Robotics: Basics and Selected Advanced Concepts
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Lecture - 41
Over-Constrained Mechanisms and Deployable Structures

Welcome to this NPTEL lectures on Robotics Basics and Advanced Concepts. In this week we will look at Over Constrained Mechanisms and Deployable Structures. So, there will be 2 lectures in this week, the first one will be on introduction to over constrained mechanisms and deployable structures.

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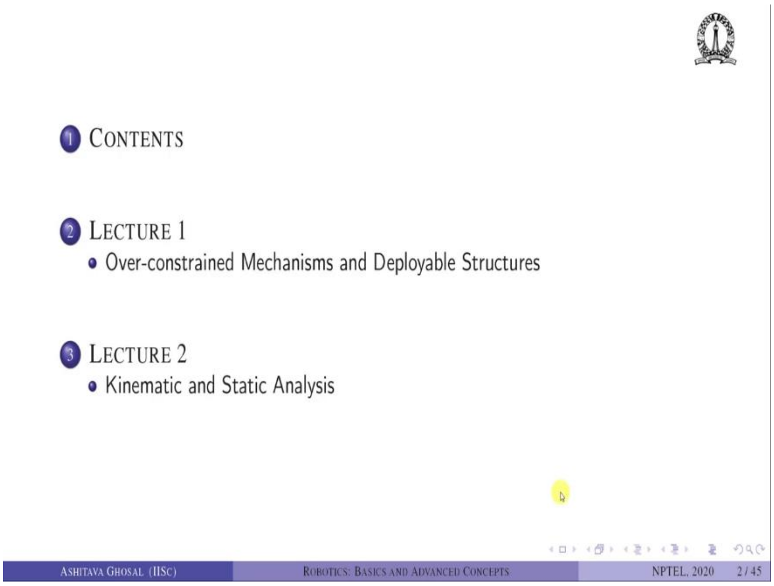

I will show you some examples and in the second lecture this week, we will look at Kinematic and Static Analysis of over constrained mechanisms and deployable structures ok. So, let us continue.

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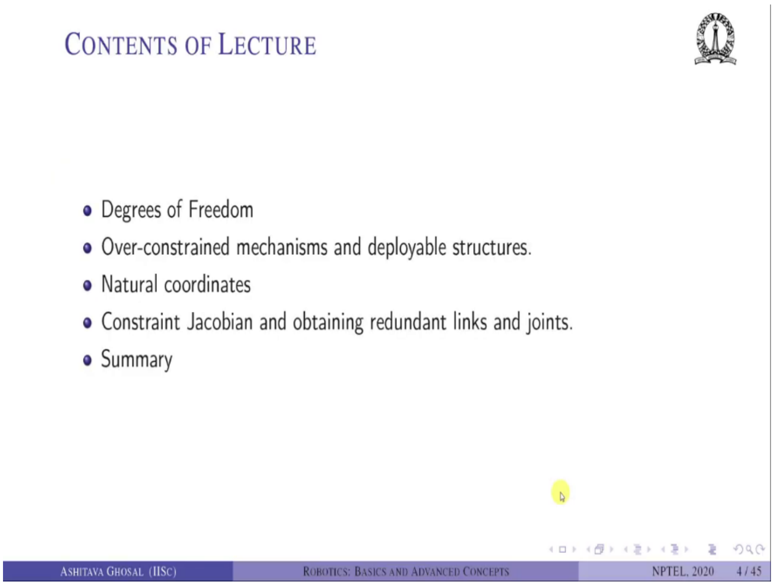

So, the contents of this lecture are we will go back and review this notion of degrees of freedom then we will look at over constrained mechanisms and deployable structures. Then I will show you how we can model and analyze over constrained mechanism, this consists of two parts first we will use a set of coordinates which are called natural coordinates.

Then we will obtain what is called as the constraint Jacobian matrix ok and then from the constraint Jacobian matrix we will find the null space of that constraint Jacobian matrix, which will tell us the actual degrees of freedom of that mechanism, over constrained mechanism and it also helps us by telling us which are the redundant links and joints in that mechanism ok.

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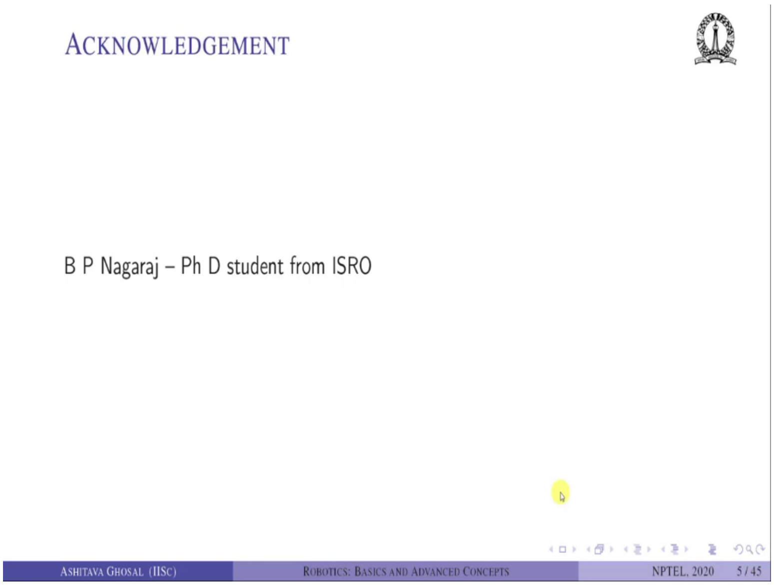

So, this work was done by my ex PhD student he is now in ISRO as a senior scientist.

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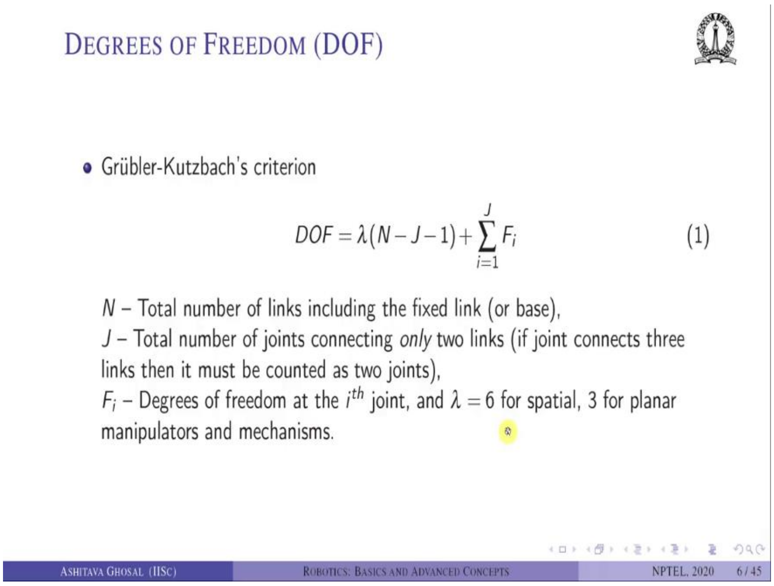

So let us continue. So, the degree of freedom of any mechanism is given by this Grubler-Kutzbach's criterion and it is given is very well-known formula by now and everybody knows it by now hopefully. So, we have $\lambda(N - J - 1)$ plus the sum of the degrees of freedom at the J joints.

So, N is the total number of links including the fixed base, J is the total number of joints connecting only two links. So, if the joint connects 3 links then it must be counted as 2 joints. And F_i is the degree of freedom of the i^{th} joint and λ is equal to 6 for spatial motion and 3 for planar motion and for planar manipulators and mechanisms ok.

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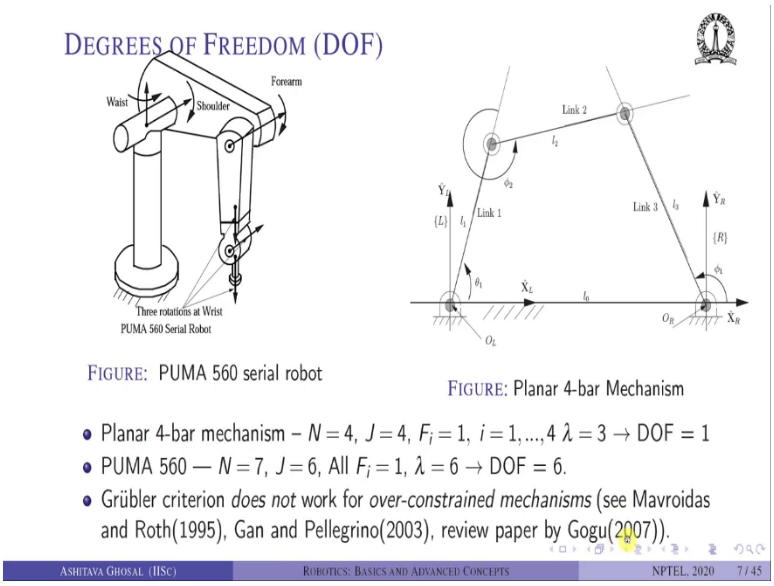

So, as an example this is a very well known PUMA 560 serial robot it consists of one joint here which is labeled as waist, then there is something called the shoulder, then there is the forearm and then there are these 3 rotations at the wrist there are 3 joints here ok.

So, if you substitute $N = 7$, $J = 6$, $F_i = 1$, and λ is equal to 6 we will see that the degree of freedom is 6. This is another very well known mechanism by now, this is a planar 4 bar mechanism, it has this fixed link first moving link second link third link ok. So, this is counted as one of the links in N.

So, $N = 4$, $J = 4$; there are 4 rotary joints 1, 2, 3 and 4 and each rotary joint the degree of freedom is 1 ok λ is 3 and if you substitute back in that formula Grubler Kutzbach's criteria we will get degree of freedom 1. It turns out that this Grubler criteria works for many many mechanisms, but it does not work always especially it does not work for what are called as over constrained mechanisms which we will see later.

Many researchers have worked on these over constrained mechanisms for example, Mavroidas and Roth, Gan and Pellegrino and there is a very nice review paper by Gogu in

2007 which gives all you know the reasons and how to tackle over constrained mechanisms.

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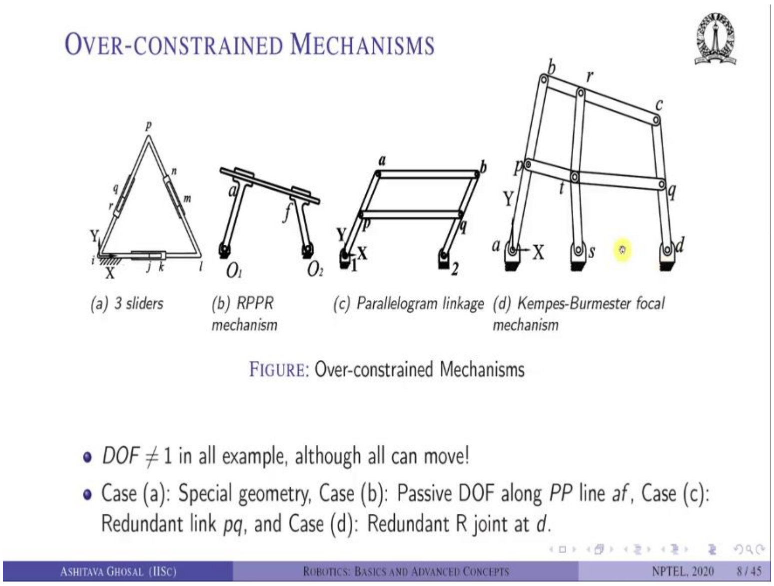

So, here is a set of examples which is an over constrained mechanism ok. So, the first over constrained mechanism is these 3 sliders. So, basically I have a fixed link here then there is one prismatic joint here and another prismatic joint here ok there is a another prismatic joint here.

So, this rq means basically these are the two different links which go into each other r and q similarly n and m are the two links which form the joints ok and j and k. So, we have 3 sliders if you substitute number of links and the number of joints which is 3 the sum of degrees of freedom is 3 and $\lambda = 3$ you will see that the degree of freedom is less than 1.

However, this mechanism moves because you can see if I pull this mechanism in this direction, this a link length will increase or this slider will change dimension, this will also move and this will also move to adjust that you are bringing it outwards ok. This is another very well known over constrained mechanism, we have a rotary joint here another rotary joint here, but then these two joints are prismatic joints ok.

So, it is an RPPR mechanism and as you can see this one has a very strange degree of freedom, this link can slide in between each other. However if I try to rotate it, it will not move ok. So, this is sometimes called as a passive degree of freedom because the configuration of the mechanism really does not change, but this link which is connecting these two prismatic joints can slide, this is another very famous linkage this is called as a parallelogram linkage.

So, basically what we have is a 4 bar linkage which is 1, a, b and 2, but let us assume we add another link which is exactly this length at this vary between in this point here as a hint joint another hint joint here. So, if you now calculate again the degree of freedom according to Grublers Kutzbach's criteria, you will see that the degree of freedom is less than 1 ok.

So, in this case there is a redundant link pq. So, first one there is a special geometry, this triangle is exactly equilateral triangle and then these lengths are chosen such that all of them are always looks like an equilateral triangle. This is another very famous mechanism which is called as a Kempes Burmester focal mechanism.

So, basically as you can see there are many links. So, there is one link there is another link there is another link there are these joints ok and then there is some special geometry of these quadrilaterals ok. And as a result as you move this link at this joint what you can see is that this link moves as if there were no d. So, the motion of this link is about d.

So, even if there is no joint at this place this link will trace an arc of a circle ok. So, that is one of the reason why it is called focal point because without this joint also if you remove this joint then this link will trace a curve a circular curve with the focus at d ok. If you apply the Grubler Kutzbach's criteria with this joint here then you will see that the degree of freedom is less than 1 ok. But nevertheless this linkage moves.

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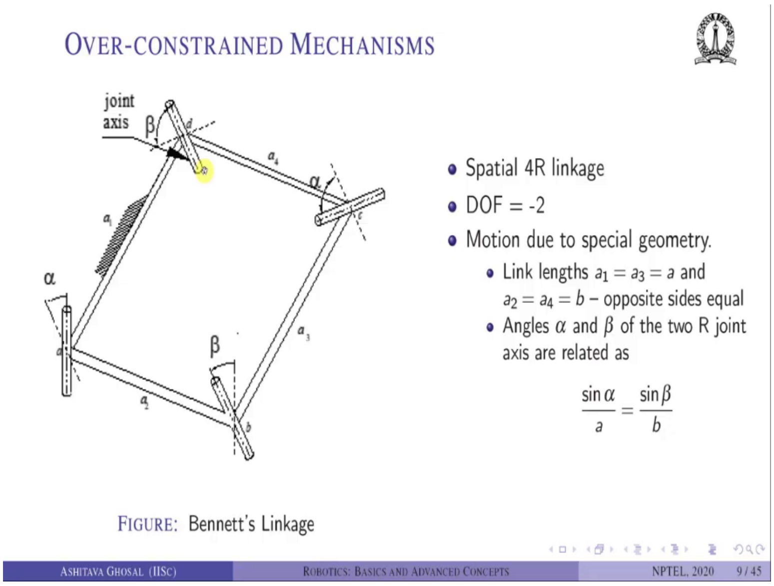

FIGURE: Bennett's Linkage

This is another very famous over constrained mechanism in 3D space, this is a spatial mechanism, this is called as the Bennett's linkage. So, basically there are 4 rotary joints ok these joint axis are in 3D space they are not coming out of this screen they are not all parallel. And there are only 4 links 1, 2, 3 and 4.

So, if you were to substitute $\lambda = 6$ the number of links as 4 and the number of joints as 4 each with one degree of freedom ok, then you will see that the degree of freedom is -2. However, there it does moves and that it moves simply because there is a special geometry of these linkages. So, this a_1 and a_3 the opposite sides are equal a_2 and a_4 are equal and these angles which these joints make.

So, this is β, this is β, this is α, this is also α and $\frac{\sin\alpha}{a} = \frac{\sin\beta}{b}$. So, where a is this a_1 and a_3 and b is a_2 and a_4. So, only because of this very special geometry very special arrangement of the joint axis it will move. And this has been very well known over constrained mechanism many researchers have worked on it and then they could figure out why it moves ok.

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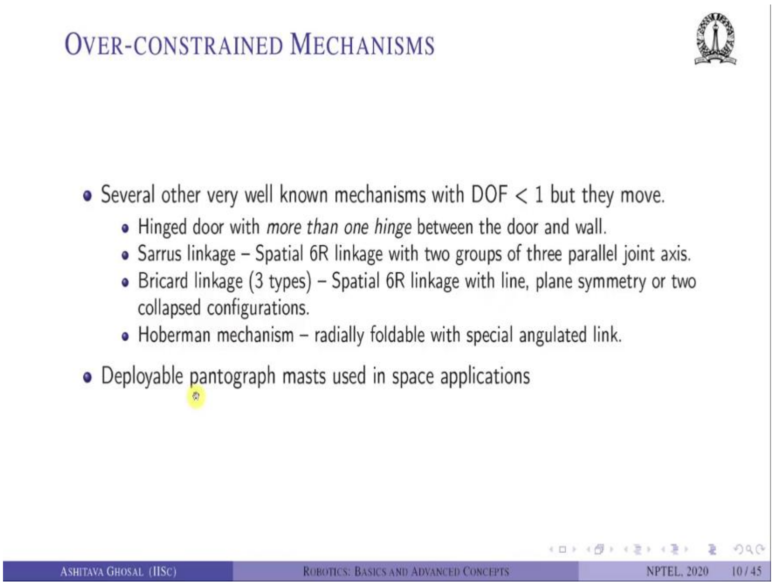

There are also many other over constrained mechanisms which have degree of freedom less than 1, but they all move. So, one obvious example is that of a hinged door. So, suppose you have the wall and there is a hinge and there is a door, but you connect thus door to the wall with more than 1 hinge ok.

So, again if you see if you use apply number of links and the number of joints in this case lets say 3, then the degree of freedom will be less than 1 ok. So, this is obvious standard example of an over constrained linkage there is also a very famous linkage called Sarrus linkage this is a spatial 6R linkage and it moves only because that are two groups of three parallel joint axis ok.

So, it does very interesting motion some linear motion of some point although the input is rotary. There is also a Bricard linkage there are 3 types these are also very well known over constrained mechanisms these are also spatial 6R linkages ok, again the degree of freedom is less than 1.

And they have this line plane symmetry or two collapsed configuration because of this symmetry or these collapsed configurations then that is the reason it moves and then there is also something called Hoberman mechanism ok. So, these are very well known mechanisms again if you go and type in Google what is a Hoberman mechanism you will see very nice videos of mechanisms where there it is radially increasing or decreasing ok.

And it happens because the link lengths are very special, these are angulated links of special dimensions ok. The last set of over constrained mechanisms are this pantograph masts and these are used extensively in space applications. So, I will look at very next slide, what are called as deployable pantograph masts?

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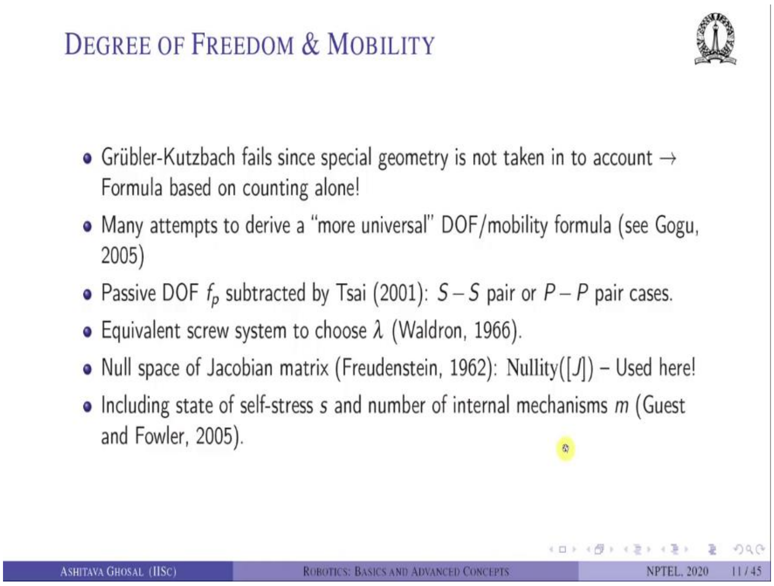

So, the Grubler Kutzbach criteria fails since special geometry is not taken into account the Grublers formula is just based on counting, you can see that is $\lambda(N-J-1)$. So, $N-1$ comes from the fact that there are $N-1$ moving links J is because J joints are adding some constraints. So, that is why $\lambda(N-J-1)$ is the total degrees of freedom minus the constraints which are joints J is adding.

And then is sum to this number the degrees of each freedom given by each joint ok. So, it is just a simple counting argument, it has nothing to do with the geometry of the mechanism. And hence many many attempts were made to derive a more universal DOF or mobility formula ok, you can see this paper by Gogu a list of such attempts are mentioned in this paper.

So, one well known attempt is that we can remove the passive degrees of freedom. So, for example, in the RPPR mechanism the PP joint the, link between the 2 prismatic joints. It is a passive degree of freedom it does not change the really the configuration of the mechanism. Similarly in a SS pair if in a mechanism there are 2 spherical joints one after another, then the link in between these two spherical joints can rotate freely without changing the configuration of the robot or the mechanism.

So, Tsai suggested that we should remove all these passive degrees of freedom or passive cases ok, Waldron earlier to that he said that we need to choose this λ properly ok. So, as I said λ is equal to 6 if it is moving in 3D space λ is equal to 3 if the motion is planar.

So, he suggested that we need to use a different value of λ depending on what are the actual degrees of freedom ok. So, if they are over constrained then λ should not be 6 or 3, the null space of the Jacobian matrix was suggested as a way to find out the actual degrees of freedom ok.

So, we find the Jacobian matrix and this was proposed by Freudenstein and we obtain the null space of the Jacobian and we will use this here ok. Of course, after 1962 many work has been done, but we are going to use a novel concept of using something called natural coordinates which makes obtaining the null space of the Jacobian matrix much simpler.

There were also attempts to include what are called state of self stress s and the number of internal mechanism. Internal mechanism is the end point and the base may be remaining fixed, but somewhere in between there is motion or there is no motion. So, those were suggested to be studied and somehow taken into account when we look at over constrained mechanisms.

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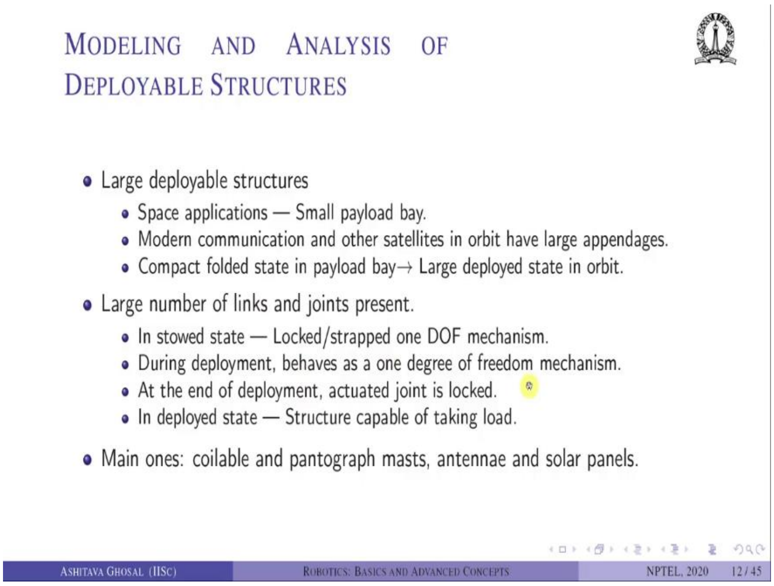

So, let us continue. So, as I said there are these masts or large deployable structures which are used in space applications ok. The main reason is in space applications the payload bay or where you will store your mechanism is very small ok. However, modern communication satellites and other satellites in orbit have very very large appendages.

So, you could have a big solar panel because you need a very big solar panel to capture or convert solar energy to electricity or you can have a big antenna, but you cannot put this huge you know say 5 meter antennae inside a space craft. So, it needs to be kept folded in a very small payload bay and once it is in orbit it is deployed ok.

So, as I was telling you there are many many appendages which are in compact folded state in the payload bay, but becomes very large when it is deployed in space, in orbit. Most of these structures or appendages have large number of links and joints ok.

So, in stowed state it is locked strapped and with one degree of freedom, during deployment it behaves as a 1 degree of freedom mechanism because you need to deploy it and make it bigger at the end of the deployment the actuated joint is locked.

So, whether there is a motor or something else it will become locked. And hence even before it was a structure because the degrees of freedom was less than 1, but once it is deployed it becomes a structure capable of taking load ok. So, the main ones are coilable and pantograph masts antennae and solar panels these are the some of the main appendages.

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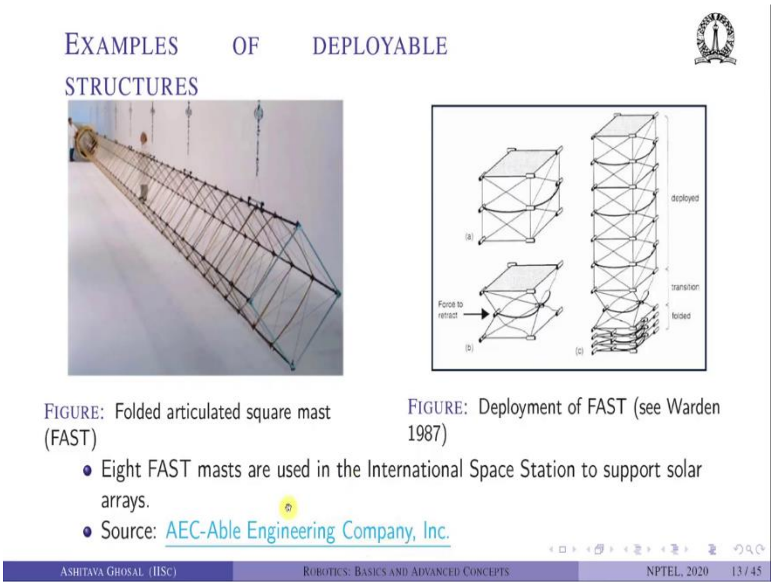

So, let me show you some pictures of deployable structures. So, one of the well known deployable structure is called FAST it is Folded Articulated Square mast. So, each section is a square there are the articulated joints and you have some kind of a cable or a arrangement which you can deploy these masts ok.

So, initially as you can see it is kept in a very compact folded manner then some in between you actuate some joints or you actuate one degree of freedom and it starts deploying and then at the end it would look like this long mast. So, these masts are used in many spacecrafts and many space stations.

So, for example, eight masts are used in the international space station to support solar arrays ok. So, these are made by many companies and one such company is this AEC Able Engineering Company.

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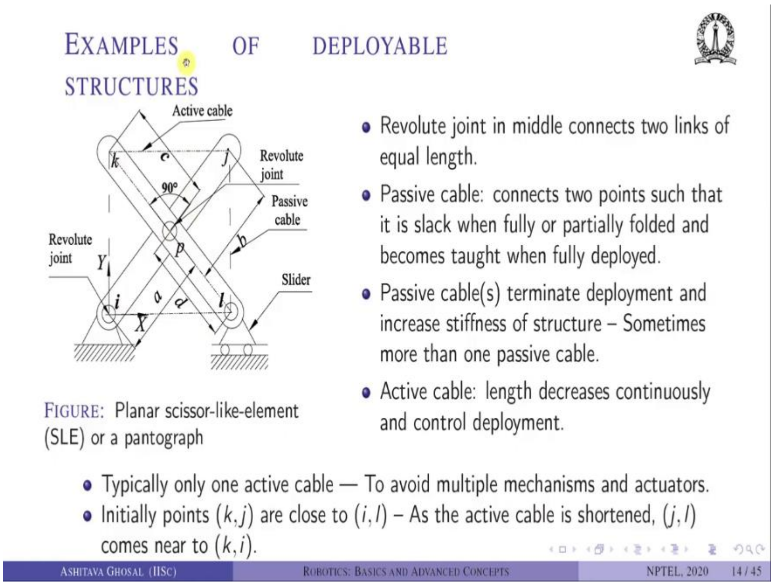

Let us look at some more examples of deployable structures. So, one of the common module in a deployable structure is this thing called planar scissor like element or a pantograph. So, basically what we have is we have a hinge joint here then we have another hinge joint here, but it is on a slider and then there are these two links and in between there is another pin joint ok.

So, what happens is there is a middle revolute joint it connects both these links which are of equal length, there is also a passive cable which connects two point such that it is slack when fully or partially folded and becomes taught when fully deployed ok. So, there is an active cable and then there are also passive cables these dotted lines horizontal and vertical dotted lines and this diagonal ones are the active cables.

So, the passive cables terminate deployment and increase stiffness of structures ok sometimes more than one passive cable is used, the active cable basically the length of this active cable decreases continuously and control deployment.

So, for example, you can have a motor which is sitting here and it will start pulling this cable. So, as you can see as it starts pulling this cable this point will go upwards this k and j will start coming closer to each other ok and this initially it might have been collapsed, but as you start pulling this cable this height of this structure will increase ok.

So, initially this points k and j are close to i and l. So, as the active cable is shortened j and l comes near to k and l. So, these points will go this way. So, this angle which is shown here 90 degrees will start decreasing and then this whole structure will go up. So, it will be going along the Y axis.

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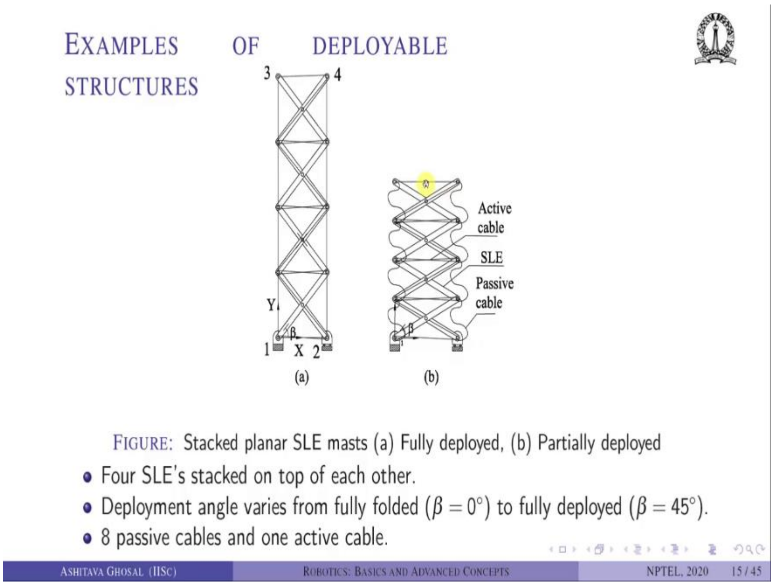

So, we can have several such SLE or Scissor Like Element masts one on top of other and then there is a cable which is running this is an active cable around the diagonal ok. So, what we want to do is as you pull this cable each of these marked points will start going towards each other and the length along the Y axis will keep on increasing.

So, as it has shown here the passive cables are not taught there is an active cable which is going like this like this and so on ok and after ending up here. So, when you start rotating and pulling this active cables either from here or from some other place. So, this points will come near to each other and the height will increase ok.

So, the deployment varies from this angle which is β initially for some value let us say very close to 0 and it will end up with 45 degrees ok, it cannot go more than 45 degrees. So, in this example there are 8 passive cables and one active cable the one active cable goes like this ok. So, this dark line is the active cable.

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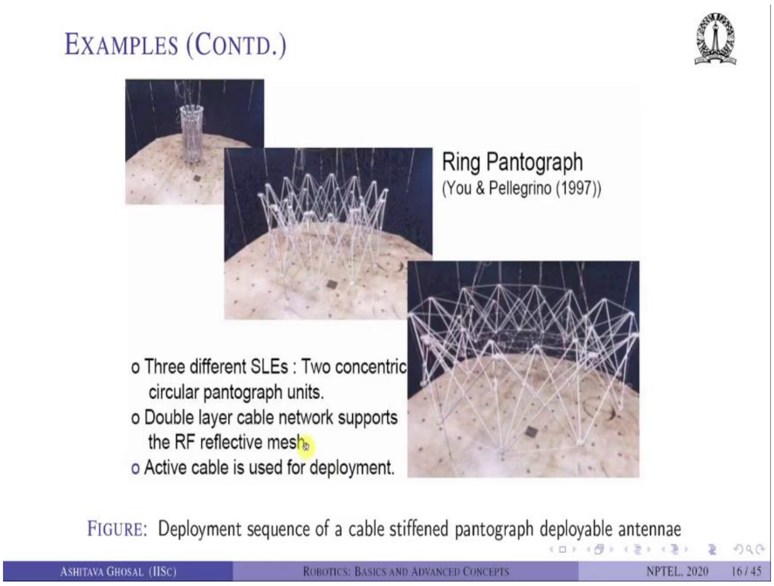

We can also organize this SLE structures SLE stands for Scissor Like Element in the form of a circle ok. So, we can have 3 different SLEs two concentric circular pantograph units ok. And as you actuate them as you can see initially it is very thin and collapsed and it looks like a tube and then as it actuates it becomes bigger and bigger and then finally, it looks like an antenna ok.

So, this can be used to make a deployable antenna ok, a cable stiffened deployable antennae because this passive cables can be made used to stiffen this whole mechanism after deployment ok. Somewhere inside here there will be a mesh which will reflect the incoming electromagnetic radiation ok. So, there will be a RF reflective mesh which basically gives the property or the function of an antennae.

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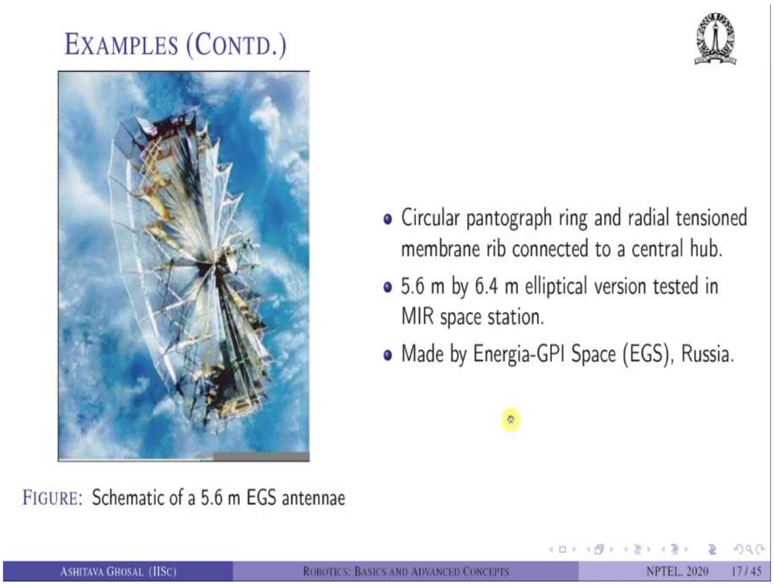

This is another one which is a well known example this is a 5.6 meter antennae ok. So, as you can see you cannot carry a 5.6 meter antennae on top of a spacecraft or a top of a rocket ok. So, it must be kept in a nice folded arrangement it goes up in space and then it is deployed ok.

So, this consists of circular pantograph ring and radial tensioned membrane rib connected to a central hub ok. We also need to make sure that this parabola or the surface which is generated after deployment is the right surface ok to act as an antennae otherwise it will not work.

So, there should be very very smooth the error between what you want and what is the deployed surface should be very small for the electromagnetic radiation and for the antennae to work ok. So, this is done by all these ribs and radial tensioned ribs and so on. So, this was a 5.6 meter × 6.4 meter elliptic version of this antennae was tested in MIR space station ok. So, this is made by a Russian company ok.

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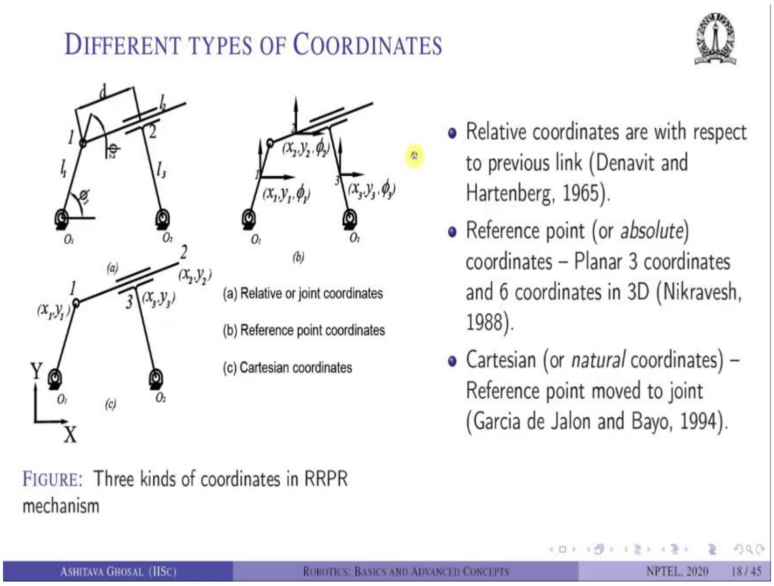

So, now, that we have seen several examples of over constrained mechanisms and also deployable mechanisms and structures let us see how we can analyze them ok. So, there are various ways to model these mechanisms and I am going to show you 3 different kinds of coordinates which are used to model a mechanism, or which can be used to find the configuration of a mechanism ok.

So, we had looked at what are called as joint coordinates ok. So, for example, in this mechanism which is RRPR ok, we can say that ϕ_2 is the rotation of the first joint ϕ_2 is the rotation of the second joint and this translation is given by d ok. So, we can have these 3 coordinates which specify the configuration of this RRPR mechanism ok.

So, of course, it is one degree of freedom, but then remember in the 4-bar mechanism we said there are 3 joint angles or 4 joint angles which give the configuration of this mechanism ok. So, maybe ϕ_1 is the single actuated degree of freedom, but we need these two to easily say this is the configuration of this RRPR mechanism.

So, we have what are called as relative or joint coordinates basically it is the rotation or translation at a joint relative to the previous link ok. So, these were actually invented by Denavit and Hartenberg in 65 we can also have what are called as a reference point coordinates ok.

So, the reference points coordinate consist of position and orientation of each link in the mechanism. So, in this case there is this is one moving link, this is second moving link, this is third moving link each of these links are moving in a plane. So, if I pick a point on this link in this point. So, I can say (x_1, y_1) is the location of this point and ϕ_1 is the orientation of that link ok.

So, in a plane I can specify the position and orientation of any link by 3 coordinates x and y coordinates, Cartesian coordinates and some orientation angle. So, these are for planar mechanisms there are 3 such coordinates, for spatial mechanism you need 6 coordinates you have seen this, any link in 3D space requires x, y, z and 3 orientation parameters ok. We can also have what are called as Cartesian coordinates or natural coordinates. So, these are reference points moved to the joint ok.

So, instead of this point at this middle of the link you move to the joint. So, hence in the case of Cartesian coordinates. So, we have one point which is at the end of this link which is at this joint. So, it just consists of (x_1, y_1) then we have another point which is (x_2, y_2) somewhere on this moving link and there is another point which is on this link which is (x_3, y_3).

So, as you can see if these 6 variables x_1, y_1, x_2, y_2, x_3, y_3 I can completely specify the mechanism I can draw this mechanism if you give me all these 6 points. Likewise using these coordinates also I can draw this mechanism ok. So, these are the 3 ways or 3 kinds of coordinates which can be used to show the configuration of this RRPR linkage.

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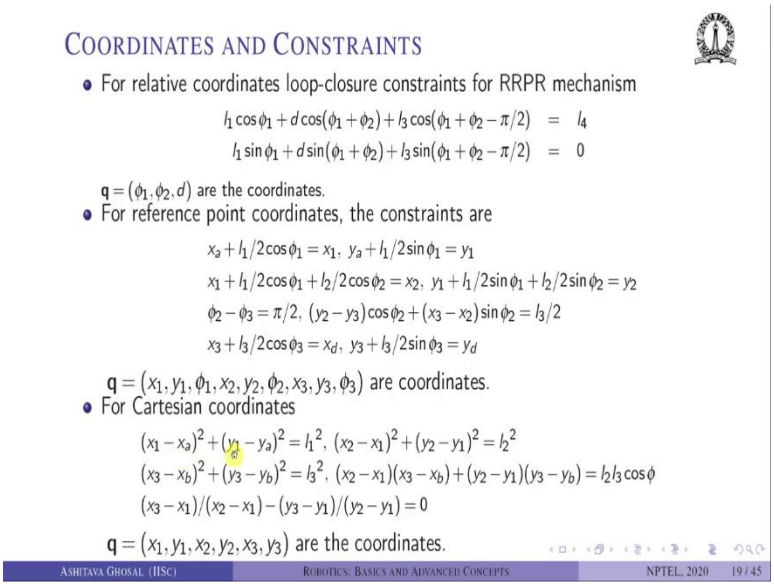

So, we need to look at the loop closure equations because not all these coordinates are independent. So, if you go back and substitute the number of links joints and everything in the Grublers Kutzbache's criteria, you will see that there is this all of these are one degree of freedom it is the same mechanism.

So, it has degree of freedom 1. So, in this case I need two constraints why because I am introducing ϕ_1, ϕ_2 and d. So, hence there must be 2 constraints because there is only 1 degree of freedom. In this case I have 6 parameters. So, I must have 5 constraints in this case I have introduced 9 parameters x, y and ϕ times 3 of them. So, there must be 8 constraints. So, we can derive these constraints.

So, for the relative point coordinates it is very easy or it is showing is that this vector, plus this vector, plus this vector should be equal to this vector ok. This standard loop closure constraint equations that we have used for 4 bar mechanism and that can be written as some $l_1 \cos\phi_1 + d\cos(\phi_1 + \phi_2) + l_3\cos(\phi_1 + \phi_2 - \pi/2) = l_4$.

And likewise the y component $l_1 \sin\phi_1 + d\sin(\phi_1 + \phi_2) + l_3(\phi_1 + \phi_2 - \pi/2) = 0$. So, we have 3 coordinates ϕ_1, ϕ_2 and d ok and we have 2 constraints and hence it has one degree of freedom consistent. So, if you have reference point coordinates, so we had 6 of them ok right.

So, you can see there were 9 of them. So, x_1, y_1, ϕ_1, x_2, y_2, ϕ_2, x_3, y_3 and ϕ_3 for Cartesians coordinates there were 6 of them. So, for this there must be 8 constraints, and these are the 8 constraints you can see that $x_a + \frac{l_1}{2}\cos\phi_1 = x_1$, $y_a + \frac{l_1}{2}\sin\phi_1 = y_1$.

Let us look at it just quickly one of them. So, we have x this and x_a is this point here. So, $\frac{l_1}{2}\cos\phi_1$ will be the coordinate of the first link x_a is the origin of the fixed coordinate system. Similarly, $y_a + \frac{l_1}{2}\sin\phi_1 = y_1$ ok and likewise we can show what is x_2, y_2, ϕ_2 and x_3, y_3 all of them are related to how they are related ok.

So, for example, $\phi_2 - \phi_3 = \pi/2$ you can see that. So, this is ϕ_2 which is the orientation of this link, ϕ_3 which is the orientation of this link. If you see they are 90 degrees apart because why this prismatic joint this angle is 90 degrees ok. So, hence there are 8 constraints 1, 2, 3, 4, 5, 6, 7 and 8. So, we have these 8 constraints, and the coordinates are 9 of them and hence it is one degree of freedom which is what is expected.

If you have Cartesian coordinates then you have 6 of these coordinates basically x_1, y_1, x_2, y_2, x_3, y_3 and x_a is the origin of the fixed coordinate system first one. So, you can see that the distance is given ok. So, $(x_1 - x_a)^2 + (y_1 + y_a)^2 = l_1^2$ that is obvious right. So, $(x_1 - x_a)^2 + (y_1 + y_a)^2 = l_1^2$, likewise this will be l_3^2 and likewise you will have some constraint involving the other points.

So, you can show that we have one constraint which is distance another one which is distance this is also a distance constraint this one involves again. So, now, these two constraints are slightly interesting. So, what it is telling you is this $(x_2 - x_1)(x_3 - x_b) + (y_2 - y_1)(y_3 - y_b) = l_2 l_3 \cos\phi$.

So, let us look at this constraint ok. So, we will come to this little later. So, basically what you can show here that this x. So, these 3 this point this point and some vector and this angle here is shown as is mentioned as $\cos\phi$, it will be related at this one ok.

So, we have 6 variables x_1, y_1, x_2, y_2, x_3, y_3 and then we can write these constraints which are 5 of them 1, 2, 3, 4 and 5 out of which 3 are length constraints and 2 are something to show that the vectors are in one plane ok. So, it has also one degree of freedom.

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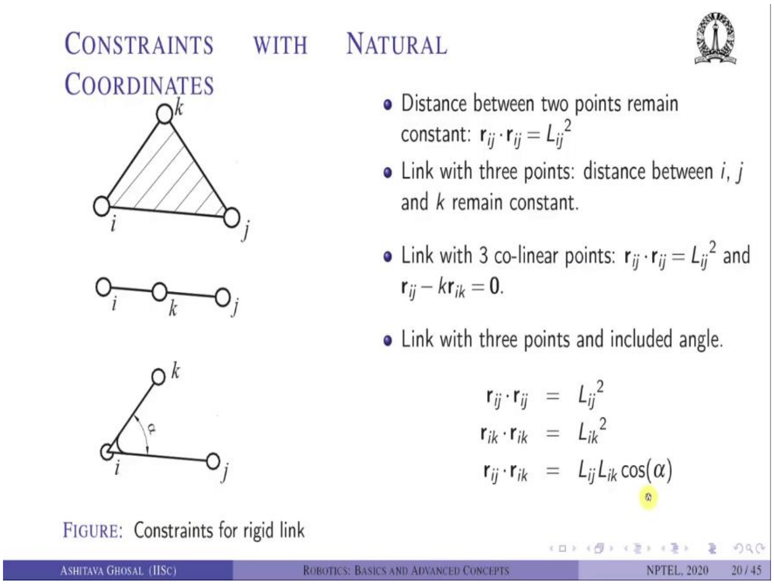

So, what are the constraints in general when we use natural coordinates? So, I have given you those equations, but where do they come from? So, basic first thing is the distance between two points remain constant. So, distance between i and j in a link or any other points in a link will remain constant.

So, $\boldsymbol{r}_{ij} \cdot \boldsymbol{r}_{ij} = L_{ij}^2$ if the 3 points are in along a line ok. So, then you have $\boldsymbol{r}_{ij} \cdot \boldsymbol{r}_{ij} = L_{ij}^2$ and also $\boldsymbol{r}_{ij} - k\boldsymbol{r}_{ik} = 0$, basically this vector from i to j is proportional to the vector from i to k ok. So, in this example there are 3 such constraints the link lengths between 3 points will be constant.

In this example these 3 points in a are in a line it is a special case of this. So, then there are that is one length constraint and one which says that this vector from i to j is equal to k times this vector from i to k. So, it is a fraction of that entire vector ok. If you have link with 3 points in an included angle.

So, I have a link with k, i and j and there is an included angle α. So, again the distance between i and k will remain constant between i and j will remain constant, but then the dot product of this vector i to k and i to j will be this quantity $L_{Ij} L_{ik} \cos\alpha$ which is nothing but the dot product formula.

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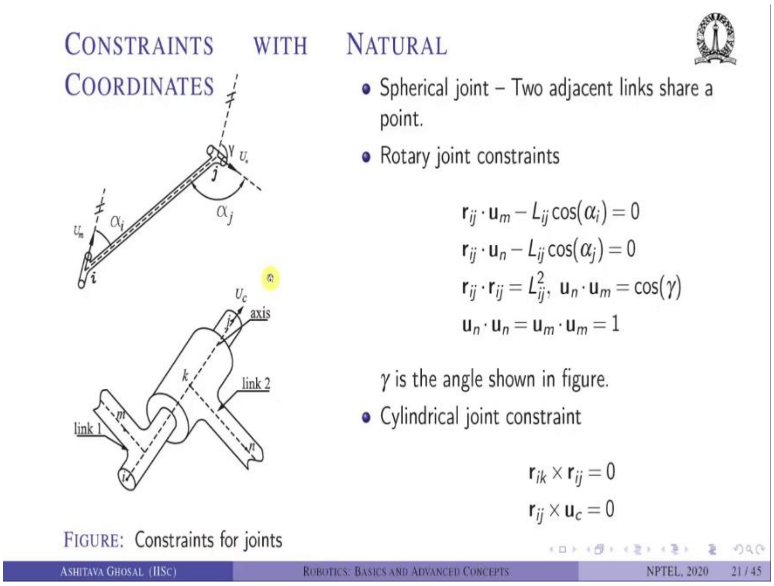

If you have a spherical joint two adjacent links share a point ok. So, the distance between two sides of the two adjacent links the distance is 0. So, they share one point if you have a rotary joint ok. So, I can write the rotary joint constraints in this form ok.

So, $\boldsymbol{r}_{ij} \cdot \boldsymbol{u}_m$. So, this is the link on both sides there are 2 rotary joints. So, $\boldsymbol{r}_{ij} \cdot \boldsymbol{u}_m$, $\boldsymbol{u}_m$ is this vector along the joint axis minus $L_{ij} \cos \alpha_i$ should be equal to 0 ok is that true? Yes, because $\boldsymbol{r}_{ij} \cdot \boldsymbol{u}_m$ will be equal to some length times cos of this angle.

Similarly, $\boldsymbol{r}_{ij}$ this vector $\boldsymbol{r}_{ij} \cdot \boldsymbol{u}_n$ product if these other joint axis will be equal to $L_{ij} \cos \alpha_j$ again standard formula for dot product. The distance between i and j should remain constant which is this $\boldsymbol{r}_{ij} \cdot \boldsymbol{r}_{ij} = L_{ij}^2$ then there is a constraint which tells how these two joint axis are oriented.

So, $\boldsymbol{u}_n \cdot \boldsymbol{u}_m$ unit vectors along the joint axis is $\cos \gamma$. So, that is the γ angle. So, this is that joint axis which is translated here and then this is the second joint and the angle between that is γ and finally, we have these two axis $\boldsymbol{u}_n$ and $\boldsymbol{u}_m$ which are unit vectors ok.

So, this is a constraint due to a rotary joint, if you have a cylindric joint which basically means that this link 2 can slide with respect to link 1 and also rotate ok you can see that $\boldsymbol{r}_{ik}$. So, i and k this is a vector from here to here and $\boldsymbol{r}_{ij}$ this is a vector from here to the other end of this get link cross product of both of them should be 0 because they are all on the same direction.

Likewise, $\boldsymbol{r}_{ij}$ from here to here and this $\boldsymbol{u}_c$ which is the joint axis of the cylindric joint that again the cross product of that should be equal to 0. So, we can derive these constraints for a rotary joint for a spherical joint ok for a link for a cylindric joint and so on.

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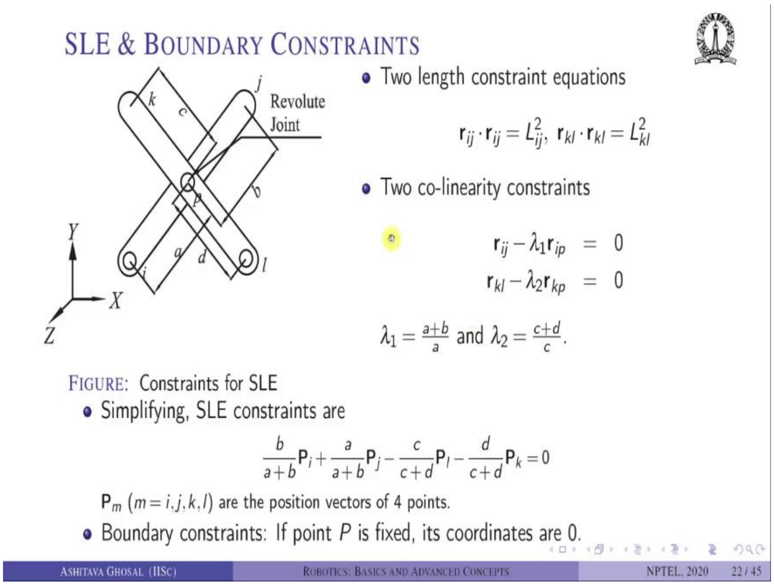

We can also derive the constraints for a SLE mechanism ok, so SLE remember consist of one link another link it is pinned here at one place and there are these two pins. So, this is the you know scissor like element arrangement. So, if I pull these two together when i goes towards l this other hinge will go up and then this whole thing will go up and down ok. So, which is what was happening when you had this SLE based masts.

So, in the case of an SLE we can write the constraints in the following form. So, we have one joint i, l, this distance is a, this distance is d, this distance is b, this distance is c and we have another point k and another point j and this is another revolute joint. So, you can see $\boldsymbol{r}_{ij} \cdot \boldsymbol{r}_{ij}$, so i to j this is single link.

So, the link length will remain constant. Similarly this link length will remain constant between k and l is L_{lk}^2, also this point this point and this point are lying on a straight line always. So, $\boldsymbol{r}_{ij} = \lambda_1 \boldsymbol{r}_{ip}$ and $\boldsymbol{r}_{kl} = \lambda_2 \boldsymbol{r}_{kp}$ this is 3 co-linear points.

And what is λ_1 and λ_2? That is $\lambda_1 = \frac{a+b}{a}$ and $\lambda_2 = \frac{c+d}{c}$. So, I can simplify all these constraints in a single vector equation which is of this form which is $\frac{b}{a+b}\boldsymbol{P}_i$, $\boldsymbol{P}_i$ is the location of this point with respect to the fixed reference coordinate system plus $\frac{a}{a+b}\boldsymbol{P}_j - \frac{c}{c+d}\boldsymbol{P}_l - \frac{d}{c+d}\boldsymbol{P}_k$.

So, if we think about it this is the simplified form of all these constraints ok. How about boundary constraints, we will see later we need to also impose what are called as boundary constraints. So, if any point is fixed its coordinates are 0. So, the (x, y) coordinate of a fixed point will be set to 0.

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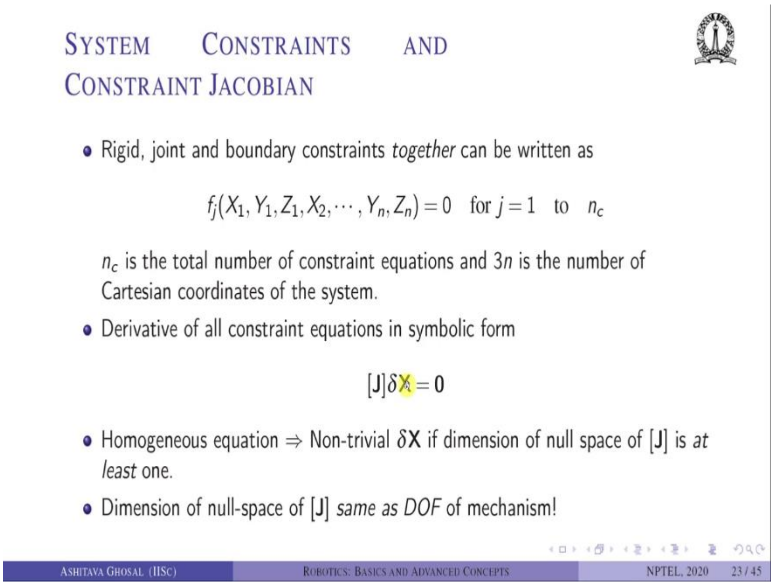

So, if you give me any mechanism we can have this link constraint, which is also called rigid constraint we can have this joint constraints ok. Rotary joints, spherical joint, SLE joint, SLE structure and we can have all these boundary constraints and we can put all of them together into a set of equations.

So, there are n_c equations where n_c is the number of constraints which now we will include all the coordinates alone ok X_1, Y_1, Z_1, X_2, Y_2, Z_2 because in the Cartesian coordinates only the x, y, z coordinates show up no angles will show up ok. So, that is clear right.

So, if I pick n points then $3n$ is the number of Cartesian coordinates of the system and then there will be n_c constraints out of this $3n$ Cartesian coordinates. So, what we can do is we can take the derivative of these n_c equations and write in a symbolic form which is

$[J]\delta X = 0$. So, δX means δX_1, δY_1, δZ_1 and so on ok and $[J]$ is the partial derivatives of these functions f_j with respect to each one of these variables X_1, X_2, X_3 and so on ok.

So, this is a homogeneous equation it is of the form $[A]X = 0$. So, this will have any non trivial solution only if the null space of this $[J]$ matrix ok is at least 1, correct? So, if this determinant of this matrix is not 0 then δX will be 0 whereas, if determinant of $[J]$ is 0 then we can have a non trivial this δX.

So, the non trivial δX if dimension of the null space of $[J]$ is at least 1 ok. So, it turns out that the dimension of the null space of $[J]$ is same as a degree of freedom of the mechanism this is Freudenstein. So, what he said was if δX is a nontrivial; that means, there is a possibility of some motion change of coordinates and that can only happen if the null space of $[J]$ is at least of dimension 1 ok.

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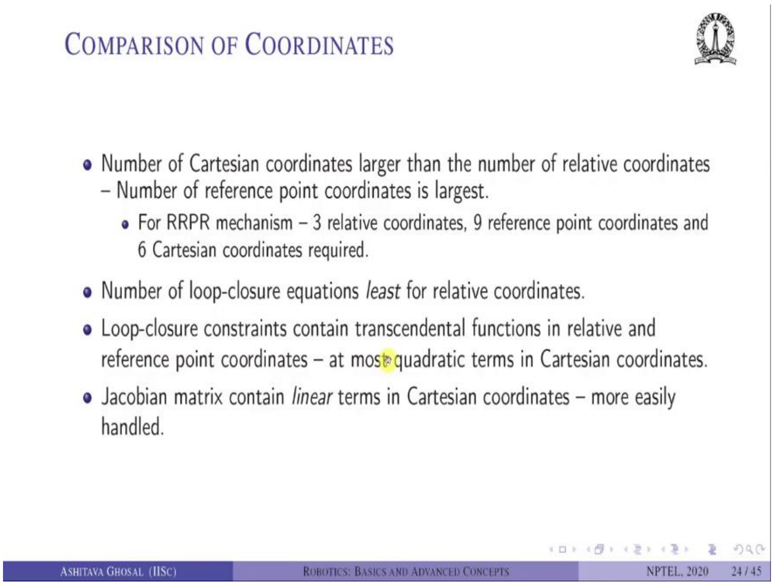

Well, quickly let us take a look at the different kinds of coordinates and their comparison. So, as I said the number of Cartesian coordinates is larger than the number of relative coordinates, why? Because the angles are you know in relative coordinates you have only relative degrees of freedom.

The number of reference point coordinates is the largest because reference points coordinates as x, y and ϕ whereas Cartesian coordinates has only x and y whereas, relative coordinates have just some variable you know either ϕ or d. So, for the RRPR mechanism there are 3 relative coordinates, 9 reference point coordinates and 6 Cartesian joint coordinates are required ok.

The number of loop closure equation is least for relative coordinates as I showed you there were 3 relative coordinates one degree of freedom. So, there were only 2 in the second case in the Cartesian case there were 6 Cartesian coordinates the degree of freedom is still 1 hence there were 5 constraint equations.

In reference point coordinates we had 9 variables or 9 coordinates and there were 8 constraints. So, the number of loop closure equation is least for relative coordinates. However, the loop closure constraints contain transcendental functions of the relative and reference point coordinates we saw there were cosine and sine of ϕ.

In the Cartesian coordinates there were only quadratic, at most quadratic terms, there are no transcendental functions. And as a result the Jacobian matrix which is the partial derivatives of each of these constraints with respect to the coordinates is at most is linear ok, when you derive the Jacobian matrix using Cartesian coordinates which is linear.

The number of rows in the Jacobian matrix will be more than the number of rows when you are using relative coordinates, but each row is very simple each element is very simple in that row.

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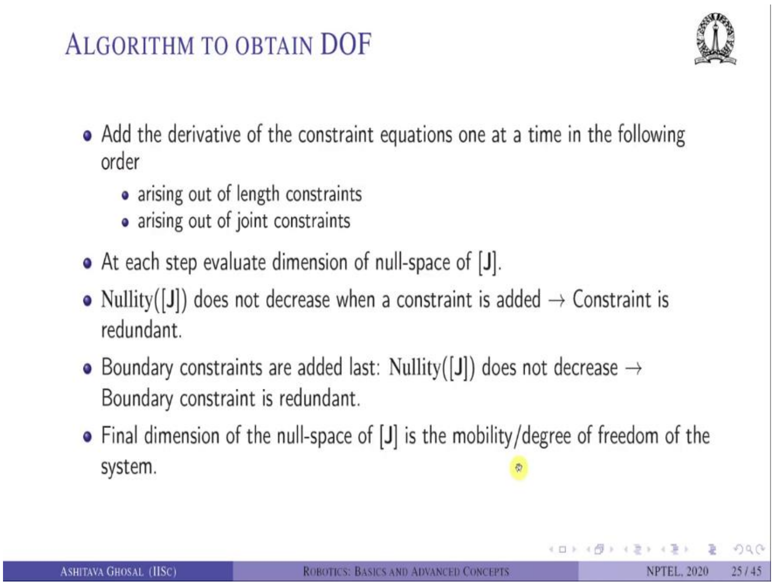

So, let us go back and see how we can obtain the degree of freedom of a mechanism with constraint equations. So, we add the derivative of the constraint equations one at a time in the following order ok. So, you first add the constraint equation arising from length constraints then arising from joint constraint at each step evaluate the dimension of the null space of the Jacobian matrix ok. So, either numerically or symbolically.

The null space of the Jacobian if it does not change if it does not decrease when a constraint is added the constraint is redundant ok, so think about it. So, I have added 2 constraint I add a third constraint, but the null space did not change. So, hence that constraint is redundant, the boundary constraints are added last and if the null space of the Jacobian does not decrease when you add the boundary constraint that particular boundary constraint is redundant ok.

So, boundary constraint means where it is fixed. The final dimension of the null space is the mobility or degree of freedom of the system ok this follows from the theory $[J]\delta X = 0$. So, as long as the null space dimension of $[J]$ after all the constraints have been added, whatever is left if the nullity is 1 the degree of freedom is 1.

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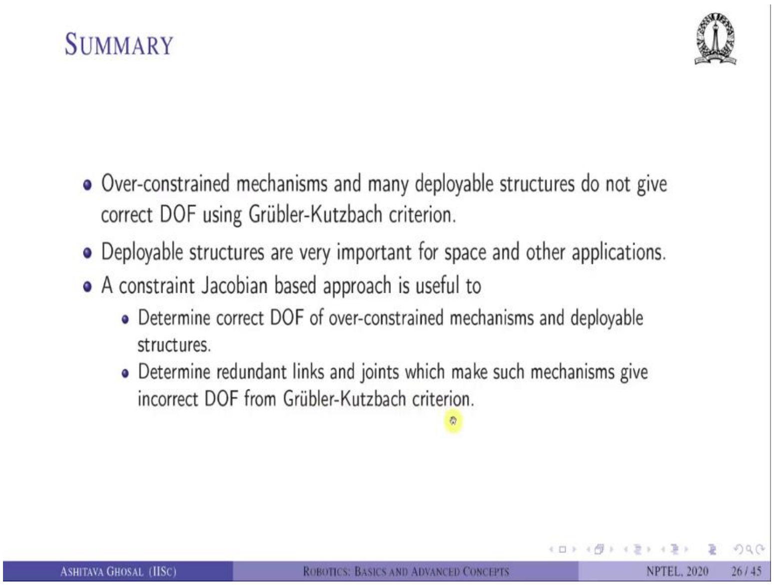

So in summary over constraint mechanisms and many deployable structures do not give correct DOF using Grubler's Kutzbach criteria ok. And the main reason is Grubler Kutzbach criteria is just a counting argument it does not take into account the special geometry or the link lengths of the mechanism.

Deployable structures are very important for space and other applications ok and I have showed you a constraint Jacobian based approach is useful to determine the correct degree of freedom of over constraint mechanisms and deployable structures. And I will show you that it can be used to determine the redundant links and joints, which make such mechanisms gives incorrect degree of freedom from Grubler Kutzbach criteria.

So, basically as I said when we had a constraint and if the nullity does not change that is redundant. So, you can relate it to the joint or the link from where the constraint came ok. So, with that we will stop in the next lecture I will look at Kinematics and Static Analysis of over constrained mechanisms.