the point of contact is on this barrel, then after a while the point of contact is on this barrel, so the distance from the centre of the platform to the point of contact will change ok. This happens in omni-directional wheels. We can also quantify these 2 rotations about the barrel rotation axis, and about this wheels axis as $\dot{\theta}$ which is a rotation of the wheel, and then this sliding speed which is ψ, this is the rotation of the barrel.

So, the 2 components of the velocity of the wheel centre for a general case of a barrel at an angle α to the wheel axis will be $\left(r\dot{\theta} + \sigma\cos\alpha\,, \sigma\sin\alpha\right)^T$. If α were 90 degrees like this drawing, then the components of the velocity are $\left(r\dot{\theta},\right)^T$, you can see $r\dot{\theta}$ is in this direction like any other wheel and σ is perpendicular direction ok.

(Refer Slide Time: 24:10)



So, if you have WMR with three omni-directional wheels ok, then we can show that it can trace any path on the plane ok. So, let us look at this WMR with 3 omni-wheel. So, we have 3 wheels, so which have σ_1 this direction $v_1 = r\dot{\theta}_1$ in this direction; similarly, σ_2 and v_2, and σ_3 and v_3. So, and these 3 wheels are 120 degrees apart. The direction of this first wheel is at an angel ψ. And this point of and this point where the omni-wheel is the centre is that (x, y).

So, no steering wheel is required ok. The wheel rotation is $\dot{\theta}_i$ for each of these three. And the kinematics with no slip at the wheels can be represented by

$$\begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{pmatrix} = \frac{1}{r} \begin{bmatrix} 0 & 1 & L_1 \\ -\sqrt{3}/2 & -1/2 & L_2 \\ \sqrt{3}/2 & -1/2 & L_3 \end{bmatrix} \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{pmatrix}$$

Where the radius of the omni-wheels is r. So, $\dot{x}, \dot{y}, \dot{\psi}$ are the linear and angular velocity of this platform ok, so ok.

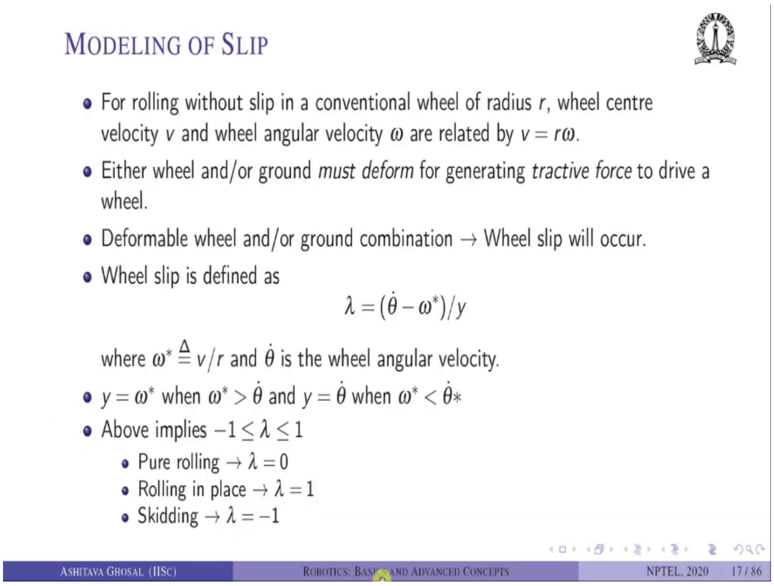
So, this L_1, L_2, L_3 are nothing but the contact points from this center of the platform. So, we can write $\dot{\Theta}$ as a vector is equal to, some matrix $[R]$ times $\left(\dot{x}, \dot{y}, \dot{\psi}\right)^T$. So, this matrix $[R]$ is analogous to the manipulator inverse Jacobian ok. Remember in the Jacobian of a normal manipulator, it was some $V = [J]\dot{\Theta}$, whereas here we have $\dot{\Theta}$ is equal to some matrix times V roughly speaking ok. So, V here correspond to linear velocity and angular velocity.

So, this matrix can be inverted. So, you can see it is looks like it can be inverted ok. So, and it is true, it can be inverted. So, hence if you want an $\left(\dot{x}, \dot{y}, \dot{\psi}\right)^T$, we can find out the $\dot{\Theta}$ and we can go backwards ok. So, we can obtain the $\dot{\Theta}$ for a desired $\left(\dot{x}, \dot{y}, \dot{\psi}\right)^T$. And we can also obtain $\left(\dot{x}, \dot{y}, \dot{\psi}\right)^T$ as some $[R]^{-1}\dot{\Theta}$. So, it. So, what this equation is telling you since the matrix $[R]$ is invertible, the WMR is controllable with $\dot{\theta}_i$.

So, you can think of it that I can move this vehicle at some desired velocity by suitably finding on what is $\dot{\Theta}$. We can also obtain the sliding speed which is $(\sigma_1, \sigma_2, \sigma_3)^T$ vector in terms of $\left(\dot{x}, \dot{y}, \dot{\psi}\right)^T$. However, that matrix relation so σ_i is equal to some another matrix times $\left(\dot{x}, \dot{y}, \dot{\psi}\right)^T$, but that matrix is not invertible ok. So, hence what it is telling you is that this wheeled mobile robot with omni-directional wheels cannot be controlled by σ_i alone ok.

So, you can think of it this way that I need to rotate the motors of three wheels to give any linear and angular velocity. But if I can somehow slide the wheels in the perpendicular direction, then this analysis is telling you that we cannot move the robot from one place to another place ok, or we cannot achieve any desired linear and angular velocities ok.

(Refer Slide Time: 28:45)



So, let us continue. We need to now look at slip ok. So, we need to model this slip firstly. So, for rolling without slip in a conventional wheel of radius r, the wheel center velocity and the wheel angular velocity ω are related by $v = r\omega$, this is obvious straightforward ok.

Now, either the wheel and or the ground must deform for generating tractive forces to drive the wheel ok. So, if you have a pure point contact and both are rigid, then we cannot generate a force which will drive the wheel ok, think about it. So, it is to achieve any tractive force where must be some deformation ok.

So, unless you have a deformable wheel or a deformable wheel and a ground ok which is deformable, wheel slip cannot occur; and without a deformable wheel and or ground combination, we cannot move.

So, wheel slip needs to happen to generate tractive force and for that we need some deformation either in the wheel or in the ground. The wheel slip can be defined as the ratio $(\dot{\theta} - \omega^*)/y$. So, let me explain. So, $\dot{\theta}$ is the wheel angular velocity, $\omega^* = v/r$. So, if $\dot{\theta} = v/r$, $\lambda = 0$. However, v/r which is ω^*; and $\dot{\theta}$ they are not equal. So, the wheel is rotating much more than if it is slipping.

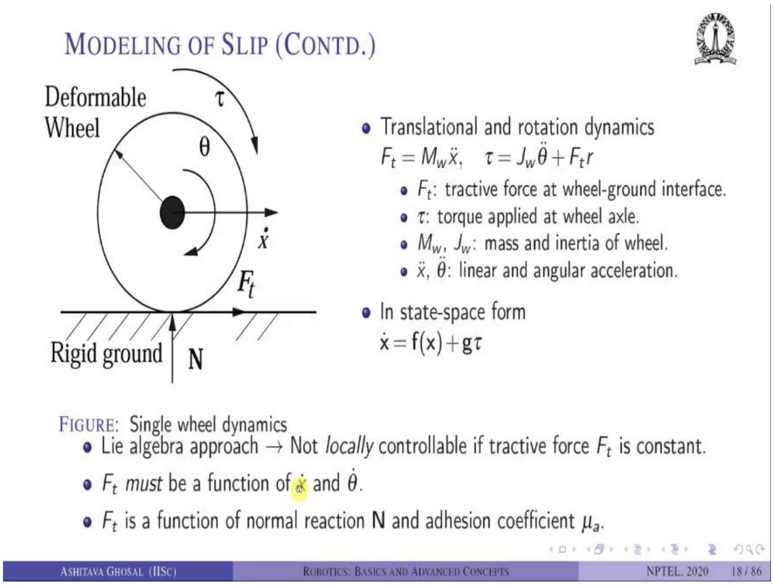
So, this quantity is some number. And we divide by y which is also ω^*. So, what it means is this is some number it is a non-dimensional number ok. So, when $\omega^* > \dot{\theta}$. So, $y = \omega^*$

when $\omega^* > \dot{\theta}$. Is that true? Yes, ok. And $y = \dot{\theta}$ when $\omega^* < \dot{\theta}\,*$. So, the above implies that this λ lies between -1 and $+1$ as I said it is a number.

So, we have pure rolling when $\lambda = 0$ which means $\dot{\theta} = \omega^*$, we have rolling in place at the same place and not moving forward when $\lambda = 1$. And we are skidding when it is not rotating but still moving forward when $\lambda = -1$.

So, in this definition, we capture all these three possible ways on wheel can move one a ground. So, it can do pure rolling, it can do rolling at one place without moving forward and it can do skidding is basically not rolling but moving forward.

(Refer Slide Time: 32:10)



So, let us look at that translation and rotational dynamics of a single wheel ok, which is deformable wheel, so that there is some traction, so there is tractive force. So, there is tractive force F_t which is applied ok. So, think of a wheel like this. If there is no tractive force, it cannot move forward ok. So, if you are on a bicycle and if there is no tractive force of the wheels, the bicycle cannot move forward because there is no external force which is driving the bicycle forward.

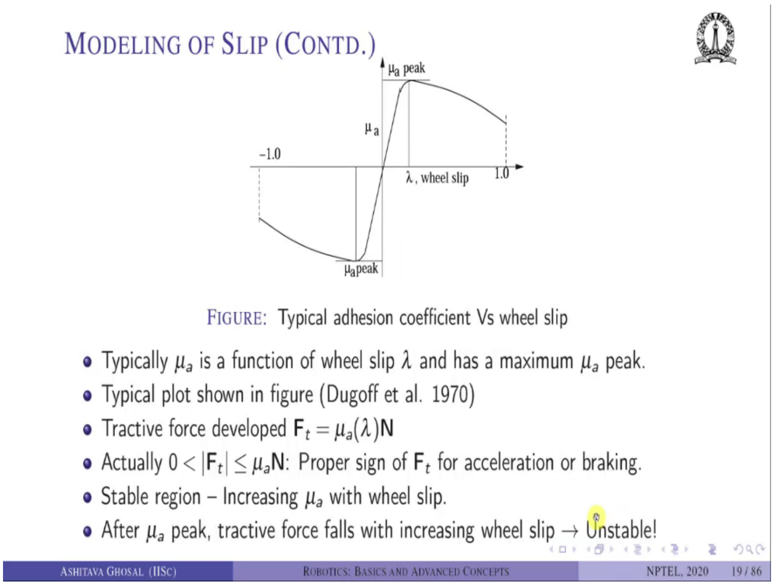
If you rotate the pedal, that is not an external force in the direction of motion. So, there must be a tractive force in the direction of motion just by Newton's Law. So, it turns out that this tractive force F_t will be equal to M_w which is mass of the wheel times $\ddot{x}$.

So, this is the direction $\dot{x}$ of the centre of the wheel. And since we are applying a torque by a motor this torque will be equal to J_w which is movement of inertia of this wheel times $\ddot{\theta}$ plus the torque due to this tractive force which is $F_t r$, where $\ddot{x}$ and $\ddot{\theta}$ are the linear and angular acceleration of the wheel. So, in state space form, this can be written as $\dot{x} = f(x) + g(\tau)$.

What else is happening? There is a ground reaction. So, as you can see this tractive force must be related to the ground reaction ok. So, it will be some coefficient of friction or adhesion coefficient times N ok or a function of N and this coefficient of friction ok. So, it turns out that in the state space form it is not locally controllable if the tractive force is constant ok. So, if you have a constant tractive force or a 0 or some number, it cannot be controlled. So, this tractive force F_t must be a function of $\dot{x}$ and $\dot{\theta}$.

So, typically what is F_t? As I said F_t is a function of the normal reaction N, and some kind of adhesion coefficient or coefficient of friction μ_a.

(Refer Slide Time: 34:39)



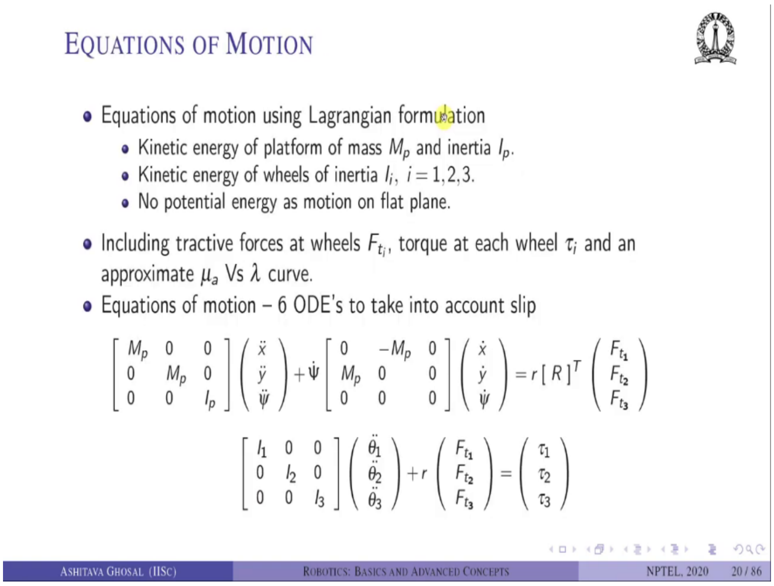
The μ_a itself will change due to wheel slip ok. So, the coefficient of friction or adhesion coefficent will increase if the wheel slip is increasing up to some peak ok. In the other direction, it will increase up to some peak value after that it starts decreasing ok. This is a typical plot of a wheel which is sliping and rolling ok. So, this was known, this has been known from automobiles and aircraft wheels which are landing and so on ok. So, typically μ_a is a function of wheel slip and has a maximum value μ_a peak.

The tractive force developed is always a function of this μ_a, which is a function of wheel slip and the normal force ok. So, it is not just simply $\mu_a N$. μ_a is not a constant. If you are doing basic mechanics problem, the friction force is μN, where μ is a constant. But in case of wheels rolling on ground ok, μ_a is a function of the wheel slip. So, actually 0 magnitude of this tractive force is less than or equal to $\mu_a N$. So, the proper sign of tractive force must be use for acceleration or breaking.

So, let us go back to this plot once more. So, we know that this adhesion coefficient or friction coefficient changes with wheel slip. So, what you can see is for some region which is from here to here as the wheel slip increases the adhesion coefficient increases. So, this region is called the stable region. Whereas, after some λ when the adhesion coefficient starts decreasing ok that is an unstable region. So, as you can see if it is slipping but the tractive force is decreasing, then it will slip more.

So, it is unstable ok. So, it is very important for any wheeled mobile robot to be operating in this region ok. And what would be the best place to operate? We would like to operate very close to the peak but on the left hand side, because remember if the μ_a is large the tractive force is large and hence we can move faster ok. So, typical region is on the left of this peak but as close as possible. We do not want to be go on the right because then it is stars slitting more, it is unstable.

(Refer Slide Time: 37:26)



We can also derive the equation of motion of this wheeled mobile robot using the Lagrangian formulation. We assume that the mass of the platform is M_p, the inertia is I_p. We can obtain the kinetic energy of the platform. We can also obtain the kinetic energy of each of the wheels, the inertia is I_i. There is no potential energy at this stage because it is moving on a flat plane. We can also include the tractive forces of the wheel F_{t_i}, torque at each wheel τ_i.

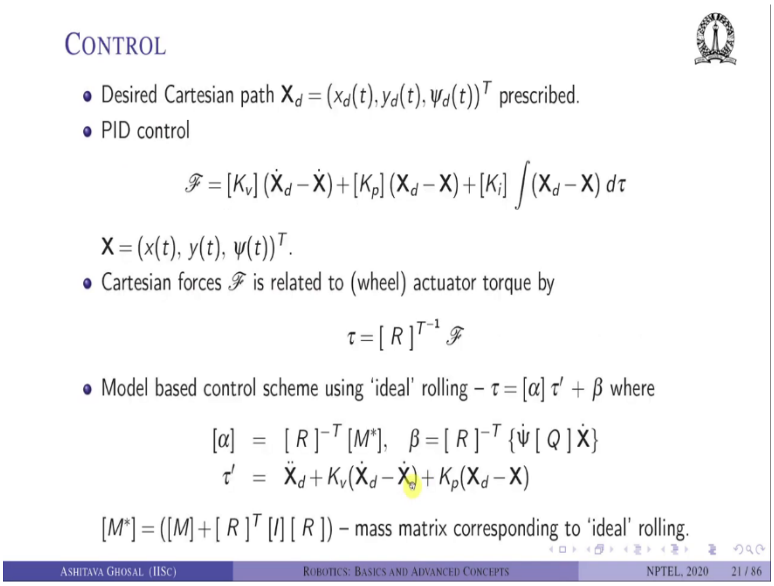
And an approximate μ_a versus λ curve ok. We need to know how this friction force or friction coefficient or adhesion coefficient changes with wheel slip. So, once we have assumed all these things, we can derive 6 ODEs ok, which take into account the wheel slip. So, we have mass time $\ddot{x}$, $\ddot{y}$ and then $I_p\ddot{\psi}$ then some Coriolis centripetal term, then we have that $[R]$ matrix which relates $\dot{\Theta}$ to the velocities of the platform.

And we will see later that this $[R]$, you can imagine that this $[R]^T F_t$ will give some kind of a torque. So, just like $\tau = [J]^T F$, so similarly here $[R]^T F$ will show up, and r is the radius of each wheel which we assume. So, this is the translational velocities and accelerations of the wheel ok.

This is a translational motion of the platform. In addition, we have an angular motion of the platform which is something like I times $\ddot{\Theta}$ plus the torque due to the tractive forces must be equal to the wheel torque ok. So, nothing else is happening. So, we have captured the effect of the tractive force by means of this μ_a versus λ curve. And then we also have capture the kinematics using this $[R]$ matrix, and then we have derived the equation of motion.

The equation of motion will be 6 of them ok, and look like this.

(Refer Slide Time: 39:44)

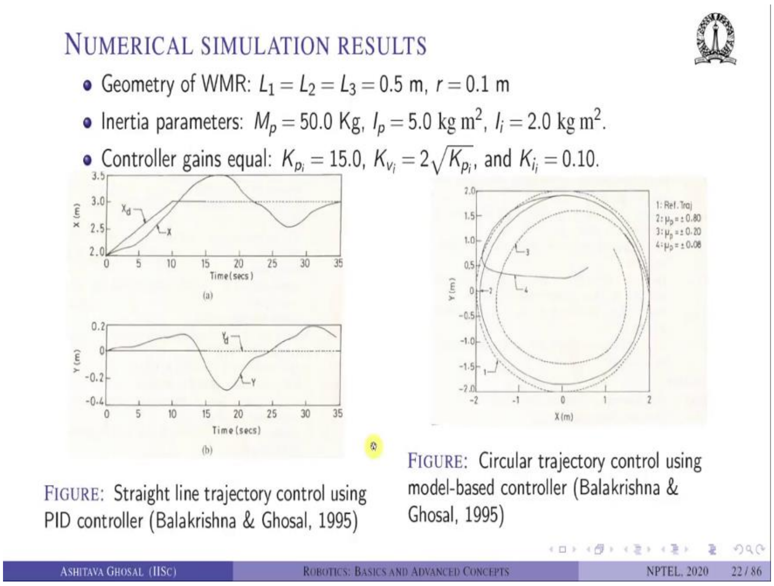


So, let us assume that we have a desired path which is $\left(x_d(t), y_d(t), \psi_d(t)\right)^T$. So, ψ is the orientation of the one of the axis with respect to the reference X-axis. We can try a PID control which is $K_v\dot{e} + K_p e + K_i \int e\, d\tau$. This is standard PID law, where X is this quantity here, $X = \left(x(t), y(t), \psi(t)\right)^T$. We can compute the Cartesian forces related to the wheel actuator torque by τ is equal to $[R]^{-T}$ times Cartesian forces ok.

So, very similar to what we did when we have Cartesian motion of a serial robot. So, we had some Jacobian matrix, and that was related to the Jacobian matrix inverses and so on. In this case, if you think a little bit about it, $[R]$ is the inverse of the Jacobian matrix. So, you have some transpose, and then again an inverse to relate the motor torques to the Cartesian forces. We can also have a model based control using ideal rolling which is $\tau = [\alpha]\tau' + \beta$, where $[\alpha] = R^{-T}[M^*]$.

$[M^*]$ was that mass matrix; $\beta = [R]^{-T}\{\dot{\Psi}[Q]\dot{X}\}$; and τ' is this PD controller scheme. So, $[M^*] = [M] + [R]^T[I][R]$. So, this is basically converting the mass matrix corresponding to ideal rolling ok. So, this is the control scheme which is basically some part of it is PID or PD. This is PID, but we use PD here. And then this is the model base term.

(Refer Slide Time: 41:51)



So, this control scheme was used by one of the students long time back. We assume $L_1 = L_2 = L_3 = 0.5$, $r = 0.1$ meters, inertia parameters was taken as 50 K g, and some I_p was 5 kg meter square. The wheels were 2 kg meter square. And we set controller gains to $K_{p_i} = 15$, $K_{v_i} = 2\sqrt{K_{p_i}}$, means critical damping, an integral gain as 0.1 ok. So, then we can see what is the mobile robot doing with this choice of controller gains, and this choice of control.

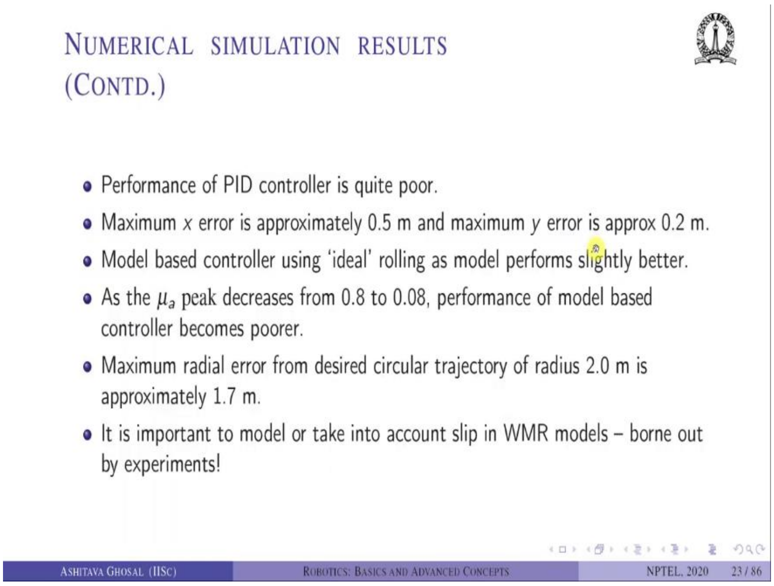
So, if I want the X_d to go up in a ramp like this and stay at the same place. So, what is the controller this particular controller doing? You can see that the motion of the platform x looks like this, the dark line. Whereas this dotted line is the desired x. Likewise, the desired y what is a constant value I want to stay at the same place. So, I want to go in some X direction. The y also changes. It does not maintain $y_d = 0$.

So, we can also see how the mobile robot traces a circular trajectory ok. So, and we can try different adhesion coefficient. So, remember μ_a is a value which you need to choose. It depends on the friction and various other parameters between the wheel and the ground. So, the reference trajectory is a circle. And for different μ_a peak, so remember μ_a has a peak value. So, if you assume that to be 0.8 ok or 0.2, or 0.08, so three different values, one very small, one medium and one very large. You can see what is happening to the trajectory.

So, 3 is with 0.2, 4 is with 0.08, and the other one, 1 is with 0.8. So, for a very large coefficient of adhesion or friction ok, it comes very close to the reference trajectory which is a circle. Whereas, as the friction coefficient reduces, we can see it deviates from the trajectory ok.

So, this is one of the findings long time back. When we showed that if you do not model slip ok, then we will not be able to track a desire trajectory in a wheeled mobile robot; so modeling or slip and taking into account the slip which is happening at the wheels is very important in a wheeled mobile robot.

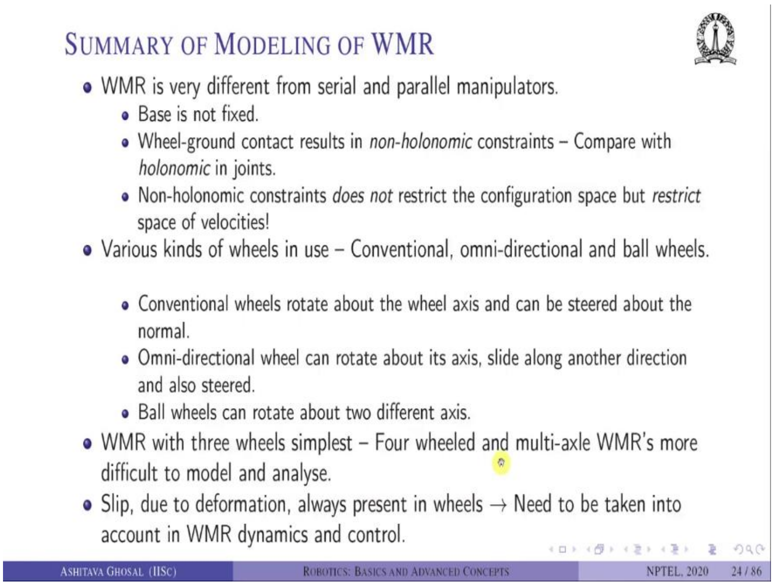
(Refer Slide Time: 44:47)



So, as I said the performance of the PID controller is quite poor. Maximum error in x is about half a meter; maximum y error is 0.2 meters. Model based controller using ideal rolling as model perform slightly better. However, as the μ_a peak decreases from 0.8 to 0.08.

So, 10 times smaller the performance of the model based controller also becomes poorer ok. So, the maximum radial error from the desired circular trajectory of radius 2 is approximately 1.7 meters; it is quite a lot. So, hence it is important to model or take into account slip in wheeled mobile robots. And this was later borne out by experiments also ok.

(Refer Slide Time: 45:40)



So, let us conclude this lecture. So, in summary, WMR is very different from serial and parallel manipulator ok. Why, because the base is not fixed, it is moving. The wheel ground contact results in non-holonomic constraints ok, it is not a holonomic constraint as what happens in let us say a rotary joint or and S-S pair.

There the constraints did not involve the rates of the generalize coordinates. So, the non-holonomic constraint does not restrict the configuration space, it does not restrict the generalize coordinate, but it restricts the space of velocities ok. So, there is a relationship with the derivatives of the generalize coordinates. There are various kinds of wheels in use. We can have conventional wheels, omni-wheels and ball wheels. Conventional wheels rotate about the wheel axis and can the steered about the normal.

Omni-wheels can rotate about this axis and slide along another direction and also can be steered ok. We can also steer the omni-wheel. I did not show you that, but it can be steered. Ball wheels can rotate about 2 different axis ok. So, the WMR with 3 wheels is the simplest configuration.

4 wheel and multi axle WMRs are more difficult to model and analyze. And finally, the slip which is due to the deformation always present in the wheels need to be taken into account in WMR dynamics and control ok. So, in the next lecture, we will look at wheeled mobile robots on uneven terrain.