Robotics: Basics and Selected Advanced Concepts
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Lecture - 39
Wheeled Mobile Robots (WMR) on Uneven Terrain

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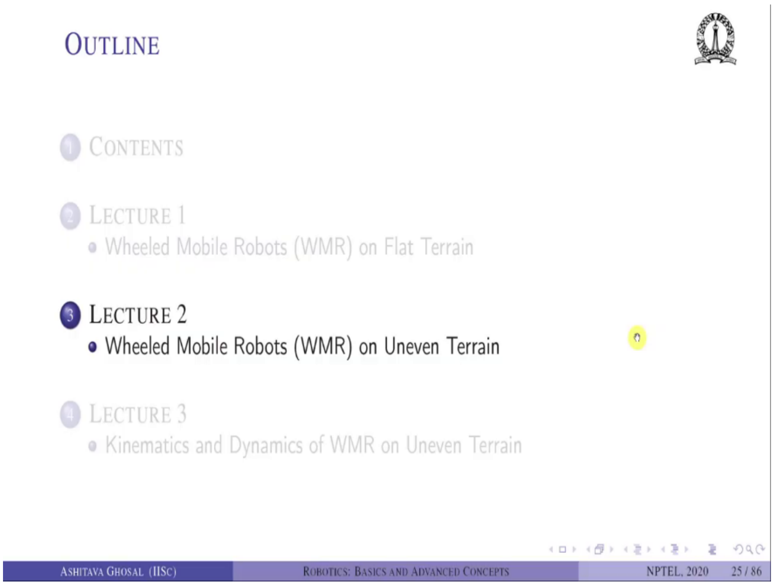

Welcome to this NPTEL lectures on Robotics. In these 3 lectures, we will look at modeling and analysis of wheeled mobile robots. In the last lecture, we had looked at wheeled mobile robots on flat terrain. And I had shown you that wheel slip is an important component of modeling and simulation. And we need to take into account wheel slip in a wheeled mobile robot. In this lecture, we will look at wheeled mobile robots on uneven terrain.

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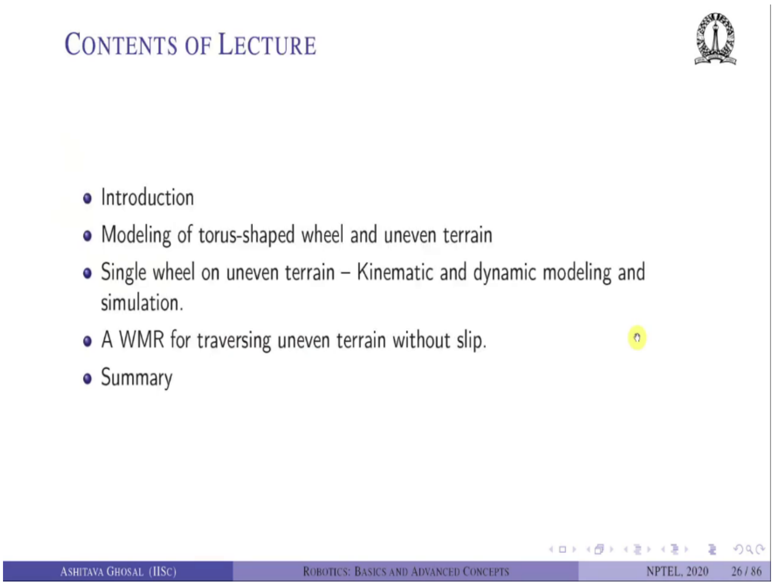

So, the contents of this lecture are, I will quickly introduce this topic. We look at modeling of a torus-shaped wheel and uneven terrain, how do you look at the uneven terrain, how do you model the uneven terrain. We will look at single wheel on an uneven terrain. We will look at the kinematic and dynamic modeling and simulations. And then we will look at the three-wheel wheeled mobile robot which can traverse uneven terrain without slip ok.

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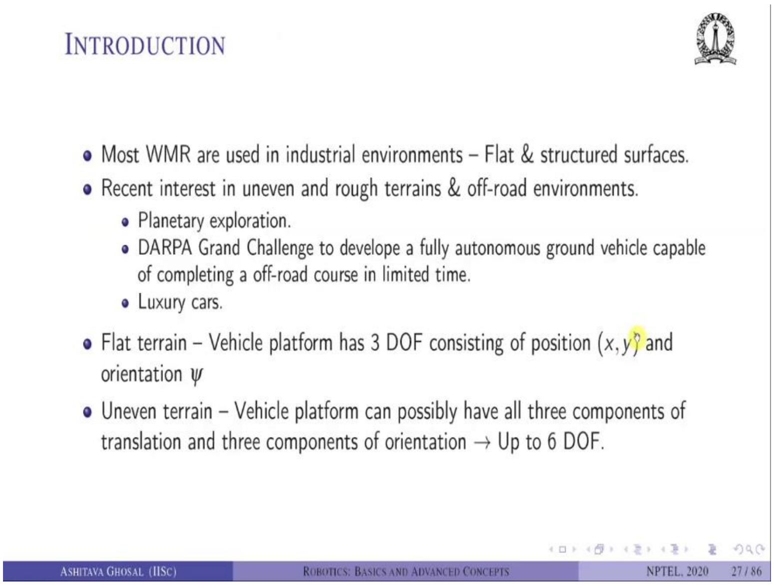

So, most WMRs are used in industrial environments which are flat and structured surfaces. However, there is a recent interest in uneven and rough terrains and off road environments. So, for example, in planetary exploration, the ground of the terrain cannot be flat like a road. A few years back there was also this DARPA grand challenge to develop a fully autonomous ground vehicle capable of completing an off road course in limited time ok.

This was a very very important channel challenge set by DARPA in USA. And then lot of technologies were developed, to make sure that a wheeled mobile robot or a car in that in this case could traverse this uneven terrain autonomously. Nowadays, uneven terrains are also showing up in luxury cars ok. So, there are cars in which when the car is going on a bend ok and the road is curved. However, the person sitting on the car will still think that it he is on horizontal plane ok.

So, in these three conditions, we need to look at uneven terrain and how to model and simulate motion on uneven terrain. In a flat terrain, the vehicle platform has 3 degrees of freedom ok. So, it consists of position x and y and some orientation of the platform with reference to a fixed coordinate axis.

In an uneven terrain, the vehicle platform can possibly have all three components of translation, and all three components of orientation. So, it can move in x and y and also z, and it can also orient in two other angles. So, we have already one side, but it can also have two other angles. So, it can have up to 6 degrees of freedom.

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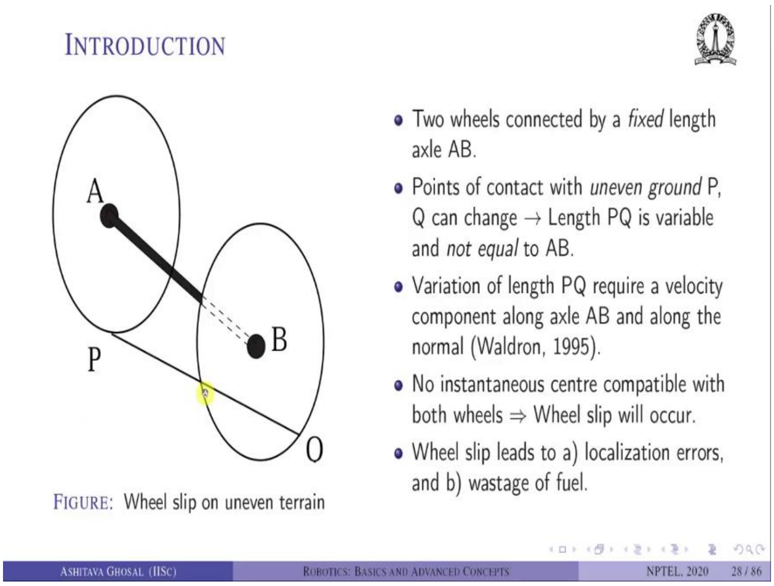

Let us continue. If you have two wheels connected by a fixed length axle AB ok, and the point of contacts are P and Q here ok, this should be Q. So, if the points of contact are on an uneven terrain, this length PQ is variable ok. So, this wheel can be on top of a little bit uneven terrain, this Q will be at some other place. At some other instant, when it is almost flat, the PQ distance will be different.

More importantly this length $PQ \neq AB$ ok. If you think of a wheel on flat surface, the two points of contacts ok between on the flat surface and the axle length would be more or less same ok, and always more or less constant. The variation of this length PQ requires a velocity component along the axle AB ok, and along the normal.

So, this was observed by Waldron – a well-known robotics researcher in 1995. And he also showed that no instantaneous center compatible with both wheels will be present ok. And hence if you have these two wheels with the fixed length axle going on an uneven terrain, wheel slip will occur ok. There is nothing you can do; there is no instantaneous center ok.

Remember in the case of the tricycle or even in the case of a three-wheeled vehicle, I showed you there must be an instantaneous center for all the three wheels ok; otherwise, there will be wheel slip. So, in this example, Waldron showed that if these two wheels if the fixed length axle is moving on uneven terrain, this point of contact is changing ok. This length is different from this A and B, and then wheel slip will occur because there is no instantaneous center.

And wheel slip leads to what is called as localization error ok, and it is waste full of fuel. So, if you are slipping, then you are wasting fuel at the same place, you are not going forward or going anywhere.

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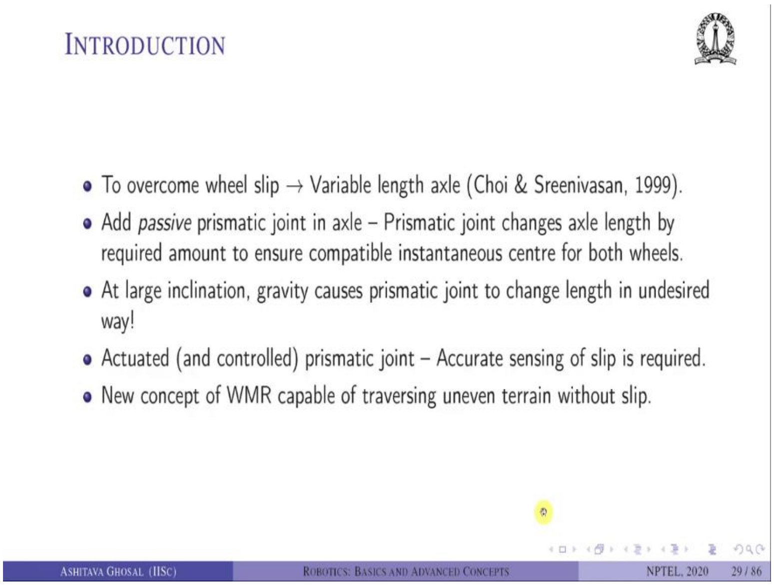

So, to overcome wheel slip, various people have suggested various techniques. One was what is called as a variable length axle ok. So, in this figure, as this point of contact P and Q are changing, if there is a way to change this length of the axle between A and B, they showed that we can overcome slip. We can also add a passive prismatic joint in the axle ok.

To change this length actively is very complicated, but we can have a passive prismatic joint. And this prismatic joint changes axle length by required amount to ensure compatible instantaneous centre for both wheels.

At large inclination, gravity causes the prismatic joint to change length on its own ok in an undesired way, so which is not a good idea. If you have actuated prismatic joint, then it is very very hard to control. Because we need to sense slip, and then we have to make sure that the length is changing properly to minimize that slip.

So, in this talk, this is the work of one of our student. We introduce a new concept of a WMR capable of traversing uneven terrain without slip ok, and this is what I am going to show you.

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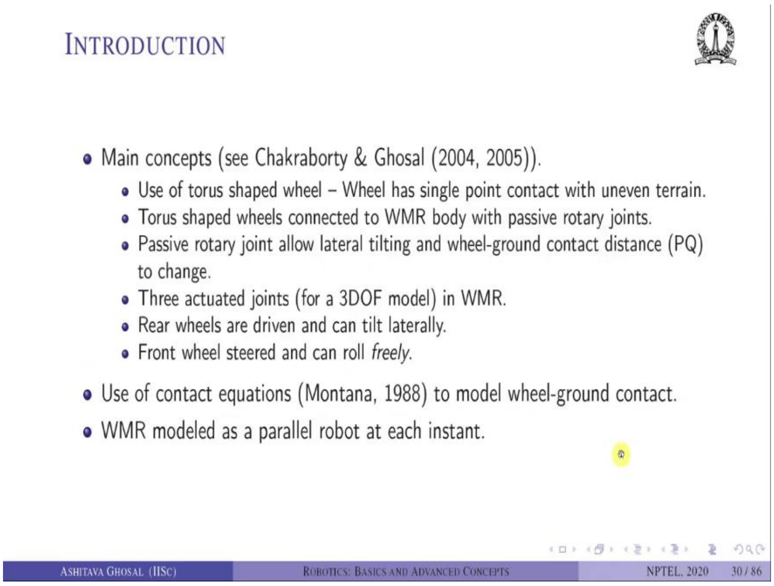

So, the main concepts which appeared in papers ok longtime back – 15 years back are the following. We will use a torus-shaped wheel ok. So, if you have a torus-shaped wheel, the wheel has a single point contact with the uneven terrain ok. And the point of contact can go along the lateral direction of the torus.

When the wheel is rotating rolling forward the point of contact can go along the circumference of the big diameter, and it can when the wheel is tilting it can go in the smaller circle ok. So, this torus-shaped wheel is connected to the WMR body with passive joints.

So, this passive rotary joints allow lateral tilting, and the wheel ground contact distance PQ to change. So, as the wheel tilts the distance between the two contact point is changing, the axle length remains the same ok. We are not changing the length of the axle.

So, we can have three actuated joints for a 3 DOF model in a WMR. The rear wheels are driven and can tilt passively not controlled, and the front wheel is steered and can roll freely ok. So, this is one big part of the idea.

So, we have torus-shaped wheels ok. This torus-shaped wheels are connected to the body of the platform using passive rotary joints. There are three of these actuated joints. The two rear wheels are driven, and can also tilt laterally and the front wheel can roll, but can be steered. So, now we need to worry about how this torus-shaped wheel contacts with the uneven terrain ok.

So, we used set of equations which was developed even older by a researcher called Montana in 1988 which models the wheel-ground contact point ok. And we will show you that we will model the wheeled mobile robot as a parallel robot at each instant of time ok. So, this is the main ideas.

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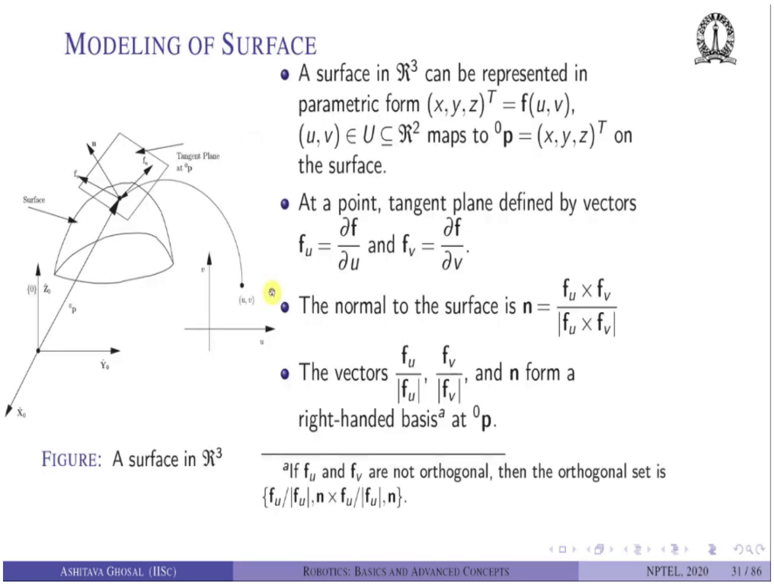

So, let us continue. How do we model the surface and uneven terrain? So, a surface in 3d space can be represented in parametric form which is $(x, y, z)^T$ as a function of u and v ok. So, u and v are the two parameters which determine the surface ok. So, if you give me any u and v on this $u - v$ plane, it match to a point on the surface which is $(x, y, z)^T$ by this equation.

At any point on this surface, we can define a tangent plane. The tangent plane is defined by two vectors f_u and f_v, $f_u = \frac{\partial f}{\partial u}$, and $f_v = \frac{\partial f}{\partial v}$. We can also find the normal to this tangent plane which is the cross product of f_u and f_v. And we can normalize it to get a unit normal.

So that three vectors, f_u made into a unit vector, f_v made into a unit vector, and n form a right handed coordinate system. It may turn out that this f_u and f_v are not orthogonal, sometimes the equation of a surface is such ok, then we need to find an orthogonal set in which we can also find.

So, we pick the unit vector along f_u as one of them. Then we pick the normal we pick the normal vector which is $f_u \times f_v$, but the f_v vector we do not use we use $n \times f_u$. So, this is like x, this is like x cross something which lies in the plane, and the $y = z \times x$. So, this gives you a right handed coordinate system. But we in some surfaces, we can directly use f_u and f_v unit vectors along the two tangents.

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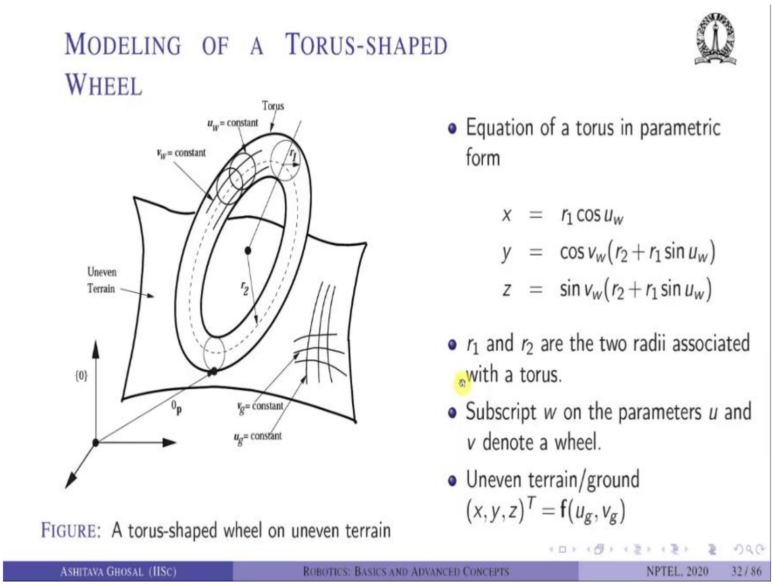

So, let us look at a typical example of a torus-shaped wheel ok. The equation of a torus can be given in this form. So, $x = r_1 \cos u_w$, $y = \cos v_w\,(r_2 + r_1 \sin u_w)$, and $z = \sin v_w\,(r_2 + r_1 \sin u_w)$. So, u_w and v_w are the parameters which describe this torus. So, u_w is equal to constant – it is a circle which is shown here; v_w equals constant is this bigger circle ok.

So, a torus can be obtained by taking this small circle and rotating about or moving it along this big circle. The r_1 and r_2 are the radius of the small circle and this big circle ok. The subscript w on the parameters u and v denote the wheel ok. So, the uneven terrain can also be represented as a surface, we can get $(x, y, z)^T$ as a function of some u_g and v_g; u_g and v_g are the two parameters which describe this uneven terrain.

So, remember u and v maps to the surface. So, this is we are going to distinguish the wheel parameters with u_w, v_w, and the ground uneven terrain parameters with u_g and v_g. So, and any point on the wheel or on the surface can be denoted by a vector from a reference coordinate system which is ${}^{0}\boldsymbol{p}$. So, the point of contact within this torus-shaped wheel and this uneven terrain is given by this vector.

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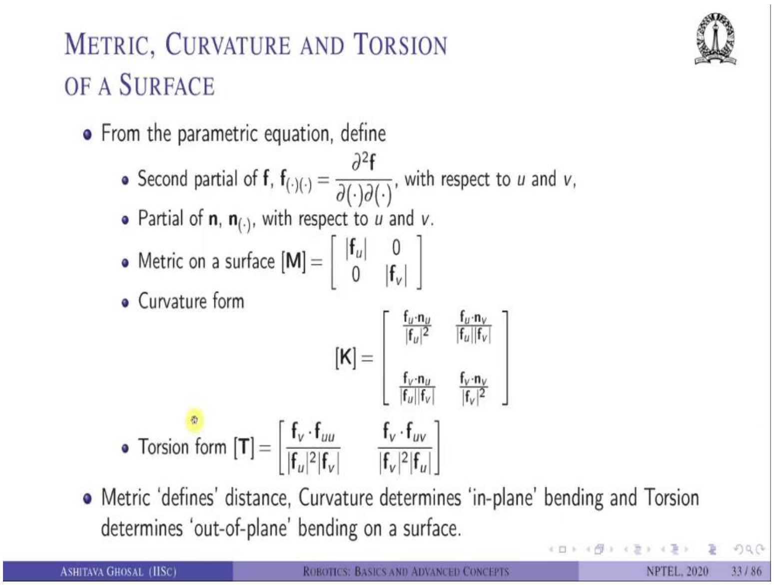

So, from the parametric equation of the surface which is $(x, y, z)^T = f(u, v)$, we can obtain the second partials of this function ok. So, we can do f_{uu}, f_{uv}, and f_{vv} which was nothing but the second partial derivatives of f with respect to u and v. We can also find the partial of n normal vector with respect to u and v. So, following Montana, he derived these equations, and he used this formulation to discuss the point of contact between two surfaces.

We can first define something called as a metric which is $[M] = \begin{bmatrix} |f_u| & 0 \\ 0 & |f_v| \end{bmatrix}$. We can also obtain a curvature form ok which is $[K] = \begin{bmatrix} \frac{f_u \cdot n_u}{|f_u|^2} & \frac{f_u \cdot n_v}{|f_u||f_v|} \\ \frac{f_v \cdot n_u}{|f_u||f_v|} & \frac{f_v \cdot n_v}{|f_v|^2} \end{bmatrix}$.

So, this is very similar to the curvature of a surface ok. We can define the curvature of a surface in terms of the derivatives of the normal with respect to u and v along this f_u and f_v directions. We can also find how the surface is bending out of the plane ok and that is typically given by the second derivative of this function ok, $\begin{bmatrix} \frac{f_v \cdot f_{uu}}{|f_u|^2|f_v|} & \frac{f_v \cdot f_{uv}}{|f_v|^2|f_u|} \end{bmatrix}$. So, this tells you how the second partial derivatives of f changes in some in directions f_v ok, out of the plane.

So, metric in a sense defines distance, curvature defines in plane bending, and torsion determines out of plane bending of the surface ok. So, those of you who have done any geometric modeling course where you have looked at the models of a surface, similar ideas are there.

So, we have the first fundamental form, and then we have the second fundamental form where you have the second derivatives. Those of you are interested can go look at some modules of how to model surfaces using partial derivatives of the function which describes the surface ok.

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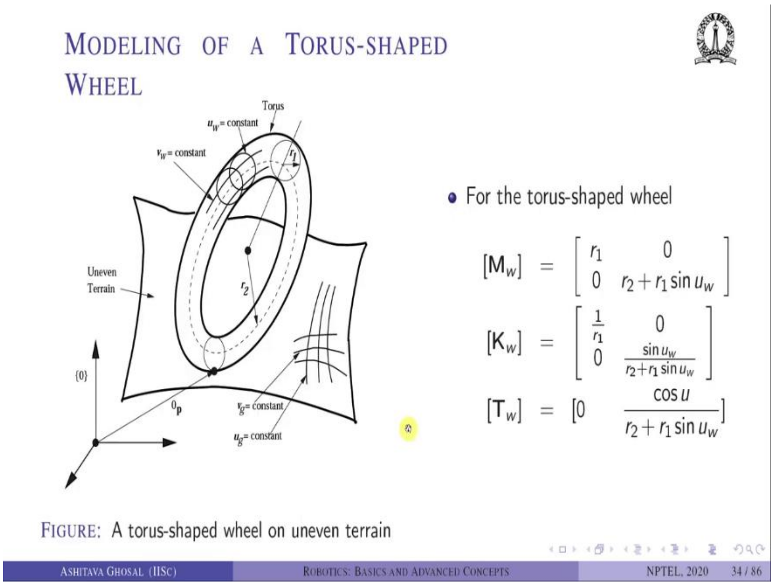

So, in our case, we have this torus-shaped wheel. We can find the metric $[M_w] = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 + r_1 \sin u_w \end{bmatrix}$. We can find the curvature form which is $[K_w] = \begin{bmatrix} \frac{1}{r_1} & 0 \\ 0 & \frac{\sin u_w}{r_2 + r_1 \sin u_w} \end{bmatrix}$.

And the torsion form, $[T_w] = \begin{bmatrix} 0 & \frac{\cos u}{r_2 + r_1 \sin u_w} \end{bmatrix}$ because we know the equations of the torus. We can easily find this metric curvature form and the torsion form for the wheel.

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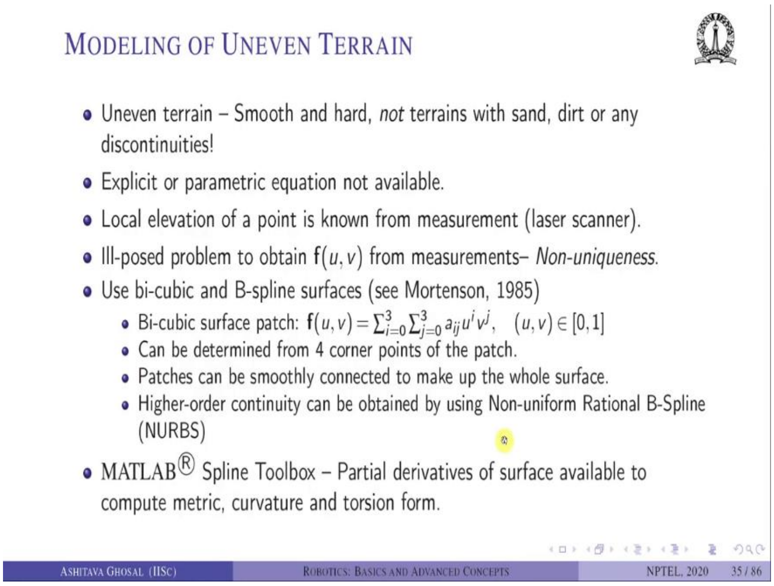

For the uneven terrain we assume that it is smooth and hard. So, basically we are not going to consider terrains which are sandy or dirt or with discontinuities ok. Most often explicit or parametric form equation of a surface is not available, especially of an uneven terrain. We can obtain what is called as a local elevation of a point from measurements ok. So, there could be a scanner on this mobile robot which looks forward ok.

And then it finds what is the height from some reference of each point ok. To obtain a function from these measurements, it is ill posed ok there are it is non-unique, but we can use what are called as bi-cubic or B-spline surfaces. And obtain the bi-cubic surface patch which is $f(u,v) = \sum_{i=0}^{3}\sum_{j=0}^{3} a_{ij}u^i v^j$, where $(u,v) \in [0,1]$.

So, if you give me four corner points of this patch, I can estimate or I can find the equation of a surface ok. And then you can connect these patches smoothly to make up the whole surface ok that is one way of doing it. So, we take measurements of four points find the bi-cubic patch, and then find the surface of this surface between these four points smooth surface, and then we can connect four in a patches like this to make up the whole surface.

You can also do higher order continuity ok. And this can be obtained using NURBS. So, we are not going to do this. So, what we have done or what the student did was he use something called MATLAB Spline Toolbox ok. So, there is a tool box in MATLAB which can find surfaces given points and so on. So, and then from this Spline Toolbox, we can find the partial derivatives of the surface to compute metric, curvature and torsion form ok.

So, we assume some points on this surface we fit this surface using bi-cubic patches or NURBS, and then we use this Spline Toolbox in MATLAB to find the partial derivatives of the surface which were generated. And then from that we find this $[M_w]$, $[K_w]$ and $[T_w]$ ok, metric, curvature and torsion form.

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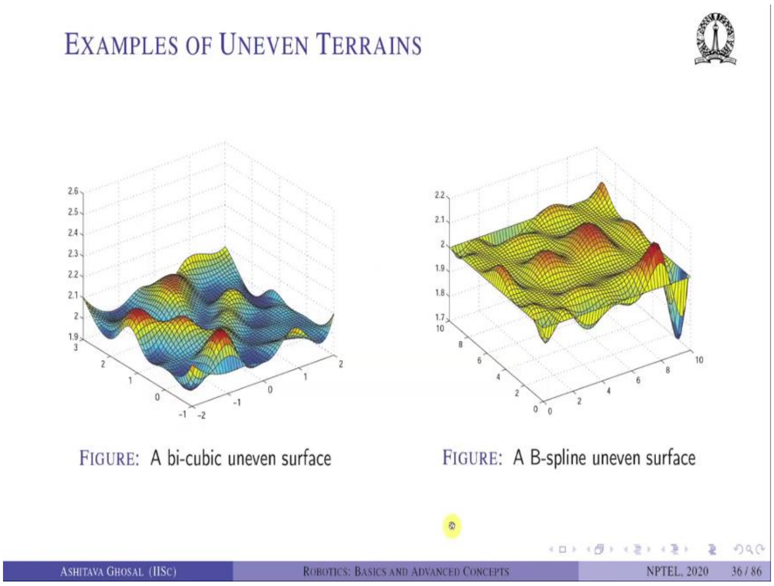

So, let us look at the few examples. So, this is the surface which is generated using bi-cubic ok uneven surface. So, some points were chosen, and then you fitted small patches, and then you put them together properly such that you get a surface. So, we can generate in MATLAB surfaces which look like this ok.

So, this is x, y and z. We can also generate slightly smoother or some other way of generating surface. So, this is the B-spline uneven surface; this is the bi-cubic uneven surface. So, MATLAB allows you to do all this ok.

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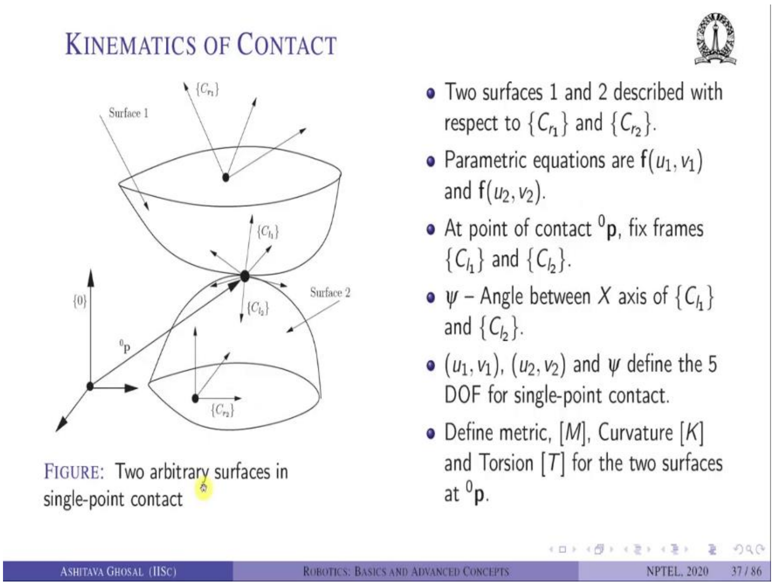

So, now once we know how to generate an uneven surface, now let us look contact between two surfaces. So, I have one surface here, and I have another surface here – surface 1, surface 2. And they are contacting at a point $^{0}\boldsymbol{p}$ from some reference coordinate system ok. So, these two surfaces can be described with respect to two reference coordinate system.

So, $\{C_{r_1}\}$ is for surface 1; $\{C_{r_2}\}$ is for surface 2. So, basically in these coordinate systems, I know the equation of the surface. So, I know all these points ok. I know what is the shape of the surface. So, we have parametric equations for these surfaces $f(u_1, v_1)$ for the surface 1, and $f(u_2, v_2)$ for the surface 2. At the contact point between these two surfaces, we fix two other coordinate systems which are $\{C_{l_1}\}$ and $\{C_{l_2}\}$.

So, $\{C_{l_1}\}$ has contained some X, Y, and Z-axis; $\{C_{l_2}\}$ also contained some X, Y and Z-axis ok. The angle between the X-axis of $\{C_{l_1}\}$ and $\{C_{l_2}\}$ is denoted by ψ, we will need this variable later on. So, what do we have? We have (u_1, v_1) at this point which relates the coordinates of this contact point with respect to the surface 1 coordinate system.

We have (u_2, v_2) which obtains the coordinates of these points with respect to the surface 2 coordinate system. And then we have this angle ψ which denotes the orientation of the two X-axis because just the point is not enough I could have rotated these two X-axis and still the point could be the same.

So, we need 5 degrees of freedom to represent the contact between these two surfaces, so basically u_1, v_1, u_2, v_2 and ψ. You can see just (u_1, v_1), and (u_2, v_2) is not enough ok. So, we defined a metric $[M]$, a curvature $[K]$, and torsion $[T]$ for the two surfaces at this point ${}^0\boldsymbol{p}$.

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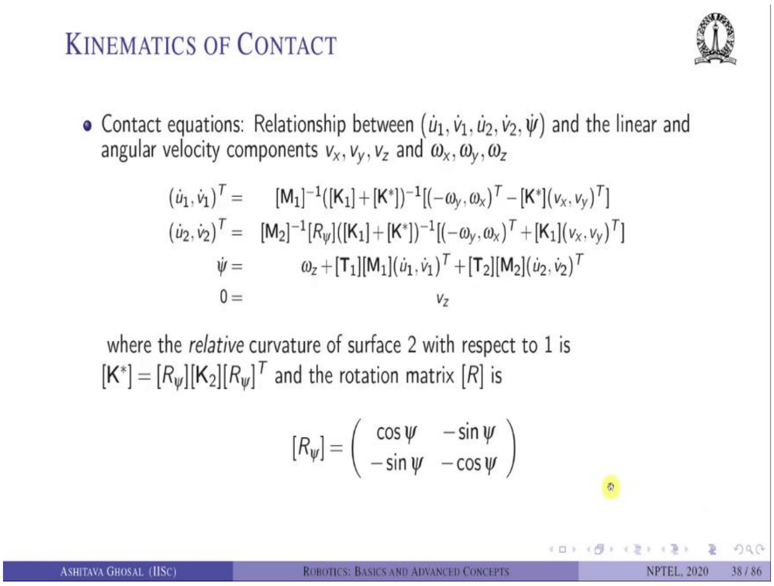

So, the contact equations obtained by Montana relates the derivative of $\dot{u}_1$, $\dot{v}_1$, $\dot{u}_2$, $\dot{v}_2$ and $\dot{\psi}$ to the linear and angular velocity components $v_x, v_y, v_z, \omega_x, \omega_y, \omega_z$. So, what are these $v_x, v_y, v_z, \omega_x, \omega_y, \omega_z$ that is the velocity of one of the surfaces with respect to the other surface ok.

And he showed that this $(\dot{u}_1, \dot{v}_1)^T$ can be related to the metric, it can be related to the curvature form, and it can be related to the relative torsion form ok between these two surfaces. And also it contains v_x, v_y, ω_x, ω_y, and also ω_z.

So, let us not go to the details of this equation, except to argue that there is a relationship between v_x, v_y, v_z – the velocity of surface 1 with respect to surface 2, ω_x, ω_y, ω_z – angular velocity of surface 1 with respect to surface 2.

And the rate of change of these parameters which describe the two surfaces, so $\dot{u}_1, \dot{v}_1, \dot{u}_2, \dot{v}_2$ and $\dot{\psi}$. And they contained the metric for surface 1, the metric for surface 2, the rotation matrix in terms of ψ which rotates the two X-axis, the curvature form of surface 1, the torsion of surface 1, the torsion of surface 2, and we have this $[K^*]$ which relates the two curvature forms ok. So, $[K^*] = [R_\psi][K_2][R_\psi]^T$.

So, and the rotation matrix corresponding to $[R_\psi] = \begin{pmatrix} \cos\psi & -\sin\psi \\ -\sin\psi & -\cos\psi \end{pmatrix}$. So, these equations were derived by Montana longtime back. And as I said it relates the rate of change of the parameters, and ψ as a function of the relative translation and angular velocity of one surface with respect to another surface. So, these equations are called kinematics of contact equations.

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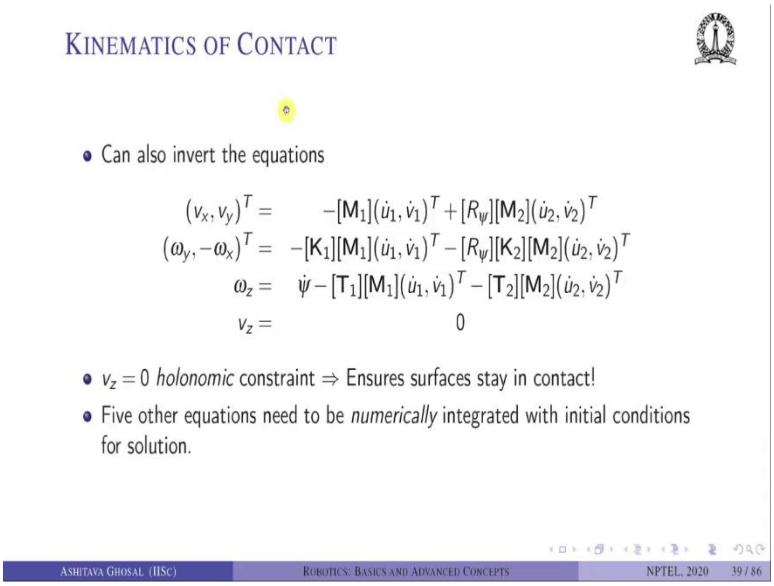

We can also invert the equation. So, if I give you $\dot{u}_1, \dot{v}_1, \dot{u}_2, \dot{v}_2$ and $\dot{\psi}$, can we find out $v_x, v_y, v_z, \omega_x, \omega_y, \omega_z$? Yes. And as you see it sort of makes sense in the contact equation, the Z component should be 0 because otherwise the two objective will not be in contact ok. So, $v_z = 0$ is a holonomic constraint to ensure that the surface is stay in contact ok.

So, these five equations need to be numerically integrated with initial conditions to solve these contact equations ok.

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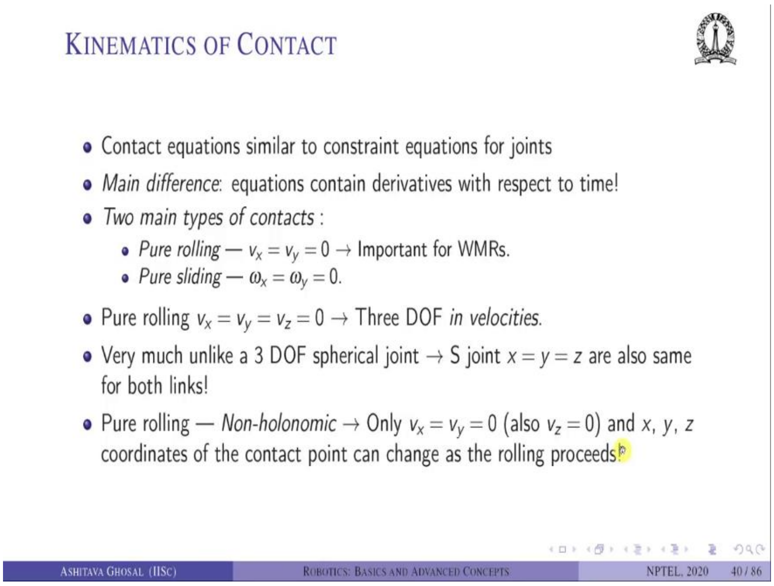

So, the contact equations are similar to constraint equation for joints. Remember for a rotary joint I showed you that there are five constraint equations; in this case two surfaces in contact generate five contact equations. The main differences these equations contains derivatives with respect to time.

In the case of a rotary joint, if you go back and remember I said that the position vector from both sides are same, and the orientation is related by the rotation matrix of one link, and the second link, and there is this rotation at the joint ok.

However, here there are derivatives of u and v and ψ. There are two main types of contacts which are possible. One is pure rolling which means that $v_x = v_y = 0$. So, the two surfaces there is no v_x and v_y, relative v_x and v_y. We can also have pure sliding which is $\omega_x = \omega_y = 0$; v_z is already 0. So, in pure rolling, there is no linear velocity at the point of contact between the two surfaces.

So, if you have pure rolling, we have $v_x = v_y = v_z = 0$. So, there are three degrees of freedom in velocities. Very much unlike a three degree of freedom spherical joints ok, in a spherical joint, we had $x = y = z$ also from both sides of the both links.

However, they were holonomic constraints ok. Pure rolling, however, is non-holonomic ok, because we have $v_x = v_y = 0$, and the x, y and z coordinates of the contact point can change as the rolling proceeds ok.

In a spherical joint, the x, y, z coordinates from the two parts or two sides are same. It does not change. Here, however, if you think of the one surface rolling purely with respect on another surface, then the point of contact will change – the x, y, z point will change. However, the velocities are 0 the relative velocity between the two contact points are 0 ok that is again a feature of non-holonomic constraint. It restricts the space of velocities, but it does not restrict the space of the generalized coordinates ok.

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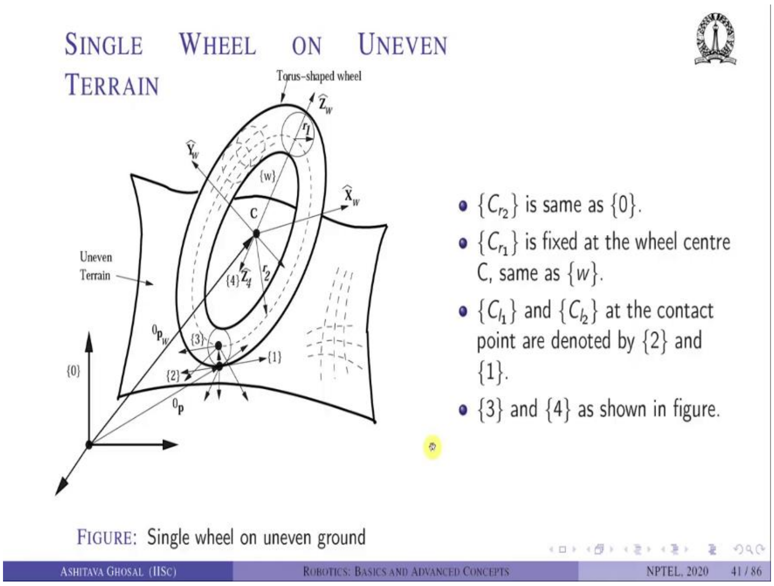

With all this math and you know machinery that we have developed, how to represent the surface, how to represent the kinematics of contact, we can now see model the single wheel on an uneven terrain. So, we have $\{C_{r_2}\}$ which was the second reference coordinate system of one of the surface.

This could be the same as the fixed reference coordinate system. $\{C_{r_1}\}$ which was the first surface ok that is fixed at the wheel center ok C, and that is also labeled as the coordinate system $\{w\}$.

The $\{C_{l_1}\}$ and $\{C_{L_2}\}$ which are the point of contact are labeled as 2 and 1 in this example. And 3 and 4 are as shown in the figure ok. So, 3 is at the center of this small circle or along the center line of this torus, and 4 is at the center of this wheel ok. So, how many coordinate systems we have?

We have a $\{0\}$ coordinate system, we have a first coordinate system, we have a second coordinate system first and second are at the point of contact, 3 is at the center of this wheel, and 4 is at the center, C at this place ok. And then we have this $\hat{X}_w, \hat{Y}_w, \hat{Z}_w$.

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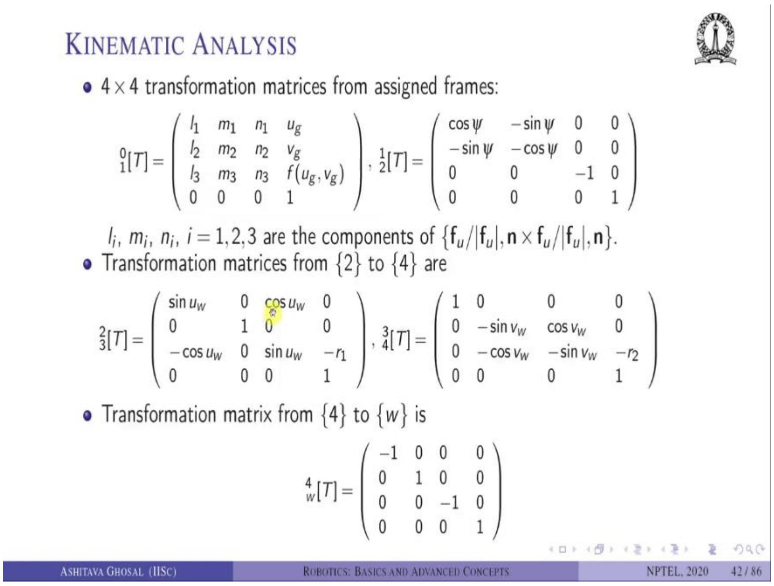

So, once we have all these coordinate systems we can find the homogeneous transformation matrices between 0 and 1, 1 and 2, 2 and 3, 3 and 4, and 4 and w ok, very similar ideas to what we have discussed earlier. So, the homogeneous transformation matrix between the fixed and 1 which is the uneven terrain are given by some rotation matrix. And this rotation matrix contains $l_1, l_2, l_3, m_1, m_2, m_3, n_1, n_2, n_3$.

This is the direction cosines of a coordinate system on the uneven terrain with respect to the fixed coordinate system. And then you have a point of contact which is described by u_g, v_g and z which is function of u_g, v_g. Between 1 and 2, we have a rotation of the X-axis. Remember the angle ψ showed the or denoted the rotation of the X-axis ok, so that transformation matrix will contain only ψ.

Between 2 and 3, now, we have this location of the wheel ok. So, we have $\sin u_w$, $\cos u_w$ and so on. So, this is let us go back and see what is 2 and 3. So, between 2 and 3, only u_w will show up ok. Between 3 and 4, v_w will show up ok; u_w and v_w are the parameters which describe the wheeled surface ok.

And then we have one r_1 which is this, this radius of the small direction; and r_2 which is this radius. And they will show up in between 2 and 3, you will have a $-r_1$; and between 3 and 4, we have a $-r_2$. So, if you think about it, this is correct. Between 2 and 3, it is like a rotation about Y-axis. So, we have rotation metrics here, and the translation along the Z-direction. Between 3 and 4, it is a rotation about an X-axis and then there is a translation of $-r_2$ along Z-direction.

And between 4 and w, it is very similar to an identity matrix except that the directions are slightly different. So, the X-axis is opposite direction in the fourth and $\{w\}$ coordinate system. The Y-axis is at the same place, the Z-axis is again in the opposite direction. This is basically due to the choice of the coordinate system.

So, we can find basically from 0th coordinate system which is the fixed coordinate system to all the way to the wheeled coordinate system. All this five 4×4 homogeneous transformation matrices ok. Remember we had used homogeneous transformation matrices for serial robots, for parallel robots, we can also use it for mobile robots.

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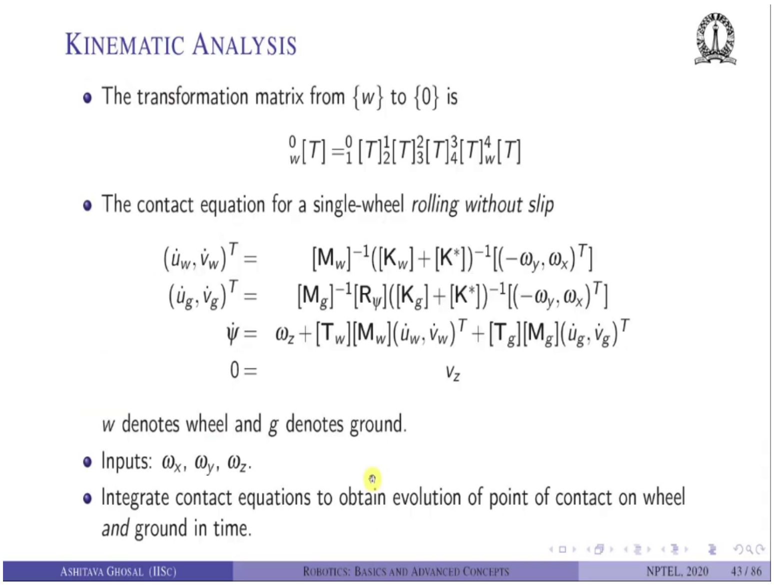

So, the transformation matrix between $\{w\}$ and $\{0\}$ can be written as product of these five matrices. And then we can also find the contact equations for the single wheel rolling without slip. So, what is rolling without slip $v_x = v_y = 0$. So, we are left with ω_x and ω_y, and then we have these 6 equations ok. So, w denotes the wheel, g denotes the ground.

So, we have a metric for the wheel, metric for the ground, a curvature for the ground, curvature form for the wheel and so on, a torsion for the wheel and torsion for the ground. These are the equations derived by Montana we are just going to use them. And assume that it is rolling without slipping.

So, hence $v_x = v_y = 0$. So, what are the three inputs in this equation? Which is $\omega_x, \omega_y, \omega_z$ ok. So, the wheel can rotate about the X-axis, Y-axis and Z-axis. So, we can integrate these equations to obtain u_w, v_w, u_g, v_g, and ψ. So, these are equations input is given, the right hand side is given, we can integrate numerically and find out u_w, v_w, u_g, v_g and ψ.

And once we find u_w, v_w, u_g, v_g and ψ, we can find the point of contact ok because the equations of the uneven surface and equation of the wheel are in terms of u_w, v_w, u_g and v_g. And ψ denotes the orientation of the X-axis of the two coordinate systems at the point of contact.

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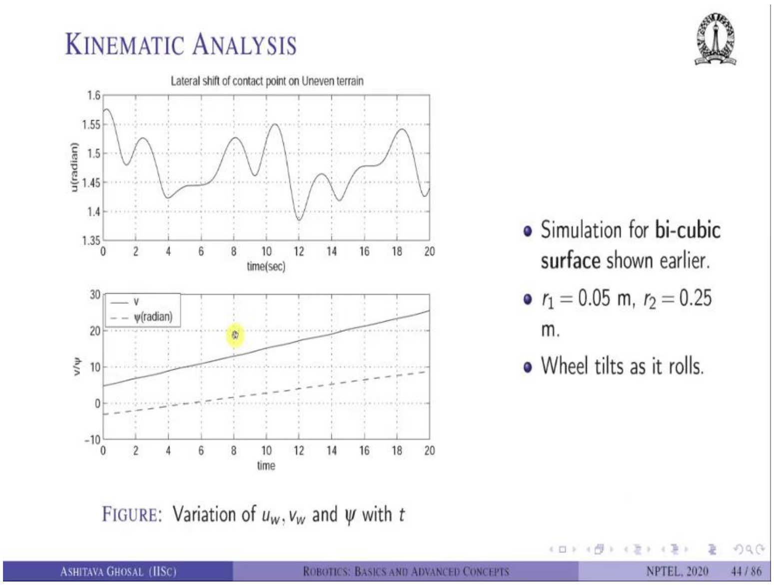

So, let us see some numerical results. So, we have generated the bi-cubic patch earlier which were shown ok. Let us assume $r_1 = 0.05$ m, $r_2 = 0.25$ m; r_1 is small, r_2 is big. And the wheel tilts as it rolls ok. So, you can plot integrate this equation you can see u as a function of time, v and ψ also as a function of time. So, u changes like this, v changes like this, and ψ changes like this ok. So, these are some numerical simulation results.

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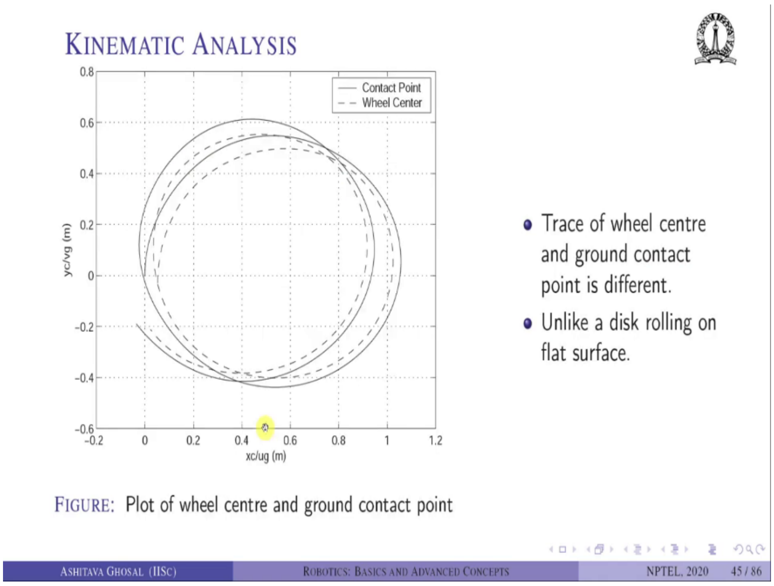

FIGURE: Plot of wheel centre and ground contact point

We can also see what is happening to the wheel as it is moving. So, we can find what is the contact point which is the dark solid line, it starts from somewhere here (0,0) and comes and ends up here in this dark solid line. The wheel center location is also known, because we know all these 4×4 homogeneous transformation matrices ok. So, using that, we can find out the wheel center with respect to the {0} coordinate system. And we can plot the wheel center.

So, what you can see is that the wheel center and the contact point do not trace the same path ok. If you had a flat disk which was rolling on the surface, it was rolling straight addressing a curve. The wheel center and the contact point will be same ok, because it is always vertical. In this case, because it is uneven terrain it is different.

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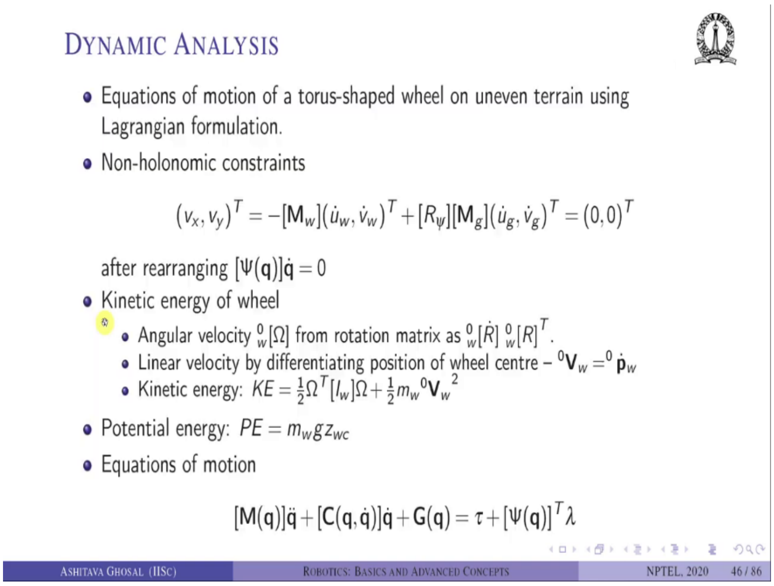

We can also perform a dynamic analysis of a single wheel. So, we can derive the equations of motion of a torus-shaped wheel moving on uneven terrain using the Lagrangian formulation ok. We find the kinetic energy, we find the potential energy, and then we take those derivatives in the Lagrangian formulation, and we derive the equations of motion.

Additionally, we have these non-holonomic constraints which are that $(v_x, v_y)^T = -[M_w](u_{\dot{w}}, \dot{v_w})^T + [R_\psi][M_g](\dot{u_g}, \dot{v_g})^T = (0,0)^T$. We can derive v_x and v_y, because remember the contact equations can go from both sides. And we are saying it is not slipping hence $v_x = v_y = 0$. So, we have these two non-holonomic constraint equations.

And this can be reorganized similar to what we had done in the dynamics lectures as $[\Psi(q)]\dot{q} = 0$. So, we have kinetic energy, we have potential energy of the wheel, and we have this constraint ok. We can find the kinetic energy of the wheel by finding the ${}^0_w[\dot{R}]{}^0_w[R]^T$, there is no propagation formula as such.

The linear velocity of the wheel is derivative of the position vector of the center of the wheel. Do we know this? Yes, because we know the 4×4 homogeneous transformation matrices. So, the kinetic energy is $KE = \frac{1}{2}\Omega^{\mathrm{T}}[I]\Omega + \frac{1}{2}m_w\ {}^0V_W^2$. So, the center or this CG of the wheel is where the $\{w\}$ coordinate system is.

Potential energy is the height of the CG from the ground and it is given by $PE = m_w g z_{wc}$. And the equations of motion will look like $[M(q)]\ddot{q} + [C(q,\dot{q})]\dot{q} + G(q) = \tau + [\Psi(q)]^T\lambda$, exactly the same form as what we had when we looked at the dynamic equations for four bar mechanism and constraint equations ok.

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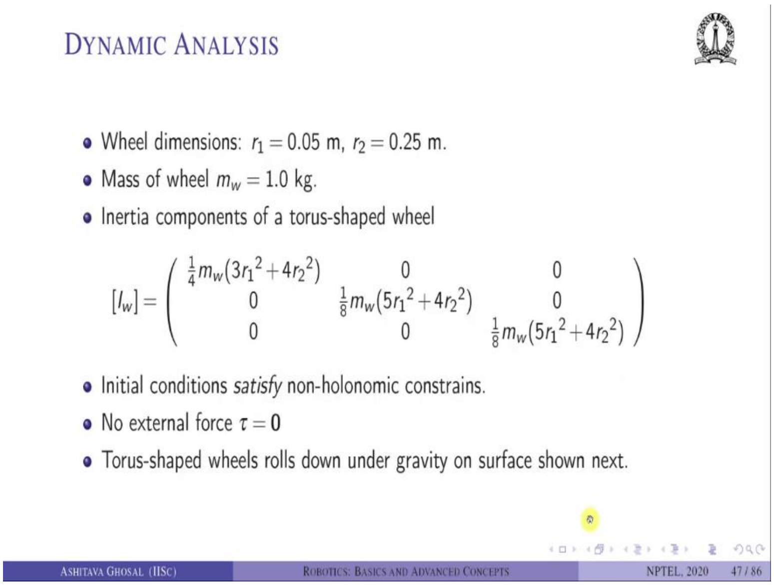

So, we can also simulate these equations. We have to assume some numbers. So, we assume the same r_1 and r_2 as for the single wheel. So, we have 0.05 meters, and 0.25 meters, but now we need some mass of the wheel which is 1 kg and we also need some movement of inertia of the torus-shaped wheel. So, we are going to assume that these are the principal movements of inertia which is some $\frac{1}{4}m_w(3r_1^2 + 4r_2^2)$, $\frac{1}{8}m_w(5r_1^2 + 4r_2^2)$, $\frac{1}{8}m_w(5r_1^2 + 4r_2^2)$.

So, these are taken from some textbook ok what are the moments of inertia I_{xx}, I_{yy}, I_{zz} for a torus. The initial conditions for simulation must satisfy the non-holonomic constraints. And we will simulate when there are no external forces. So, basically the torus-shaped wheel rolls down under gravity on the surface as shown next ok.

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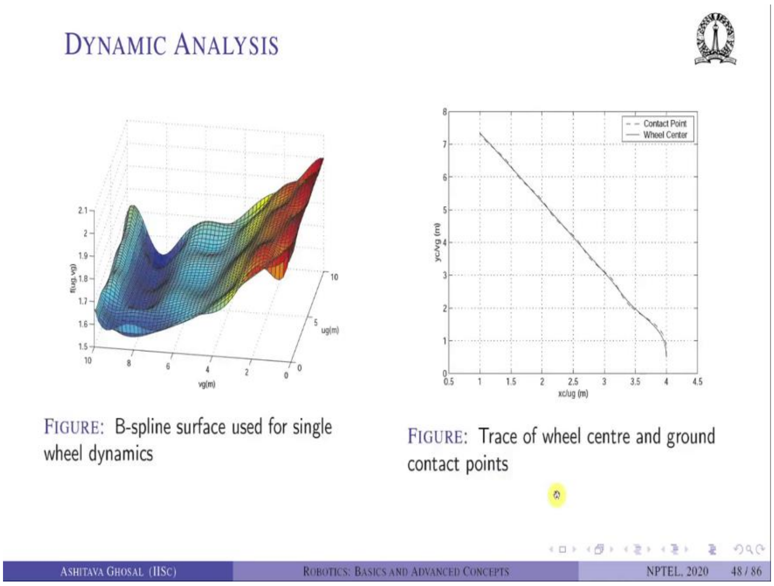

So, we have this B-spline surface which was created earlier ok or we can create again. We will put the wheel at some place and see how it rolls down ok. So, there is no wheel torque, but we are going to solve the equations of motion for this surface. And then we can plot various things.

So, the first plot is the contact point how it is changing as it is moving on this surface, and also the wheel center. So, as you can see they are not exactly same. So, the wheel is tilting and moving little bit while it is rolling down.

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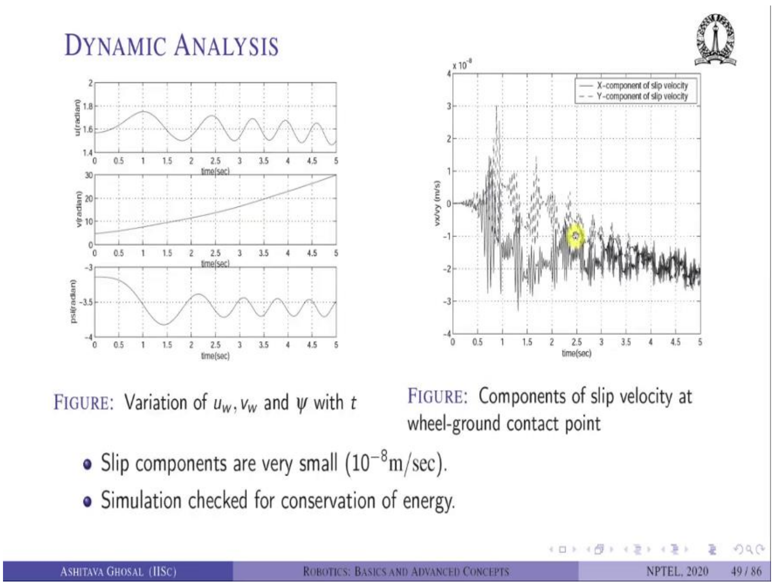

We can also see what is u, v and w for the wheel we can plot the u which looks like this, v which looks like this, and ψ which looks like this ok. We can plot the parameters u_w, v_w and ψ as a function of time. We can also see whether the slip velocities which is v_x and v_y are indeed 0. So, we can plot v_x and v_y because we have the expressions for v_x and v_y.

And it turns out that these are indeed very close to 0 it is like 10^{-8}. So, the slip components are very small 10^{-8} meters per second. And we also check that there is a conservation of energy ok. So, when it is rolling down the surface, there should be a conservation of energy potential plus kinetic energy should be constant. So, it was checked.

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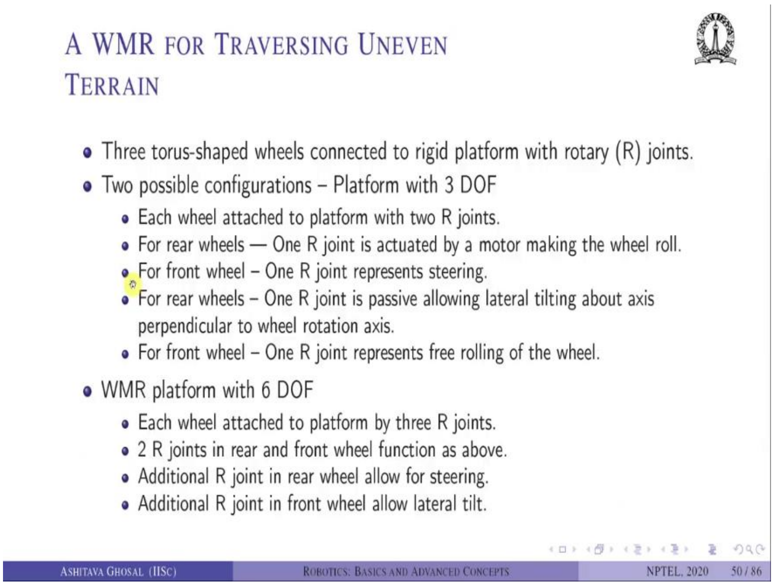

Next that we have now verified that we can simulate the motion of a single wheel on an uneven terrain, we will try to see and model a three wheeled mobile robot for traversing uneven terrain ok. So, what do we have? We have three torus-shaped wheels connected to a rigid platform with rotary joints ok. So, there are two possible configuration a platform with 3 degrees of freedom.

In the 3 degree of freedom model, each wheel attached to the platform with two rotary joints ok. For rear wheel, one rotary joint is actuated by a motor making the wheel roll. For the front wheel, one rotary joint represents steering. For the rear wheel, one rotary joint is passive allowing lateral tilting about its axis perpendicular to the wheel axis. Remember it is a torus-shaped wheel.

So, it can tilt laterally it with a passive degree of freedom. And for front wheel, one rotary joint represents free rolling of the wheel. So, the front wheel can be steered and it can roll freely. The lateral wheels can be driven, but tilt passively. If you have a 6 degree of freedom model, each wheel attached to a platform by 3 rotary joints 2 R joints in rear and front wheel as above.

Additional R joint in the rear wheel allows for steering ok. So, in the 6 degree of freedom model all the wheels can be steered. An additional R joint in the front view allows lateral tilt. So, all the 3 wheels can be steered and also tilt allowed.

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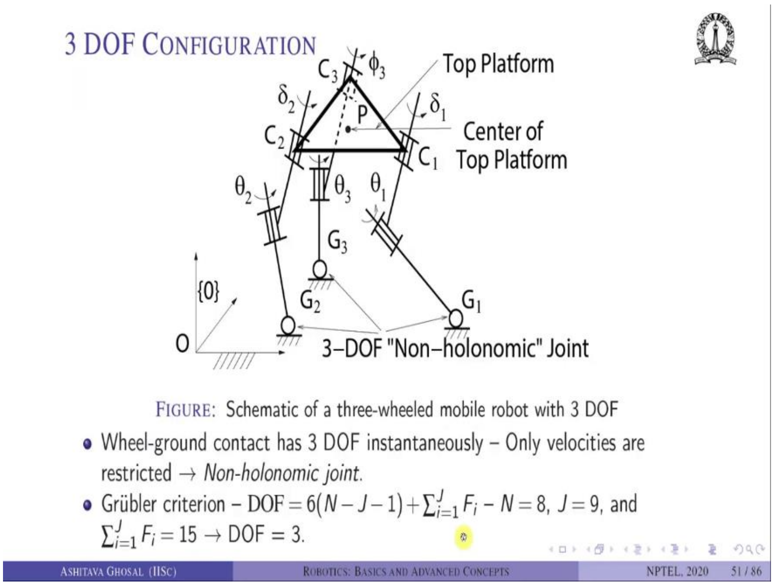

So, 3 degree of freedom model, there is a top platform, there is no bottom platform, but there are these three ground contact points G_1, G_2, G_3 we will call this as a ground platform. So, except that the constraint between the ground and the wheel is non-holonomic. So, we are going to call it as non-holonomic joint ok. It is not a spherical joint.

So, between the platform, there are two rotary joints; one in the rear wheels one of them is for driving and one of them represents lateral tilting. In the front wheel, one of them is rolling freely and steering ok. So, the wheel ground contact point has 3 degrees of freedom instantaneously right, $v_x = 0$, $v_y = 0$ because of no slip, and $v_z = 0$. So, only the velocities are restricted.

So, this is the non-holonomic joint. So, if you apply Grubler's criteria to this mechanism to this parallel mechanism which is $6(N - J - 1)$ plus the summation of degrees of freedom. You can see that this model of a mobile robot with wheel ground contact ok with no slip has 3 degrees of freedom instantaneously.

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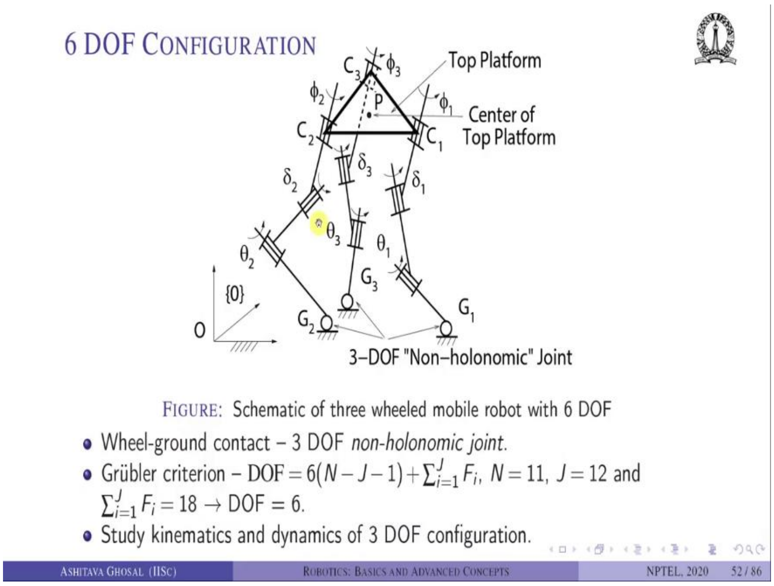

If you have three rotary joints at each leg or each wheel here, then you can see that the top platform has 6 degrees of freedom. Again the wheel ground contact points are 3 degrees of freedom. So, they are not really joints, but we are going to call them as non-holonomic joints. They restrict the velocities at the wheel ground contact point. So, we are going to study kinematics and dynamics of a 3 degree of freedom configuration.

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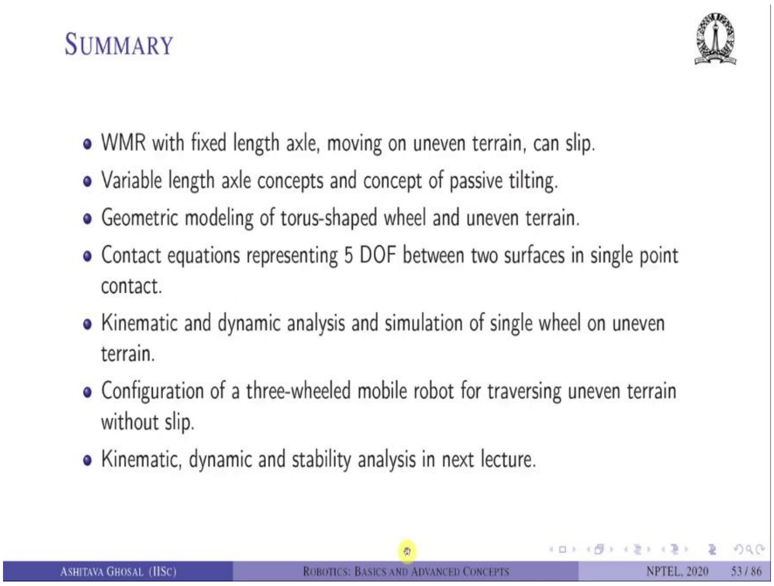

So, in summary, a WMR with fixed length axle moving on uneven terrain will slip ok, because there is no instantaneous center. We can avoid the slip using variable length axles ok, and by allowing the wheel to tilt passively.

We have used a torus-shaped wheel moving on an uneven terrain which we can model the torus-shaped wheel using simple kinematic equations for this torus, or the x, y, z as a function of u_w and v_w. And similarly we can model the uneven terrain in terms of parameters u_g and v_g.

Then I showed you how we can use the contact equation which represents the 5 degree of freedom between two surfaces in a single point contact ok. So, I showed you how the rate of change of u, v, ψ, so (u, v) of the wheel, (u, v) of the ground, and ψ can be related to v_x, v_y, v_z and ω_x, ω_y, ω_z. Then I showed you how we can derive the kinematics and dynamics of a single wheel on uneven terrain. I showed you some simulation results.

And then I showed you how we can model a three-wheeled robot which can traverse uneven terrain ok. So, it has a wheel ground contact point which is represented using non-holonomic constraints, and then we have rotary joints which represents passive tilting or which allows steering and also rolling of the wheels.

In the next lecture, we will look at kinematic, dynamic and stability analysis of a three-wheeled robot.