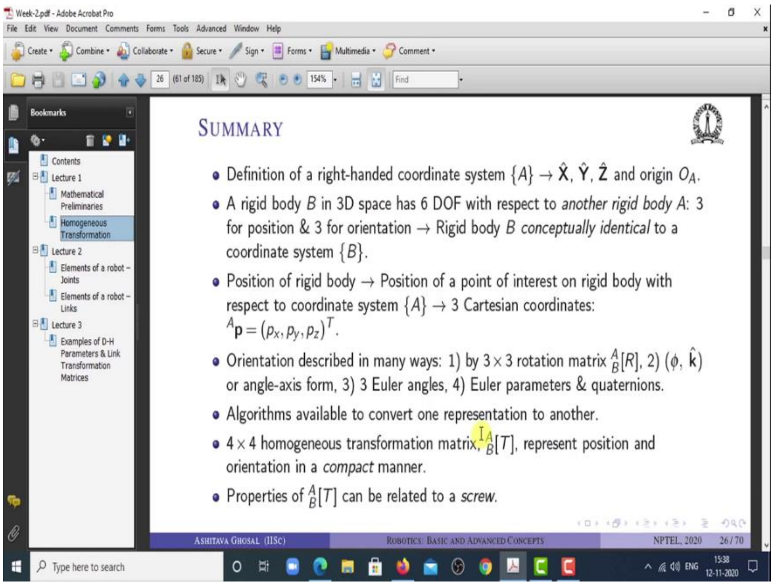
Robotics: Basics and Selected Advanced Concepts
Prof. Ashitava Ghosal
Department of Mechanical Engineering
Centre for Product Design and Manufacturing (CPDM)
Indian Institute of Science, Bengaluru

Lecture - 05
Elements of robot - Joints, Elements of robots – Links

(Refer Slide Time: 00:19)



Welcome to this NPTEL lectures on Robotics: Basics and Advanced Concepts. So, in the last lecture we had looked at mathematical preliminaries and an important concept called homogeneous transformation matrices. So, in the mathematical preliminaries basically we looked at how to represent a rigid body in 3D space.

So, first important thing which we need to do to represent a rigid body in 3D space is to have a reference coordinate system. We are going to use a right-handed coordinate system; we label these coordinate system as {A}, {B} and so on. Each coordinate system is defined by means of 3 coordinate axes an X axis a Y axis and a Z axis and an origin O_A.

A rigid body in 3D space has 6 degrees of freedom with respect to another rigid body A. So, these 6 degrees of freedom are 3 for a position vector - position of a point on the rigid body and 3 for orientation. So, in a sense the rigid body B is conceptually identical to a coordinate system {B}. At this stage we are not really interested in the shape and size of the rigid body.

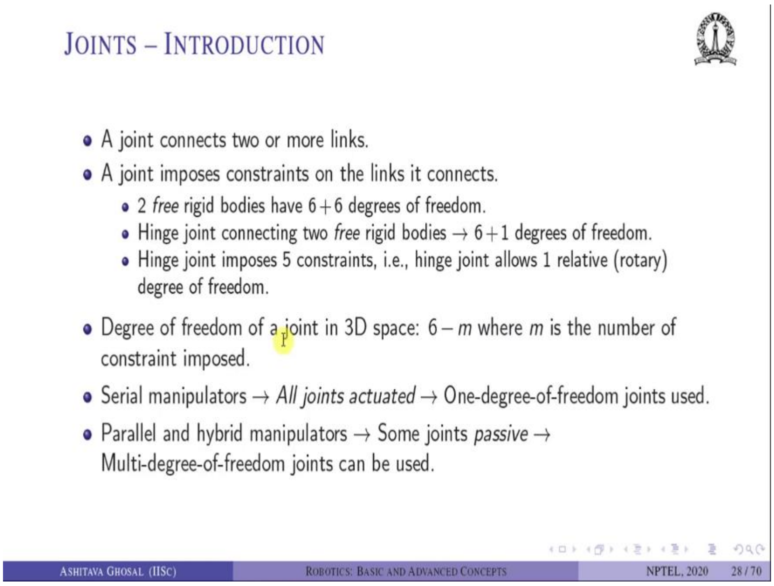
The position of a rigid body is one of the important concepts - the position of a rigid body is basically the position of a point of interest on the rigid body with respect to the coordinate system {A}. We are going to use 3 Cartesian coordinates p_x, p_y, p_z. One more way of looking at it is we pick a point on the rigid body and from the origin of the reference coordinate system we draw a vector ${}^{A}p$ - A denotes that it is with respect to the {A} coordinate system and this is the column vector with 3 components p_x, p_y, p_z.

The orientation of a rigid body can be described in many ways, we looked at a 3 by 3 rotation matrix, we also looked at an angle and axis form ϕ and k, and then we looked at Euler angles - 3 minimal- 3 Euler angles have basically 3 angles; and that is the minimal representation of orientation. And we also looked at what are called as Euler parameters and quaternion's. So, the 3 by 3 rotation matrix had 9 quantities and 6 constraints, the angle axis form has 4 quantities and 1 constraint whereas, the Euler angles are minimal. And I showed you that we can derive algorithms to convert one representation of orientation to another representation.

When we had a 4 by 4 homogeneous transformation matrix ${}_{B}^{A}[T]$ it represented position and orientation in a compact manner ok. The top 3 by 3 represented the rotation part and the last column represent the translation part and the bottom row is 0 0 0 1. And I showed you that these properties of ${}_{B}^{A}[T]$ in some sense relate to the general motion of a rigid body, which is nothing but rotation and translation along a line in 3D space. It is similar to a screw motion.

So, as I said this is a summary of the last lecture. Now, let us look at this lecture. In this lecture we will look at the elements of a robot. There are two main elements of a robot - joints and links. So, we will start with elements of a robot and again the idea is that we would like to represent joints and links in a mathematical form, such that we can do analysis with them later on.

(Refer Slide Time: 04:18)



So, briefly introduction to joints, basically a joint connects two or more links. I have two links in or more links in 3D space, I can connect them using joints. A joint imposes constraint from the link it connects. So, for example, if you have 2 rigid bodies in 3D space, if they are free then have 6 + 6 degrees of freedom.

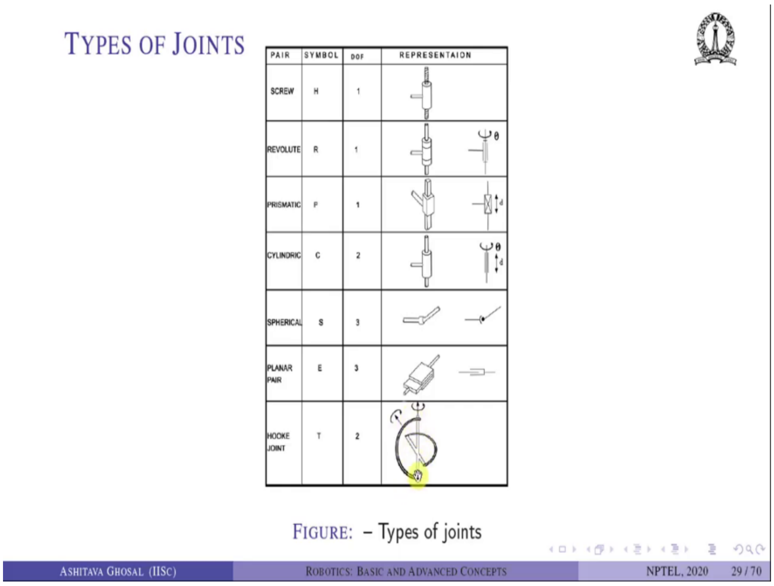
So, each rigid body is position plus orientation which is 6 and 2 of them means 12. If I now connect these two rigid bodies using a hinge joint, then the second rigid body can only rotate with respect to the first rigid body. So, the total system degrees of freedom are now 7, 6 + 1.

So, another way of looking at it is that the hinge joint imposes 5 constraints, or a hinge joint allows only one relative degree of freedom. So, the degree of freedom of a joint in 3D space in general can be written as 6 - m, where m is the number of constraints imposed.

In a serial manipulator all joints are actuated most of the time and as a result only one degree of freedom joints are used. Because it is sort of difficult to put more than one actuator at one place to implement a multi degree of freedom joint, we do not use multi degree of freedom joints in a serial manipulator.

However, in a parallel and hybrid manipulator some of the joints are passive. We will see this later in as we go along in this course. That we can have passive joints in a parallel manipulator and since these joints are passive and hence we do not need to put actuators, we can easily use multi degree of freedom joints. So, we will see later that a parallel manipulator a spherical joint is used or even a hook joint is used ok.

(Refer Slide Time: 06:22)



So, continuing what are some of the main types of joints. Symbolically we have these joints screw is denoted by an H, revolute joined by R, prismatic by P, cylindric by C, spherical by S, planar by E and hook joint by T ok. A screw joint has 1 degree of freedom. So, what is the 1 degree of freedom?

Think of this white portion has one rigid body and the hatch portion has another rigid body. The white portion can rotate about this hatch portion and also translate up. But the rotation and translations are related by the pitch of the screw. Whereas, in a revolute joint also sometimes called as a rotary joint.

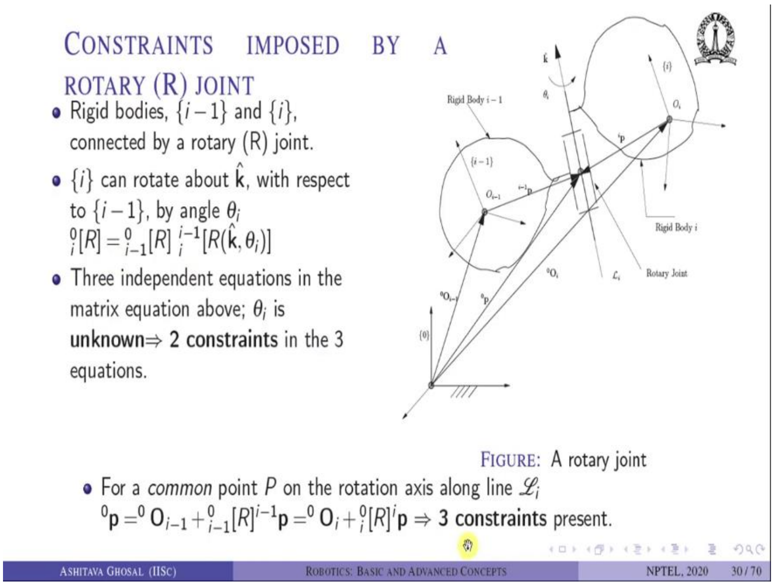
These are the 2 rigid bodies one is at the centre and one along these lines or axis; and symbolically shown as one rod here and another axis here. The rigid body here can rotate with respect to the other rigid body by an angle θ, it cannot translate. A prismatic joint on the other hand the 2 rigid bodies shown here and here they can translate with respect to each other, and the variable used in d.

So, rotary joint is typical variable used is θ in a prismatic joint typical variable used is d. We can also have a cylindric joint, which is in some sense a generalization of a screw joint, where the θ and d are not related they are 2 independent. A spherical joint has 3 degrees of freedom - a spherical joint is also sometimes called as a ball and circuit joint.

So, basically, I have one link here and another link here, the second link can rotate with respect to the first link about 3 different axes. So, it has 3 degrees of freedom. The hook joint is sometimes the symbol used is T sometimes U; sometimes U it is 2 degrees of freedom ok.

So, U comes from universal. So, basically, we have one rigid body connected to this one ring and the other rigid body connected to the second ring and in between there are these two bars which sort of connect each other. And what you can see is this both of these rigid bodies can have 2 rotations about 2 different axes, one along this and one along this vertical. So, hence it is 2 degrees of freedom.

(Refer Slide Time: 09:02)



As I said one of the important thing is a joint imposes constraint when you connect two rigid bodies by a joint. So, let us look at the constraint imposed by a rotary joint. So, in this figure what we have is a rigid body *i* and a rigid body *i - 1*, it is connected by a rotary joint. So, the rotary joint is represented by an axis or a line along which it rotates. So, this is labelled as *k* and it can rotate by an angle θ_i.

The rigid body is as usual represented using a coordinate system X_{i-1} Y_{i-1} Z_{i-1}. If the rigid body is labelled *i - 1* ok. And there is an origin O_{i-1}. The next rigid body, the *i* th rigid body also has 3 axis X_i, Y_i, Z_i and an origin O_i. The origin of the i th coordinate system can be represented by a vector O_i with respect to $\{0\}$ coordinate system. The origin of the $i - 1$ th coordinate system or the $i - 1$ th rigid body can be represented by a vector O_{i-1}.

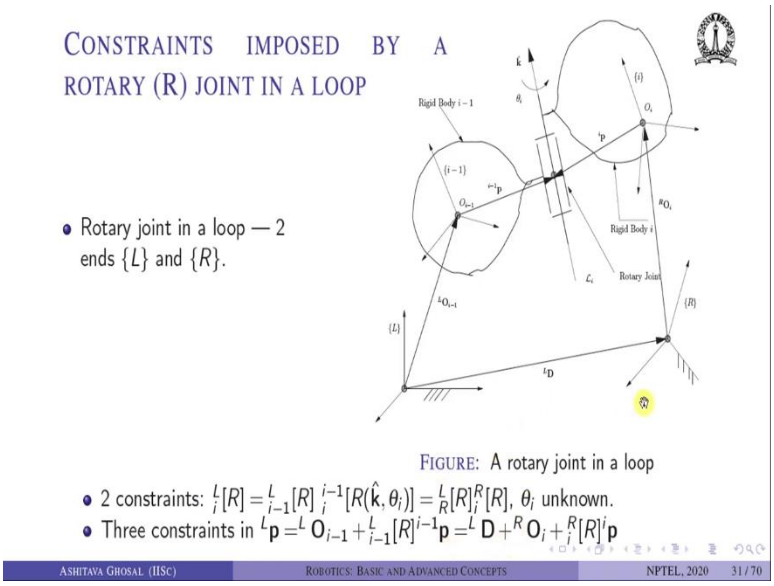
Now, let us pick a point on the joint axis ok. So, I can find out this point from the reference coordinate system, the zeroth coordinate system, by a vector ${}^{0}p$ ok. Whereas, the same point can be obtained by going from the origin of the reference coordinate system to the origin of the i – 1 th rigid body and a vector ${}^{i-1}p$. Likewise, I can reach the same point first by going to the origin of the i th rigid body and then a vector from there to that point on the joint axis. So, this is the story. So, we want to know, what are the 5 constraints which are imposed by this revolute joint when connecting 2 rigid bodies $i - 1$ and i.

So, first thing that we can see is this rigid body i can rotate about k with respect to $i - 1$. So, hence the rotation matrix which describes the orientation of rigid body i can be obtained from the rotation matrix of rigid body $i - 1$ by simply pre multiplying or post multiplying this is ${}_{i-1}^{0}[R]$ with the rotation about k by θ_i. So, this relationship is straightforward because the revolute joint allows only one rotation about this k axis.

Now, this is a matrix equation; however, it is a matrix equation - on both sides of this matrix equation we have rotation matrices. So, there are only 3 independent equations ok. Rotation matrix has 3 independent parameters which is what we saw in the first lecture. θ_i in this matrix equation is unknown. So, hence there are only two constraints; so, if I relate these 3 independent equations parameters on each side, but one of those equations will contain θ_i which is an unknown. So, there are basically 2 constraints in these 3 independent equations.

Likewise, this point on the joint axis, as I said, can be reached from 0 origin of the reference to the origin of the $i - 1$ rigid body and then this vector. And likewise, this origin of the i th rigid body and this vector and then both of them must be equal. So, this is a vector equation ${}^{0}p$ is $O_{i-1} + {}_{0}^{i}[R]\,{}^{i-1}p$ and so on ok. Why do we need this rotation matrix? Because if you want to add these two vectors O_{i-1} and ${}^{i-1}p$, we need to write it in the same coordinate system. So, this is a vector equation with 3 constraints. So, how many constraints we get? We get 2 constraints from the matrix equation and 3 constraints from the vector equation, so hence we have 5 constraints imposed by this rotary joint.

(Refer Slide Time: 13:31)

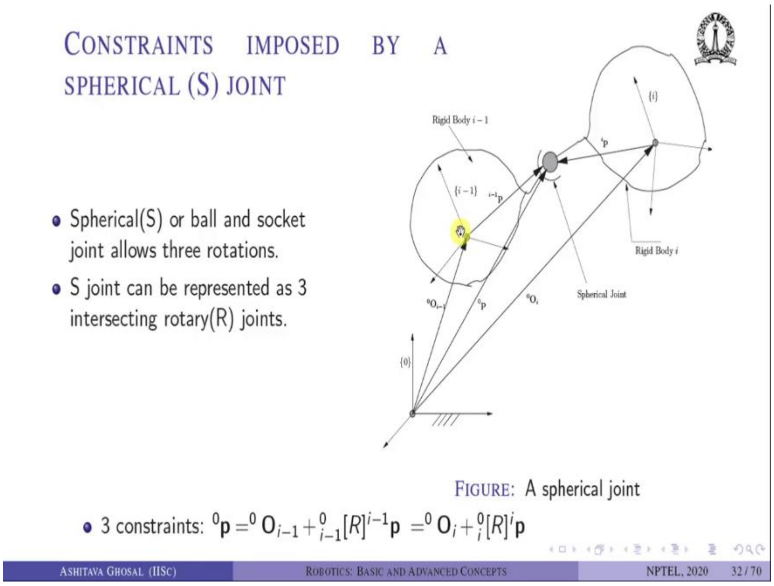


Same story happens when you have a rotary joint in a loop. So, for example, if you have a 4 bar mechanism and we have a rotary joint connecting 2 links in that loop or any other mechanism or a closed loop mechanism with rotary joint. So, I can again show you, that I can reach this point on the joint axis either by going from a reference coordinate system, in this case the left coordinate system let us say, and the {R}. We can go from ${}^L O_{i-1} + {}^{i-1}p$, likewise I can also reach this point by going from the left fixed coordinate system to the right fixed coordinate system because in a loop - there will be more than one coordinate fixed coordinate system - and then we can go from {R} to *i* th and then ${}^i p$.

So, from the origin of the *i* th coordinate system to the point on the joint axis. So, we have now three constraints which has ${}^L O_{i-1} + {}^{i-1}\underline{p} + {}^L D$ plus this is equal to this. So, again we have a vector equation with three constraints. Likewise as in the first case we have a rotation matrix of this *i* th rigid body can be related to the rotation matrix of the $i - 1$ th rigid body by simply post multiplying ${}^{L}_{i-1}[R]^{i-1}_{i}[R]$ again a rotation by *k*, θ.

So, this is also a matrix equation, but with 2 constraints because θ_i is unknown. So, if the rotary joint is in a loop or just 2 rigid bodies in space, we always get 5 constraint equations - 5 constraints. So, hence the rotary joint has one degree of freedom.

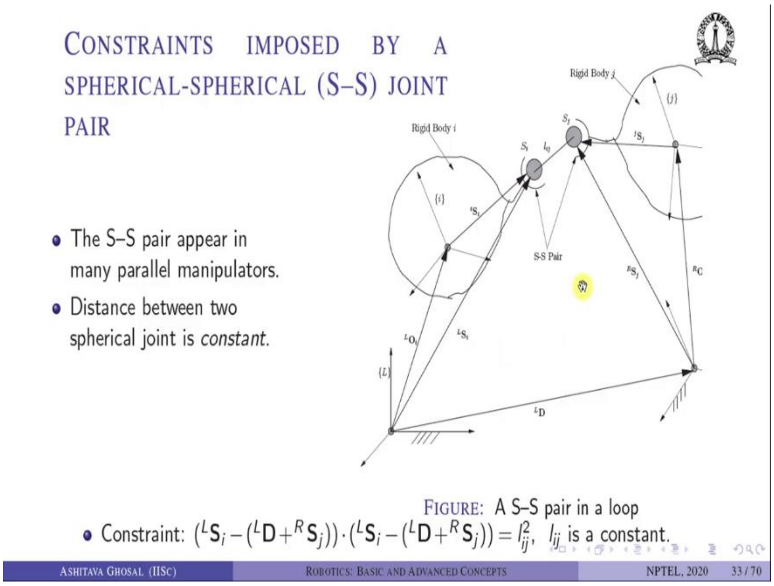
(Refer Slide Time: 15:31)



Let us look at a spherical joint, a spherical joint is also called as a ball and socket joint allows three rotations ok. So, there must be three constraints ok. So, one way of looking at it a spherical joint is representing a spherical joint as 3 intersecting rotary joints ok, that is one way. The other way is let us look at it directly as a spherical joint. So, I have a rigid body $i - 1$ and a rigid body i. So, they are connected by a spherical joint. So, I can reach the centre of the spherical joint or some point on the spherical joint. First by going to the origin of the i th coordinate system and then from the origin to the center of the spherical joint; so, $O_{i-1} + {}^{i-1}p$ must be equal to O_i, origin of the i th rigid body, plus this vector ^ip. Again, we have to pre multiply properly using rotation matrices so that we can add these two vectors properly.

So, we will get one constraint equation which is a vector equation and there are 3 components, hence there are 3 constraints. So, the with spherical joint imposes 3 constraints on the relative degrees of freedom of this i th rigid body with respect to the $i - 1$ th rigid body.

(Refer Slide Time: 16:58)

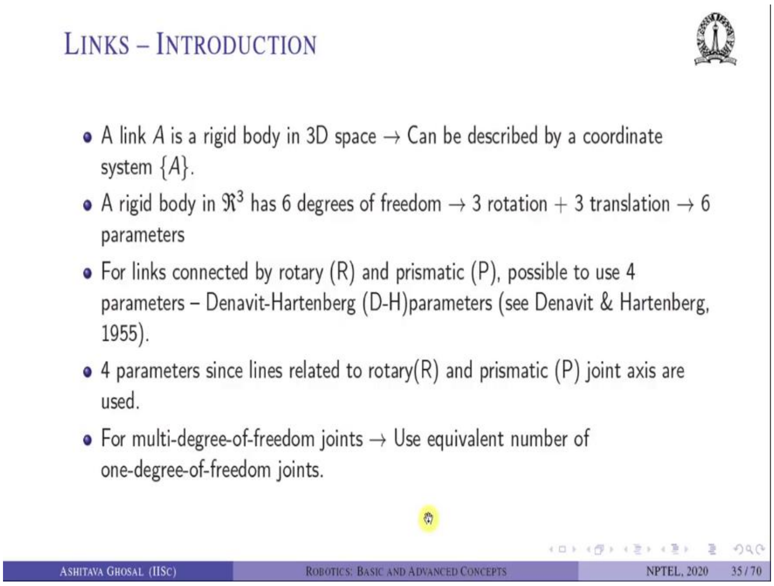


In many parallel robots will come across this S-S pair. What is an S-S pair? Basically, we have a rigid body *i* and we have a rigid body *j* and they are connected by one S joint which is connected to the rigid body *i*, another S joint which is connected to the rigid body *j* and in between there is a link, in between there is a another rigid body of length l_{ij}.

So, this happens very often. So, it is worthwhile to see what the constraint is posed or generated by S-S pair. So, the basic idea is the distance between these 2 spherical joints is constant ok. And mathematically what can we say? We can write that this vector which is on one of the spherical joints which is ${}^L S_i$ and the vector which is the other spherical joint which is ${}^L D + {}^R S_j$, {R} and {L} of the 2 reference coordinate systems and the distance between these two is constant.

So, ${}^L S_i$ - ${}^L D$ + ${}^R S_j$ the dot product with itself should be equal to l_{ij}^2. So, this is a scalar equation and hence it has one constraint. So, with this we will come to a stop about representing a joint and its constraints, but we can find the constraints imposed by other joints if you so desire - if it is really required. Now let us look at the elements of a robot the second main important element of a robot which are the links.

(Refer Slide Time: 18:42)



As I have mentioned earlier a link A is a rigid body in 3D space ok. So, any link is basically a rigid body in 3D space, and it can be represent using a coordinate system {A} ok. So, rigid body requires a point on the rigid body and some 3coordinate axis which describes the orientation of the rigid body and that is the same as the origin of the coordinate system and the X, Y and Z axis attached to the coordinate system.

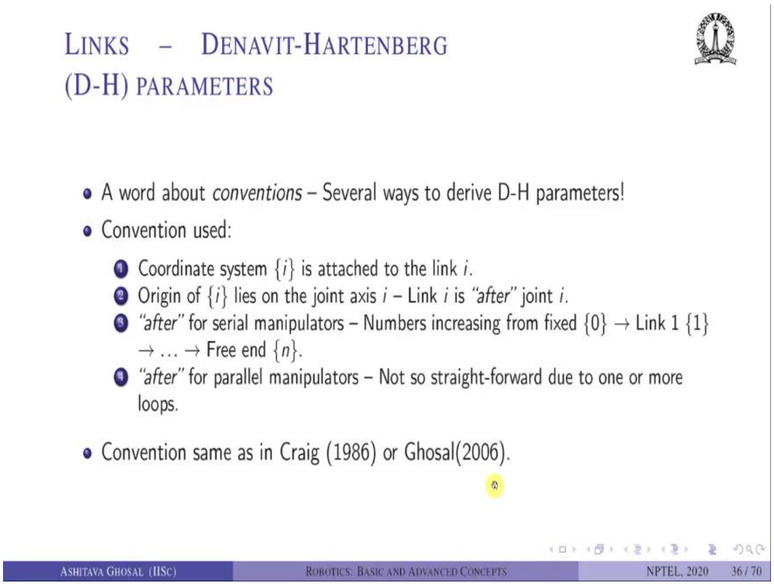
So, a rigid body in 3D space has 6 degrees of freedom 3 rotations plus 3 translation. So, it must have 6 parameters. So, if you have two rigid bodies as I have mentioned ok. So, you need 6 parameters to represent one rigid body with respect to the reference coordinate system or with respect to another rigid body.

If the 2 rigid bodies are connected by rotary or prismatic joint, it is possible to use 4 parameters and these were invented by Denavit and Hartenberg in 1955. These are the very well known D-H parameters 4 parameters of a link. The important thing to remember is this is not 2 generic rigid bodies in 3D space, these are 2 rigid bodies connected by either a rotary or a prismatic joint.

So, these 4 parameters are related rotary and prismatic joint because we will see later that we are looking at joint axis, we are looking at lines in 3D space. And those of you who know study little bit about lines in 3D space; a line in 3D space is referred or represented by 4 parameters. Nevertheless, let us not worry about lines and points in 3D space we look at this D-H parameters now.

For multi degree of freedom joints we cannot use these D-H parameters directly. One way of using D-H parameters is to make use of equivalent number of one degree of freedom joints. So, for example, a spherical joint which is the 3 degree of freedom joint can be represented by 3 intersecting rotary joints and hence we can again use D-H parameters for the rotary joints.

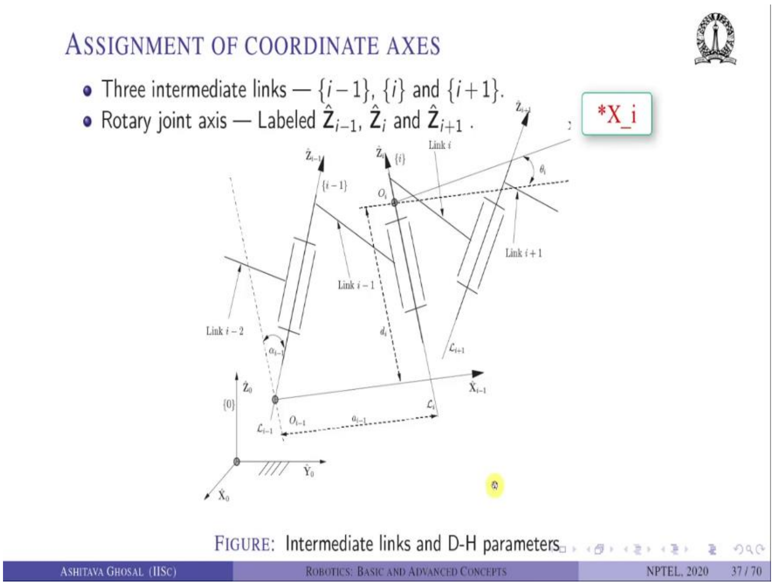
(Refer Slide Time: 21:19)



So, we continue with Denavit-Hartenberg parameters for a link. A word about the conventions there are several ways to derive the D-H parameters ok. Initially there were many many people who derived their own D-H parameters. Nowadays, it is there are not that many ways D-H parameter conventions; however, there is still a few.

So, we will use the convention in the following way ok. So, in our convention the coordinate system *i* is attached to link *i* the origin of *i* lies on the joint axis *i* and the link *i* is after the joint. So, in a serial robot after make sense because we have numbers increasing from a fixed reference coordinate system, which is 0, link 1 has a label 1 and so on. All the way till the free end which has a label *n*; in a parallel robot this notion of after is not so straightforward because there are several loops. So, I can go to one particular link by more than one ways - in more than one loop. So, it is not very clear what is after; nevertheless, we will stick to this notion of after. So, the joint axis is labelled *i* and the link *i* is after the joint *i* this is the same convention that has been used in Craig as well as in Ghosal 2006.

(Refer Slide Time: 22:56)



So, let us look at three intermediate links which are labelled as $i - 1$, i and $i + 1$. So, we want to assign coordinate axis to these links. So, this is very useful and very important - this is one of the most important part of the D-H convention, how do we find various dimensions from such a drawing or such a figure?

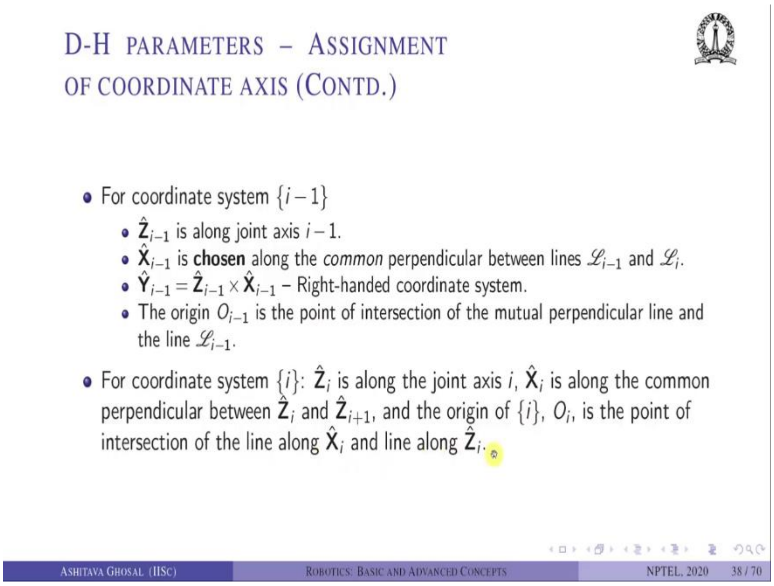
So, we have 3 rotary joint axis these are labelled Z_{i-1}, Z_i and Z_{i+1}. So, this is one rotary joint axis which is Z_{i-1}, the next rotary joint is Z_i and the last rotary joint is Z_{i+1}. So, as I have said the link $i - 1$ is after the joint axis Z_{i-1}; link i is after the joint axis Z_i and link $i + 1$ is after the joint axis Z_{i+1}.

So, in this figure we look at several things which are of useful ok, quantities. First is between this Z_{i-1} and Z_i there is a twist - there is an angle which is denoted by α_{i-1}. So, how do I find this angle? We project this Z_i axis onto here, then we have a line which is mutually perpendicular to Z_{i-1} and Z_i -that is labelled as X_{i-1}. So, we find the angle between Z_{i-1} to Z_i about axis X_{i-1} ok. So, this angle is α_{i-1} as shown here with this figure. Then the distance between the foot of this common perpendicular between Z_{i-1} and Z_i ok, that distance is a_{i-1}. And this foot of this common perpendicular intersects this joint axis at some point, that is the origin of the $\{i - 1\}$ coordinate system ok.

So, there are these lines which is along the Z_{i-1} joint or Z_{i-1} direction and then there is another line L_i along the i th joint axis and we can always find the common perpendicular between L_{i-1} and L_i and that is the direction of X_{i-1}. So, we have Z_{i-1}, X_{i-1} and the origin fixed. *Y* axis is common is following the right-hand rule. So, *Y* axis will be somewhere here which makes *X* x *Y* as Z. Likewise for link *i* we have a common perpendicular which is this dotted - some dotted line here, which is not really shown ok. So, sorry this solid line here and then we have this origin of the *i* th coordinate system at the foot of the common perpendicular. So, this is O_i the *X* axis is along this solid line. So, this is X_i; X_i is not being shown here very clearly. So, again we have a coordinate system which is X_i, Z_i and again Y_i will form the right-handed coordinate system.

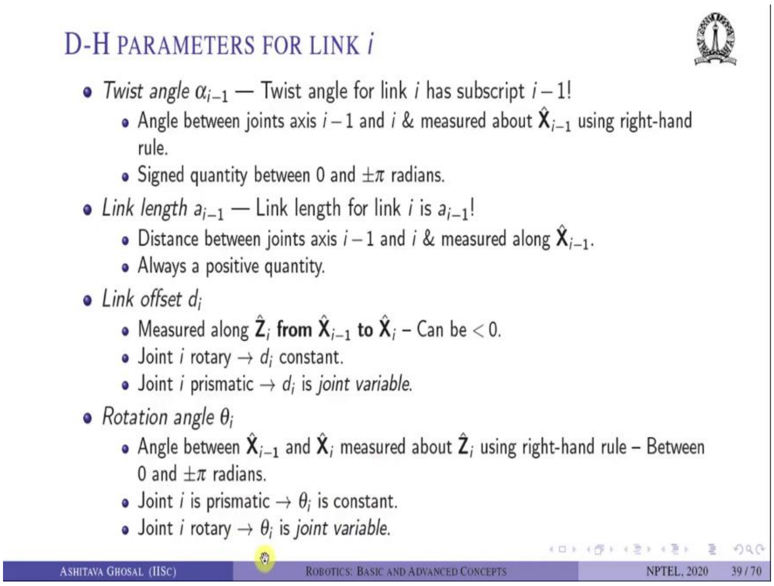
So, for every link say for example, link *i*, I have a coordinate system label *i*, for link *i - 1*, I have a coordinate system labelled $\{i - 1\}$ ok. So, between this *X* axis X_{i-1} and the next which is that X_i the distance is called d_i. So, this is the link offset likewise if I project this X_{i-1} to this dotted line here then the angle between these two lines is called θ_i. So, we have the assignment of axis, which are the *Z* axis is along the joint axis the *X* axis is along the common perpendicular, the origin of the coordinate system is on the joint axis and so on ok.

(Refer Slide Time: 27:20)



So, as I was saying for coordinate system $\{i - 1\}$, Z_{i-1} is along the joint axis *i - 1*; X_{i-1} is chosen along the common perpendicular between lines L_{i-1} and L_i; Y_{i-1} is Z cross X which form the right-handed coordinate system. The origin O_{i-1} is the point of intersection of the mutual perpendicular line and line L_{i-1}. So, origin O_{i-1} is on the joint axis along Z_{i-1}. Likewise for coordinate system { *i* }, Z_i is along the joint axis *i*, X_i is along the common perpendicular between Z_i and Z_{i+1} and the origin of $\{i\}$which is O_i is the point of intersection of the line along X_i and line along Z_i ok.

(Refer Slide Time: 28:21)



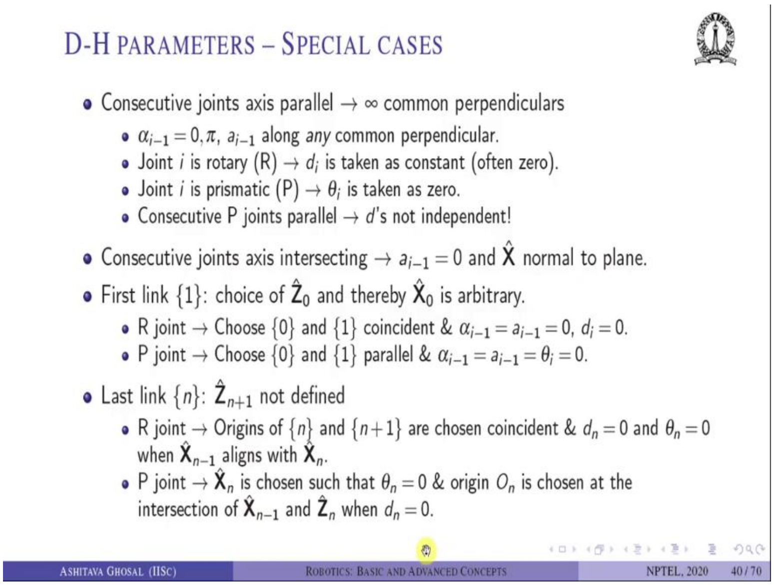
Again, to recapitulate we have one we have defined as something called as a twist angle. Twist angle for link i has a subscript $i - 1$ ok, this is a result of the you know convention. Although we are talking about the D-H parameters for link i and one of the is called twist angle it has a subscript $i – 1$. We have to live with this in this convention.

So, the angle between joint axis $i - 1$ and i measured about X_{i-1} using the right-hand rule is the twist angle α_{i-1}. So, it is a signed quantity between 0 and $\pm\pi$ radians. Then we have the link length. Again it is the link length for link i, but with the subscript $i - 1$ ok.

So, we need to get used to this. It is nothing but the distance between joint axis $i - 1$ and i measured along X_{i-1} and its always a positive quantity because it is a length. Link offset d_i - this is measured along Z_i from X_{i-1} to X_i. So, hence depending on where these common perpendiculars are intersecting the joint axis, it can be either plus or minus or 0 also if it is parallel.

The joint axis i if it is rotary then d_i is constant, if the joint i is prismatic d_i is the joint variable ok. Rotation angle θ_i - this is the angle between X_{i-1} and X_i. measured about Z_i using right hand rule between 0 and $\pm\pi$ radians. So, joint i is prismatic θ_i is constant. If joint i is rotary θ_i is the joint variable.

(Refer Slide Time: 30:34)



So, now let us look at some special cases the first special case - there are 4 of them. The first special case is if this consecutive joint axis are parallel ok. If there are two lines in space which are parallel, then there are infinitely many common perpendiculars ok. So, if you have two joint axis which are parallel, then the twist angle or α_{i-1} is 0 or π and a_{i-1} is same for all the common perpendiculars. So, you pick any of them ok.

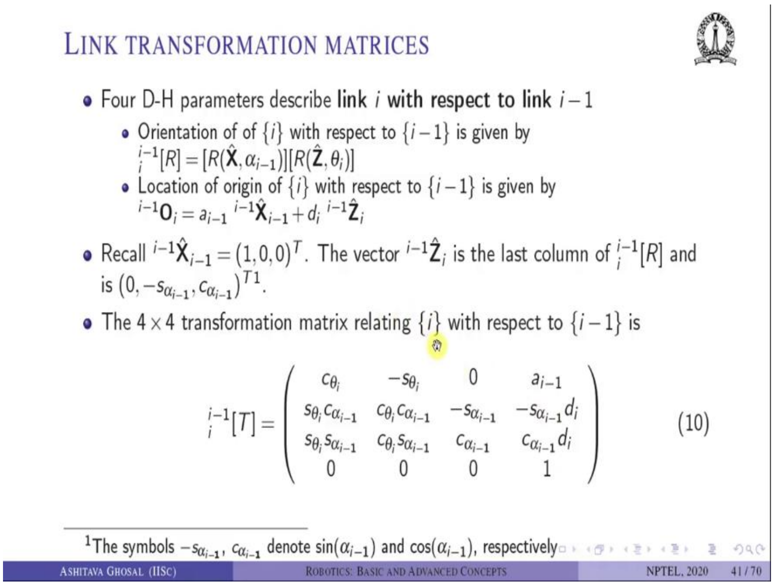
So, a_{i-1} is along any common perpendicular. If the joint *i* is rotary d_i is taken as constant - often 0. If joint *i* is prismatic θ_i is taken as zero ok. So, if two consecutive joint axis are parallel and that joint axis is rotary, d_i is taken as constant. You cannot have two consecutive prismatic joints. So, prismatic joint allows basically slide translation along the joint axis. So, if we have 2 prismatic joints and they are parallel then the third rigid body after the 2 consecutive joints we cannot distinguish between the motion of these 2 prismatic joints. So, it will be both you know it does not matter. So, 2 parallel prismatic joints is basically equivalent to 1.

If you have consecutive joint axis which are intersecting - this is another special case a_{i-1} is 0 and the X axis is normal to the plane. That is obvious -- right if you have 2 lines which are intersecting, they determine a plane and the a_{i-1} is the distance between the 2 joint axis which in this case is 0.

The other special case is for the first link, why is it a special case? Because we do not know Z_0 and hence X_0 is arbitrary, remember X_0 is required to find the a_{i-1} ok. So, a_{i-1} is the distance along X_0 between Z_0 and Z_1. So, since there is arbitrariness in choosing Z_0 , X_0 is also arbitrary. So, we have a convention which says that the rotary joint we choose 0 and 1 coincident. Which basically means α_{i-1}, a_{i-1} is 0 and d_i is 0. For prismatic joint choose 0 and *i* parallel. So, which basically means α_{i-1}, a_{i-1} and θ_i are zero.

Last special case is the last link, the last link is special because there is no $Z_n + 1$ ok. So, we cannot find the common perpendicular between say let us say Z_6 and the next one - there is no Z_7. So, in order to resolve this problem - if the last joint is rotary then the origins of n and $n + 1$ are chosen coincident, d_n is 0 and θ_n is 0. So, there is no problem of a_{i-1} and α_{i-1} because the Z_5 is known, assuming Z_6 is the last joint. And we choose $X_n - 1$ to align with X_n when θ_n is 0. So, basically this forms a reference of, what is θ_n , how do we measure θ_n . If the last joint is prismatic then X_n is chosen such that θ_n is 0 and origin O_n is chosen at the intersection of $X_n - 1$ and Z_n when d_n is 0, this is another way of specifying what is the 0 position of the last prismatic joint ok.

(Refer Slide Time: 34:46)



So, we now have these four D-H parameters α_{i-1}, a_{i-1}, d_i and θ_i. So, what do we do with them? So, one obvious thing is to represent the position and orientation of the *i* th link with respect to the *i – 1* th link ok. So, what is the orientation of link *i* with respect to *i - 1*? We will go back to the figure but let us just write mathematically first.

So, what you can see is the rotation matrix is nothing, but a first a rotation about X axis by α_{i-1} and then a rotation about Z axis by θ_i. And the location of the origin of *i* th coordinate system is first go along X axis by a_{i-1} and then go along Z axis by d_i. We have to make sure that all the vectors are written in the same coordinate system.

So, let us quickly go back and see the picture to convince ourselves. So, this is the link *i*, this is link *i - 1*. How do I go from the origin of link *i - 1* to the origin of link *i*? I have to go along X_{i-1} by a_{i-1} and then d_i by Z_i. So, this vector this plus this.

How about rotation or the orientation of link *i* with respect to link *i - 1*? There are two angles ok. Remember α_{i-1} and θ_i. So, first I have to rotate - make a simple rotation about X axis by α_{i-1} then I have to make another simple rotations by angle θ_i about Z axis ok. So, these are the two rotations which are happening, and these are the two translations which are happening - these are the four D-H parameters basically ok.

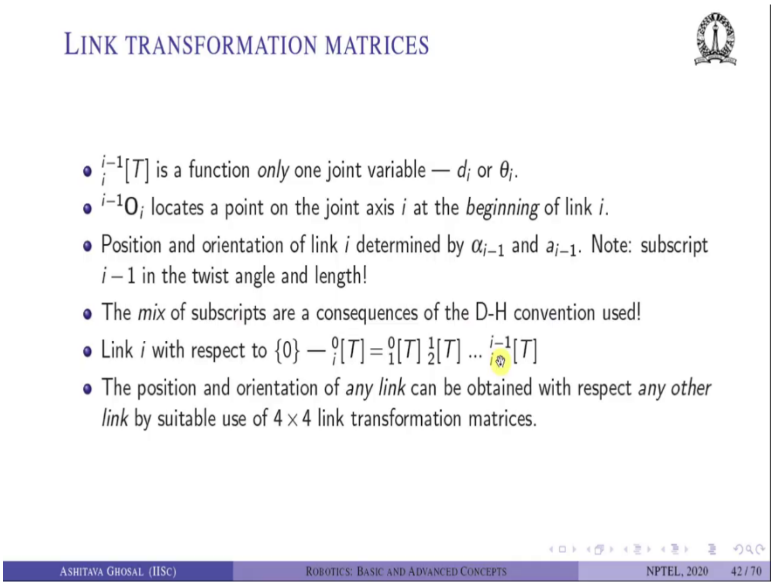
Let us go back to the equation once more. So, ${}^{i-1}_{i}[R]$ is nothing, but simple rotation about X axis by α_{i-1} followed by a rotation of θ_i about Z axis. And the location of the origin *I* with respect to {*i - 1*} coordinate system is this; a_{i-1} along X_{i-1} and d_i along Z_i.

So, we recall that this ${}^{i-1}X_{i-1}$ is 1 0 0. How about ${}^{i-1}Z_i$, where can we get this from? This is nothing but the last column of this rotation matrix ${}^{i-1}_{i}[R]$ ok. Remember that the rotation matrix of a coordinate system {B} with respect to {A}. The first column is the X_B with respect to {A}, second column is Y_B with respect to {A}, third column is Z_B with respect to {A}. So, here we have Z_i with respect to {*i – 1*} which will be the third column of this product ok. And it turns out that this is given by 0 - $s\,\alpha_{i-1}$ $c\,\alpha_{i-1}$. So, what is $s\,\alpha_{i-1}$? It is sin α_{i-1} and cosine α_{i-1} ok.

So, now we can assemble the 4 by 4 homogeneous transformation matrix which relates coordinate system {*i*} with respect to {*i – 1*} or describes link *i* with respect to link *i - 1*. So, as usual there is a top 3 by 3 rotation matrix - where is the rotation matrix coming from? From this equation - product of rotation about X and product which followed by a rotation about Z. So, you can see that you will get $c\,\theta_i$ - $s\,\theta_i$ 0 and various sine and cosine θ and sin α

The last column is the translation which is a_{i-1} (1 0 0) + d_i (0 - $s\,\alpha_{i-1}$ $c\,\alpha_{i-1}$). So, we will get this, and the last row is 0 0 0 1 ok. So, as you can see that I can describe link *i* with respect to link *i - 1* using these four D-H parameters. What are those 4 D-H parameters? Just to recapitulate θ_i, a_{i-1}, d_i and α_{i-1} - 2 angles and 2 distances.

(Refer Slide Time: 39:37)



The important thing to note is that this 4 by 4 homogeneous transformation matrix is function of only one joint variable either d_i or θ_i. So, if the joint axis *i* is rotary then θ_i is the variable if the joint axis *i* is prismatic then d_i is the variable all others are constant ok. Why? Because again remember we are dealing with one degree of freedom joints. So, actually there should be only one variable which is the joint rotation or the joint translation.

This vector $^{i-1}O_i$ locates a point on the joint axis at the beginning of the link *i* that is important ok. So, the origin is always on the joint axis and the link is after the joint axis ok. So, it is at the beginning of the link. So, it does not capture the length of the link after the joint - that is what I am trying to say here.

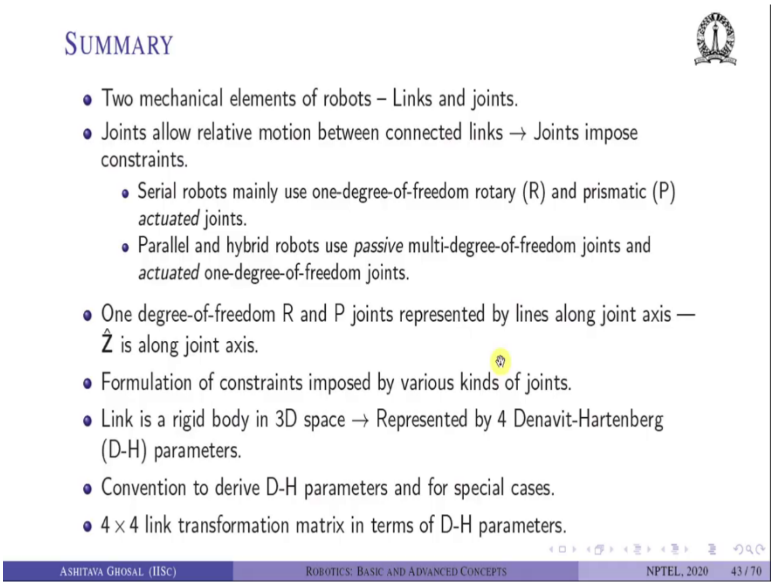
Position and orientation of link *i* is determined by α_{i-1} and a_{i-1} ok. So, this is the confusing part of the convention. We are talking about position and orientation of link *i*, however, the symbols it contains are α_{i-1} and a_{i-1}. So, is this very serious? If you think about it, it is no because whenever we are talking about position and orientation of link *i*, it is always with respect to link *i - 1*. So, this carries that information α_{i-1} and a_{i-1} refer to what is happening to the previous link.

So, the mix of this subscripts are a convention - of D-H convention which is being used. You cannot avoid it. There are other kinematicians, who have developed their own version of D-H convention. In all of them some problem like this always happens. So, in some others it is *i* and *i + 1* ok.

Now, if I want to find link i with respect to some other reference coordinate system lets say $\{0\}$ ok. All I need to do is multiply all these transformation matrices - just by the rule of matrix multiplication. So, ${}^{0}_{i}[T]$ is nothing, but ${}^{0}_{1}[T]{}^{1}_{2}[T]$ ... ${}^{i-1}_{i}[T]$ ok. We can also obtain the position and orientation of any link with respect to any other link ok.

So, if I want to find let us say 5 th link, i equals 5 with respect to 2, 2nd link. So, ${}^{2}_{5}[T]$ is nothing, but ${}^{2}_{3}[T]{}^{3}_{4}[T]{}^{4}_{5}[T]$ ok. So, it is always possible to obtain position and orientation of any link with respect to any other link by just suitably multiplying relevant 4 by 4 transformation matrices.

(Refer Slide Time: 42:54)



So, in summary we have two main mechanical elements of a robot these are links and joints. The joints allow relative motion between the connected links another way of saying joints impose constraints, a serial robot or a serial manipulator mainly uses one degree of freedom rotary and prismatic actuated joints.

However, in parallel and hybrid robots we can use passive multi degree of freedom joints and actuated one degree of freedom joints. We cannot use multi degree of freedom joints in serial robot because it is difficult to put 3 actuators, let us say we want a 3 degree of freedom joint somewhere.

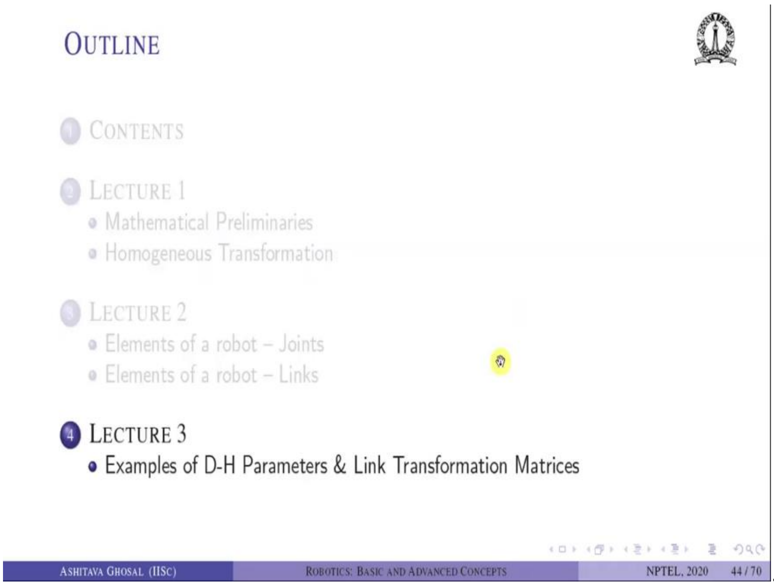
One degree of freedom rotary and prismatic joints are represented by lines along joint axis and we always label the Z axis along the joint axis ok. We choose or label the joint axis as

Z axis. The formulation of constraint imposed by several kinds of joints as discussed. So, for example, the rotary joint imposes 5 constraints. So, I showed you there is a rotation matrix equation which imposes 2 constraints and then there is a position based vector equation which imposes 3 constraints.

Link is a rigid body in 3D space it is represented by 4 Denavit-Hartenberg parameters. So, basically they are α_{i-1}, a_{i-1}, θ_i and d_i. There are 2 lengths and 2 angles and I showed you that there are there is a convention to derive the D-H parameters and then there are special cases in the D-H parameters.

With these DH parameters we can go ahead and determine this 4 by 4 link transformation matrices in terms of the D-H parameters; so, basically these 4 by 4 homogeneous transformation matrices contain all the information about the position and orientation of one link with respect to the previous link.

(Refer Slide Time: 45:17)



So, let us we will stop in this lecture in the next lecture we will look at examples of D-H parameters and examples of link transformation matrices for different commonly available robots ok.

Thank you.