Robotics: Basics and Selected Advanced Concepts
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Lecture - 06
Examples of D-H parameters and Link transformation matrices

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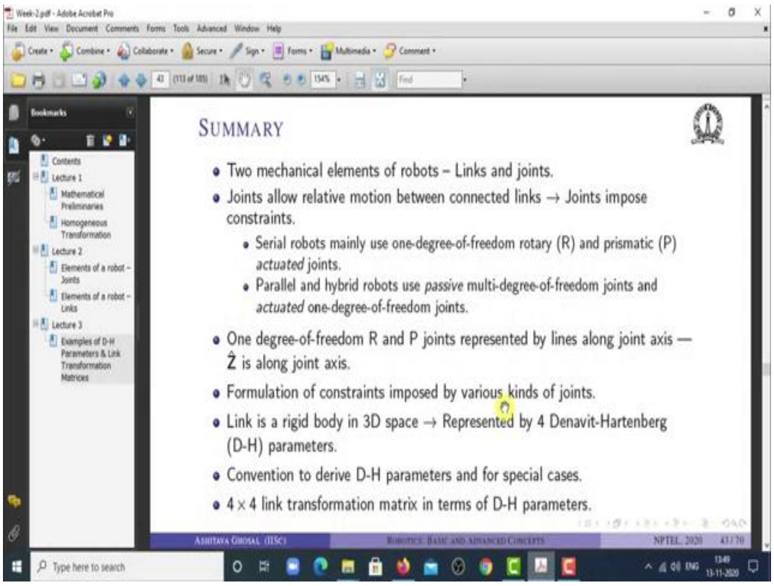

Welcome to this NPTEL lectures on Robotics: Basic and Advanced Concepts. So, this is Week 2 and the 3rd Lecture of this week. In the previous Lecture 2, we had looked at how to model and represent the links and joints of a robot. In this lecture, we will look at examples of D-H parameters, link transformation matrices etcetera.

So, to recapitulate, in the previous lecture, I had shown you that there are two main mechanical elements of a robot - the links and joints. A joint allows relative motion between the connected links or another way of saying, a joint impose constraints. So, a rotary joint allows relative rotation between two links or another way of saying that from the six-degrees of freedom of the second link there are five constraints which are imposed by the rotary joint.

To continue, in a serial robot we mainly use one-degree of freedom rotary and prismatic actuated joints and that is simply because it is sort of hard to put more than one actuator at one place and implement a multi degree of freedom joint. However, in a parallel and hybrid robot, we have passive multi degree of freedom joints and actuated one-degree of freedom joints . So, we have I will show you later that in a parallel robot, we can have spherical joint which is not actuated.

The one-degree of freedom rotary and prismatic joints are represented by lines along joint axis, the Z axis is always along the joint axis ok. So, as far as the link is concerned, we will have a Z axis which is along the joint axis. And I showed you how to formulate the constraints imposed by various kinds of joints. So, I showed you how to obtain the five constraints for the rotary joint connecting two rigid bodies - two came from a matrix equation relating the orientation of the second rigid body with respect to the first rigid body and three came from the position vector of a point on the joint axis.

The link as I mentioned in my previous lecture is a rigid body in 3D space. Typically, a link or a rigid body in 3D space would require six parameters, three position and three orientation. However, because they are connected by rotary and prismatic joints, they can be represented by 4 D-H parameters and I showed you that they in these 4 D-H parameters, two were angles and two were distances. So, there was a twist angle and there was a link rotation angle and then there was a link length and a link offset.

Typically for arbitrary joint axis, none we can easily find the D-H parameters. However, there are some special cases and namely, I showed you there were four special cases; when the two consecutive joint axes are parallel, what happens when the two consecutive joint axis intersect and what do we do for the first and the last link? So, I showed you there is a convention of how to obtain the D-H parameters for these special cases.

Once we have the D-H parameters for each link, we can use these D-H parameters to obtain the 4 by 4 link transformation matrix ok. Recall again, a link transformation matrix is a 4 by 4 homogeneous transformation matrix which contains - the top 3 by 3 is the orientation of link i with respect to i minus 1 and the last column is the position of the origin of link *i* with respect to *i - 1*.

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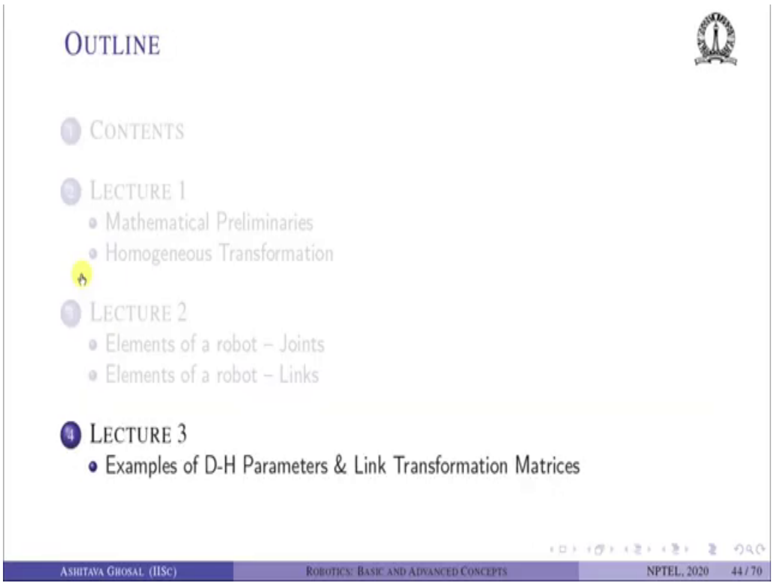

Let us continue. We will now look at examples of D-H parameters and link transformation matrices.

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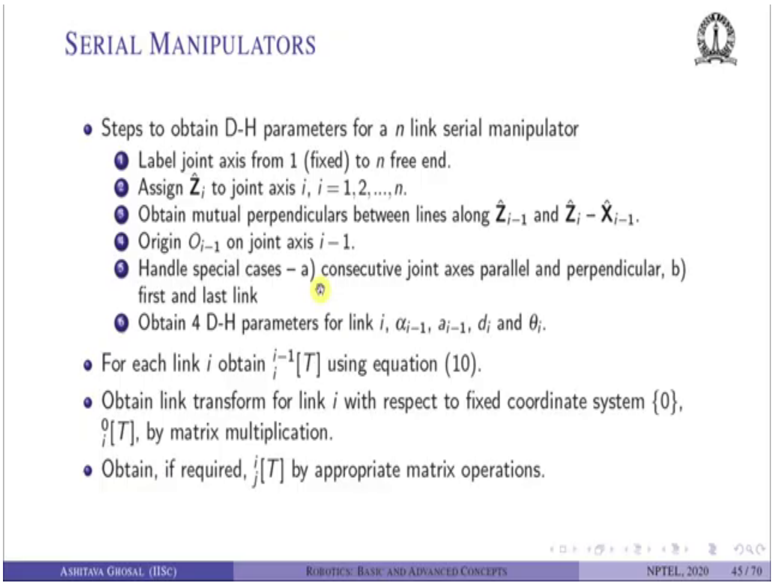

So, just to recapitulate again, what are the steps to obtain the D-H parameters for a n link serial robot? First label all the joint axis from 1 to the free end. So, 1 is near the fixed - 1 is connecting the fixed coordinate system with the first coordinate system first link and n is the last or the free end.

Assign Z_i to joint axis i for each one of those joints - i equals 1 through n. Obtain mutual perpendicular between lines Z_{i-1} and Z_i between two consecutive joint axis Z_{i-1} and Z_i and we call them as X_{i-1} and the origin O_{i-1} on the joint axis $i - 1$. So, Z_{i-1}, X_{i-1}, O_{i-1} determine the coordinate system which is attached to link $i - 1$.

Then, I showed you that we can handle special cases which are two consecutive joint axes being parallel or intersecting or perpendicular. So, all these special cases we can handle and finally, what happens when we have first and last link. So, eventually after these steps, we can obtain the 4 D-H parameters for link i and they are called α_{i-1}, a_{i-1}, d_i and θ_i.

So, α_{i-1} is the twist angle, it is angle between two consecutive Z axis, a_{i-1} is the link length, that is the distance along the common perpendicular between Z_{i-1} and Z_i, d_i is the link offset - it is the distance along Z_i axis from X_{i-1} to X_i and θ_i is called the link rotation, it is the angle between X_{i-1} and X_i about Z_i. So, with these 4 D-H parameters, I can obtain the link transformation matrix i with respect to $i - 1$ using some equation (10). We will come back to this equation then once more. And once we obtain this link transformation matrix, we can find any link with respect to the fixed coordinate system ok. So, if I want to find the transformation matrix of link i with respect to 0 so, all we need to do is multiply matrices 0 to 1, 1 to 2 all the way till i ok.

And what does this ${}^{0}_{i}[T]$ contain? It contains the orientation and position of link i with respect to the 0 th coordinate system. We can also obtain that link transformation matrix for any link j with respect to any other link i. So, if I want to find the link transformation matrix between *2* and *5*, then we have to go from *2* to *3* then *3* to *4* and then *4* to *5*.

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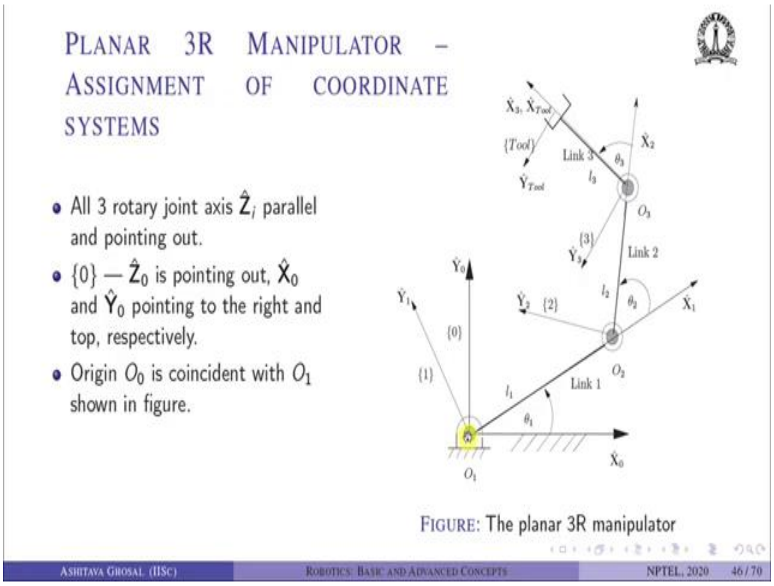

Let us take our example. The first example is that of a planar 3R manipulator. So, 3R here means that there are 3 revolute joints. So, there is this is one revolute joint, this is the second revolute joint and this is the third revolute joint. All 3 rotary axis of the revolute joint axis are parallel, and they are pointing out. So, Z_1 is here, Z_2 is here, and Z_3 is pointing out ok.

The 0 th coordinate system is described by Z_0 and X_0 and Y_0 is to the right here ok. So, this is a choice we have made of the 0 th coordinate system. The origin of O_0 and O_1 is at the same place. So, the first link origin or the first coordinate system origin is at the same place as the 0 th coordinate system. So, the second origin is shown here O_2 and the third origin is shown here as O_3 ok. So, we will go over this once more carefully, but let us quickly show you that the X_1 axis is the common perpendicular between Z_1 and Z_2 so, this is the line along X_1 axis.

The X_2 axis is along the common perpendicular between Z_2 and Z_3 as shown here ok. X_3 is a special case; we will show how to do it. So, once X_1 is known, Y_1 is perpendicular to this as shown here, Y_2 is perpendicular to X_2 as shown here ok. So, the first coordinate system {1} is determined by X_1, Y_1 and the origin O_1 which is here. Second coordinate system is determined by X_2, Y_2 and the origin which is shown here ok.

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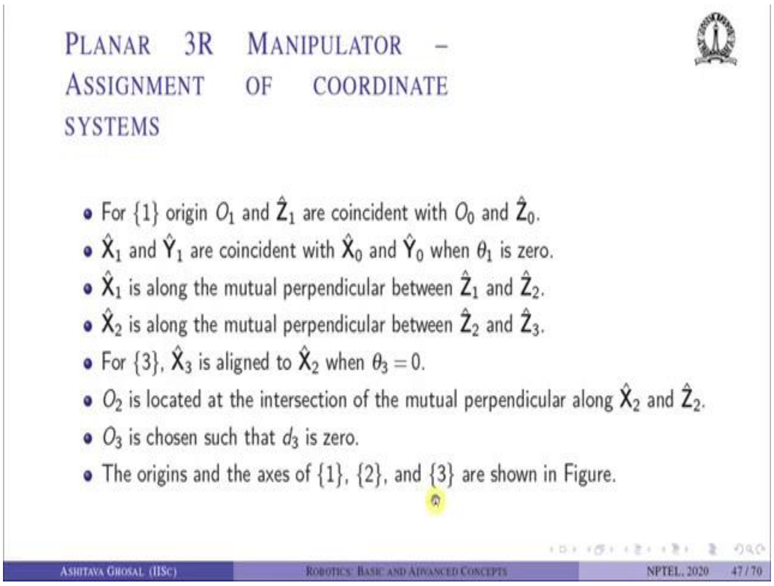

So, as I said for first coordinate system, the origin O_1 and Z_1 are coincident with O_0 and Z_0. So, X_1 and Y_1 are coincident with X_0 and Y_0 when θ_1 is zero. So, this is important that this is how the first coordinate system is assigned. It is a special case and X_1 is along the mutual perpendicular between Z_1 and Z_2 as I have shown.

Likewise, X_2 is along the mutual perpendicular between Z_2 and Z_3. For the 3rd coordinate system, which is also a special case, X_3 is aligned to X_2, when θ_3 is 0 ok. So, as you can see here X_2 is this way, X_3 is this way, the angle θ_3 is between X_3 and X_2 or X_2 and X_3 and when θ_3 is 0, X_3 and X_2 will be aligned to each other.

O_2 is located at the intersection of the mutual perpendicular along X_2 and Z_2. O_3 is chosen such that d_3 is zero and likewise, the origins of {1}, {2} and {3} are as shown in the figure. So, once we have assigned the origins and the axis for each of the links, then we can determine the D-H parameters ok.

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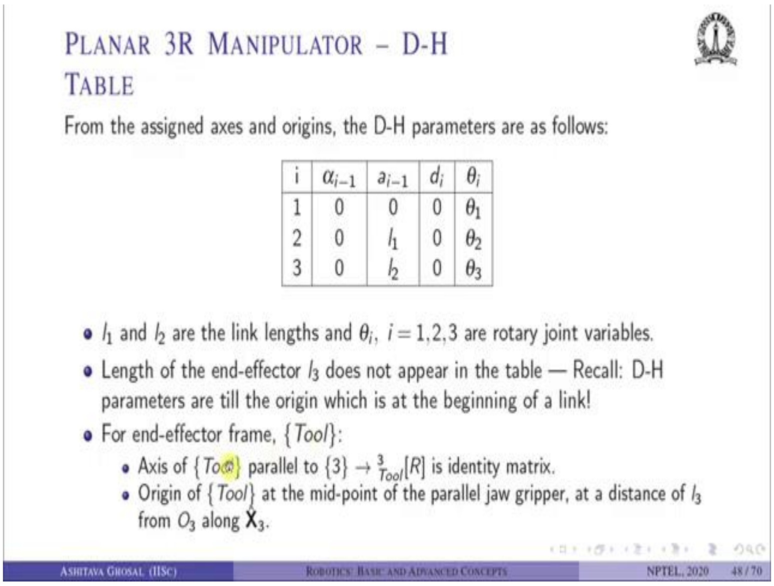

So, for the first link i equals 1, α_{i-1}, a_{i-1} and d_i is 0 - this is part of the convention. For the first link and the last link, we have some special way of assigning the 4 D-H parameters. Only the joint variable θ_1 is shown here. For i equals 2 which is the second link, α_{i-1} is 0 why? Because the joint, all the joint axes are parallel and hence, all the α_{i-1} are 0.

What is a_{i-1}? Ok. a_{i-1} is the link length 2 remember; however, it is the distance between Z_1 and Z_2 ok. So, this the link is here, but a_{i-1} is this distance so, this is the part of the convention. So, this is l_1 ok. So, a_{i-1} is l_1, d_i is 0 because this is a planar example, and the joint angle is θ_2 or the link rotation of link 2 is θ_2.

Likewise, for link 3, i equals 3, α_{i-1} is 0, the link length is l_2, d is 0 and this is θ_3. So, let us go back again and see carefully. This is because of this convention; we have this unusual subscript and subscripts. So, the link length of link 2 is actually this distance l_1. The link length of 3 is actually this distance l_2. So, that is what the D-H convention are showing.

So, for i equals 3, l_2 is here. Actual l_3 does not show up anywhere in these D-H parameters, the distance l_3 that we will see later, we have to assign some other coordinate system at the tool and then, we can assign this l_3 ok. So, recall that the origin is on the joint axis and the link is after the joint axis. So, hence, this is the way the D-H parameters come up.

So, in this example of a planar 3R manipulator, most of the D-H parameters are 0 except this l_1 and l_2 and of course there are these 3 joint variables, 3 rotary joint variables. So, as

I have mentioned, the length of the end effector l_3 does not appear in the table. Recall D-H parameters are till the origin which is at the beginning of the link.

So, if I want to show an end effector coordinate system whatever the robot is gripping, we have to define a coordinate system called {*Tool*} and how do we define this coordinate system {*Tool*}? The axis of the {*Tool*} is parallel to {3}, the last coordinate system. So, another way of saying is the rotation matrix between {*Tool*} with respect to {3} is identity and the origin of the {*Tool*} is the midpoint of this parallel jaw gripper which is shown here at a distance l_3 from O_3 along X_3 ok.

So, let us go back and see the picture once more. So, I have assigned the {*Tool*} coordinate system at the midpoint of this parallel jaw gripper. The X_3 and X_{Tool} coordinate system axis are parallel ok. So, Y_{Tool} is here Y_3 is here. So, the {*Tool*} coordinate system is parallel to the 3rd coordinate system. However, it is located at a distance l_3 from the last origin which is O_3.

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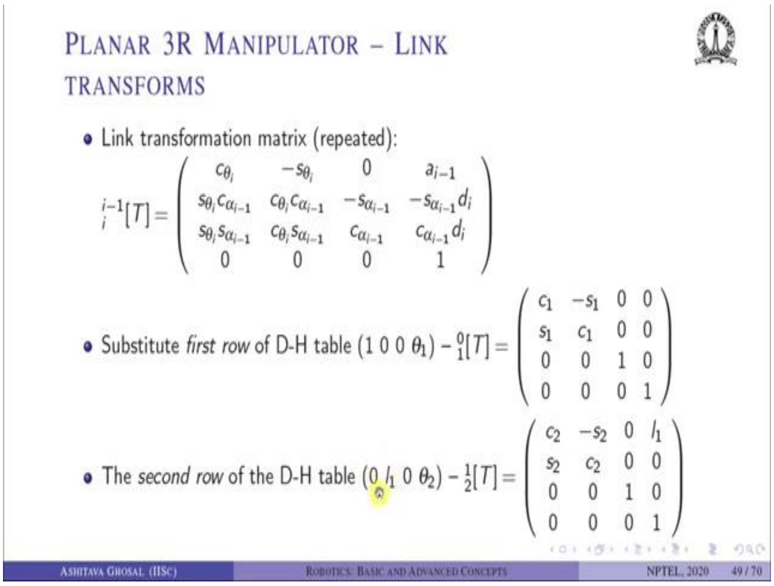

So, once we have all these D-H parameters, we can go back and substitute in the link transformation matrix. So, this is what was equation (10). So, the general form of a link transformation matrix of a link i with respect to i - 1 is given by the $c\ \theta_i$, - $s\ \theta_i$, 0 and then a_{i-1}, then - $s\ \alpha_{i-1}\ d_i$, $c\ \alpha_{i-1}\ d_i$. So, the last column represents the location of the origin of link i with respect to link i - 1. The top 3 by 3 is the rotation or the orientation of link i with respect to link i - 1.

So, if I now substitute 0, 0, 0, θ_1 in this equation, I will get the transformation matrix ${}^0_1[T]$. So, remember θ_1 is so, this will be c_1 which is corresponding to cos θ_1, this is - sin θ_1, this is 0 anyway, a_{i-1} first row was 0 so, we will get this 0, $c\ \alpha_{i-1}$ is 1 because α_{i-1} is 0. So, the second row will be s_1, c_1, 0, 0. The third row is 0 because sin α_{i-1} is 0 so, 0, 0, 1, 0 and the last row for any transformation matrix, link transformation matrix is 0, 0, 0, 1 ok.

To obtain the transformation matrix 2 to 1 ok, we substitute the second row. So, what was the second row? 0, l_1, 0, θ_2. If you substitute 0, l_1, 0, θ_2 here, you will get cos θ_2, - sin θ_2, 0, l_1 because a_{i-1} is l_1 and then, s_2, c_2, 0, 0, 0, 0, 1, 0 and then three 0's, 1 so, it is very straightforward. We just pick whatever is in the row of the D-H table and substitute in this link transformation matrix.

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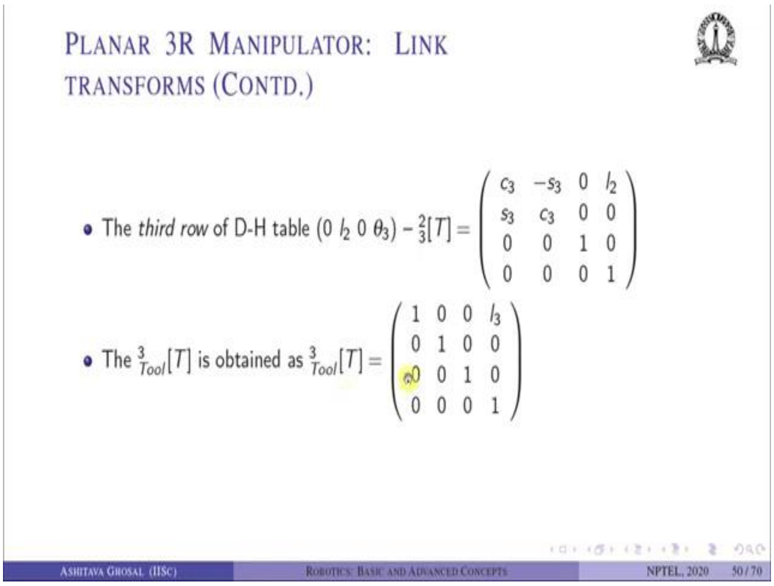

Third row is 0, l_2, 0, θ_3. So, ${}^2_3[T]$ is given by this. And as I said I have defined a {*Tool}* coordinate system at the midpoint of the parallel jaw gripper. The {*Tool}* coordinate system, the rotation matrix is parallel to the third coordinate system. So, hence this is an identity matrix. However, it is along a distance, along the X axis by l_3. So, this is the ${}^3_{Tool}[T]$ transformation matrix. The position and orientation of the tool with respect to the third coordinate system is given by this.

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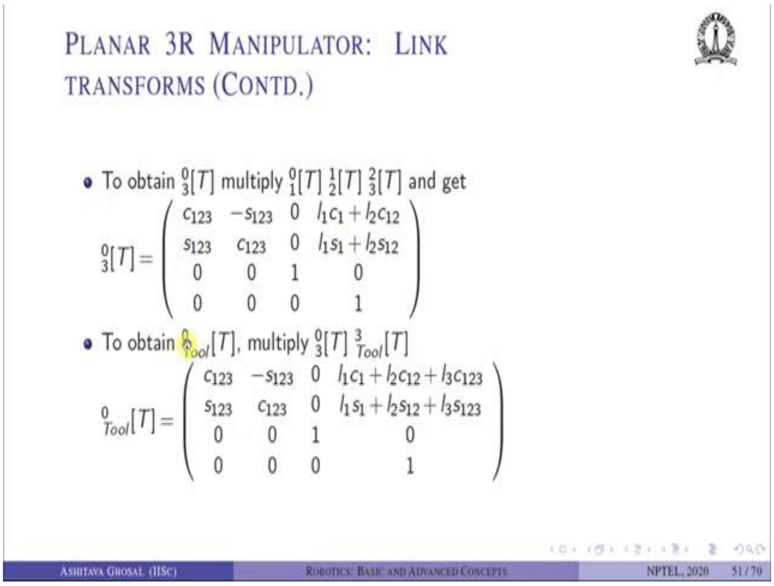

If I want to obtain let us say what is the third coordinate system with respect to the 0th coordinate system, all I need to do is multiply ${}^0{}_1[T]\ {}^1{}_2[T]\ {}^2{}_3[T]$ and what you can see is you will get a expression like this. So, in this case c_{123} means cosine of $\theta_1 + \theta_2 + \theta_3$ ok, c_{12} means cosine of $\theta_1 + \theta_2$.

So, does this make sense? Let us go back and see the figure for once because this is a planar example we can see. So, please remember that the X coordinate of the third origin with respect to the 0 origin is $l_1\ c_1 + l_2\ c_{12}$. The Y coordinate is $l_1\ s_1 + l_2\ s_{12}$. The orientation of the X axis and the Y axis is determined by the sum of the three angles ok.

So, let us go back and see this figure. So, I want to know what is the position of this point O_3 with respect to the 0th coordinate system. So, from basic high school trigonometry, what you can see is the X coordinate is nothing, but l_1 into cos θ_1 so, the projection along X axis is $l_1\ c_1$. The angle with respect to the horizontal of this link is $\theta_1 + \theta_2$. So, this point will be $l_2\ c_{12}$ or cosine of $\theta_1 + \theta_2$. So, the X coordinate, this will be $l_1\ c_1 + l_2\ c_{12}$.

Likewise, the Y coordinate will be l_1 sin $\theta_1 + l_2$ sin $(\theta_1 + \theta_2)$ and what is this angle with respect to the horizontal? This will be $\theta_1 + \theta_2 + \theta_3$ so, which is the orientation of this link.

So, at least for this example, we can clearly see that whatever we know from basic high school trigonometry, it matches with whatever we have derived using link transformation matrices. All this D-H convention and all these link transformation matrices and all these advanced concepts basically tells you the same thing as what we know from basic trigonometry, at least for this planar example. This makes sense.

If you want to obtain the {*Tool}* coordinate system with respect to the 0th coordinate system so, we can just multiply ${}^{0}_{3}[T]\ {}^{3}_{Tool}[T]$ ok. So, again because the rotation matrix of this {*Tool}* coordinate system is identity with respect to the third - nothing happens to the this 3 by 3 part. However, the position now, we will have one term which is $l_3\ c_{123}$. So, if you go back and see the picture, this again makes sense.

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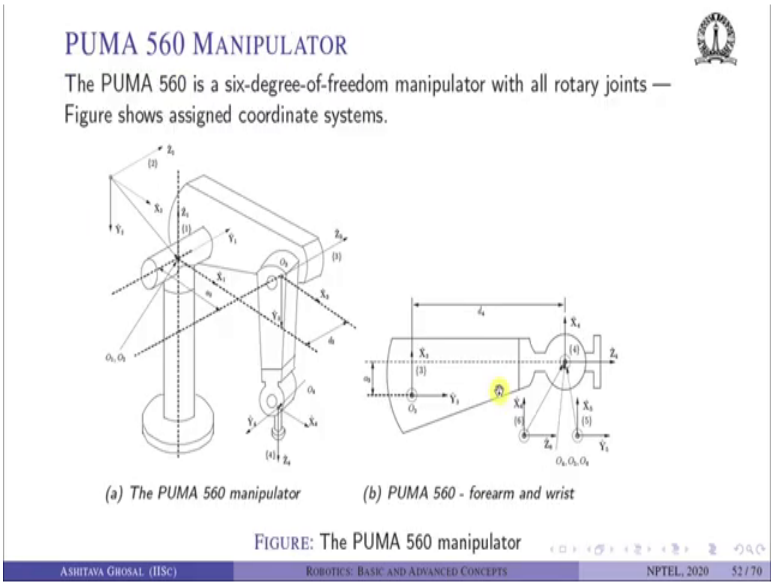

Let us take a three-degree of freedom spatial robot ok. So, we in the previous example is that our planar 3R robot. This is the very well-known PUMA 560 manipulator ok. So, it is a six-degree of freedom manipulator with all rotary joints.

So, first thing is we need to assign the coordinate axis. So, in this picture, what you can see is there is one degree of freedom here as I have if you follow my cursor. The second joint is here which is along this direction which is perpendicular to the first joint, the third joint is here which is parallel to the second joint and the fourth, fifth and sixth joint are intersecting at one place.

So, this figure here shows what are the fourth, fifth and sixth joint axis. The Z_4 axis is along this direction. Z_5 axis is perpendicular going into the page - X, Y, Z going into the page and the Z_6 axis is again parallel to the Z_4 axis. So, we have the Z_1 vertical, Z_2 along perpendicular to that, Z_3 is parallel to Z_2 and then, Z 4, 5 and 6 are along the directions as shown here.

So, once we have all this Z_1 through Z_6, we can now determine what is X_1, X_2, X_3, X_4, X_5 and so on. So, what is X_1? X_1 is the common perpendicular between Z_1 and Z_2. So, Z_2 is in this direction, Z_1 is in this direction so, these two axes are intersecting. So, this is a special case. So, Z_1 and Z_2 lie in a plane so, X_1 is the common normal to the plane, this is normal to the plane. So, this is X_1.

How about X_2 ok? X_2 is the common perpendicular between Z_2 which is this way and Z_3 which is this way ok. So, now, these two axes are parallel to each other. So, again this is a special case ok. So, there are infinitely many common perpendiculars. So, we can choose any one of them. So, in this example, X_2 is this way as shown here. So, Z_2 is this way, Z_3 is this way, X_2 is along the common perpendicular to this.

Likewise, what is X_3? X_3 is the common perpendicular between Z_3 and Z_4 so, Z_4 is here. So, this is in 3D. So, it is sort of hard to visualize and this is one of the toughest problem in kinematics of robots how to assign the coordinate axis, the origins and obtain the D-H parameters. So, if you can do that, then everything else is sort of mechanical.

So, let us go back to what is X_3. X_3 is the common perpendicular between Z_3 which is here, and Z_4 is here. So, it is not very clear how Z_3 and Z_4 are related. We have to look at this figure, the right-hand side. So, in this case, Z_3 is going into the page and Z_4 is like this. So, the common perpendicular is from this direction, this is X_3 ok.

Next, we have to find out what is X_4? So, X_4 is the common perpendicular between Z_4 and Z_5 which is here, they are intersecting. So, we need to choose the normal to the plane formed by Z_4 and Z_5. So, in this case, X_4 is shown here. What is X_5? X_5 is the common perpendicular between Z_5 which is going into the page and Z_6 which is again shown here, they are intersecting ok. So, in this case again, we have to this is a special case - we have to choose what is X_5.

And what is X_6? That is the last link, again we have to follow one of the rules for the special case. So, we have assigned Z_1 through Z_6, X_1 through X_6 and where is the origin? The origin is at the intersection of the common perpendicular with say for example, O_1 is at the common perpendicular between along joint axis 1 and the common perpendicular so, O_1

is here ok. O_2 is also at the same place if you think about it. The origin of the third coordinate system is as shown here; it is at the foot of the common perpendicular along X_3 with Z_3. Then, the origins of 4, 5 and 6, because all three joint axes are intersecting are at the same place. So, this is at slightly difficult example, we have to think in 3D, we have to visualize the joint axis in 3D ok.

So, again very quickly, Z_1 through Z_6 are along the joint axis, X_1 through X_6 are along the common perpendiculars and the origin is on the joint axis. So, origin O_1 is on the joint axis 1, origin O_2 is at the joint axis 2, both are at the same place because 1 and 2 are intersecting and likewise for all other joints.

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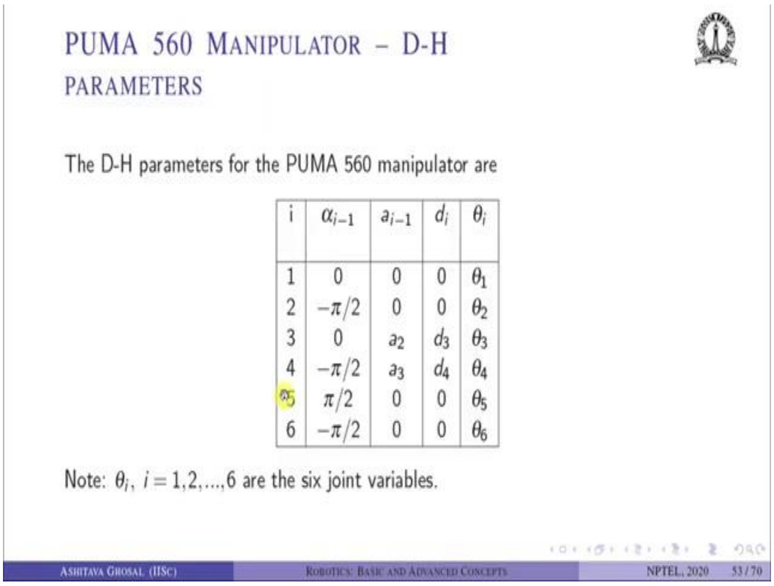

So, once you have assigned the axis and the origins, we can quickly find the D-H parameters. So, I will do this little slowly because this is the first example of a 3D case. So, for link 1, this is a special case. So, as I have shown you, first joint is a rotary joint. So, hence, the twist angle α_{i-1}, link length a_{i-1}, d_i is all 0 and we have θ_1.

For link 2 ok, see here I have written α_{i-1} is $-\pi/2$, a_{i-1} is 0, d_i is 0 and this is θ_2. So, let us check whether we actually have $-\pi/2$ or not ok. So, what is α_{i-1} for link 2? It is the angle between this angle between Z_1 and Z_2 so, Z_1 is like this, Z_2 is also like this as you can see here and about X_1. So, Z_1 if you do the right-handed rule, you can see Z_1 to Z_2, if I have to do I have to go by 270 degrees or $-\pi/2$ so, Z_1 is like this and I have to go all around to Z_2 about X_1 so, this is $-\pi/2$. How about a_{i-1} for link 2? It is 0 because it is intersecting and then, we have θ_2. So, this is $-\pi/2$, 0, 0, θ_2.

Let us look at link 3. Recall that α_{i-1} is 0, link length is a_2, d_i is d_3 and this is θ_3 ok. So, let us verify. So, for link 3, what is a_{i-1} ? It is the distance between Z_2 and Z_3 about X_3. So, Z_2 is this way, Z_3 is this way and it is the distance from Z_2 to Z_3 about this so, this is a_2. What is d_3? It is the distance between the two X axis - between X_2 which is this way, X_3 which is this way so; we have d_3 here ok. So, this is the distance X_2 and X_1 are at the same place here so, this is d_3. The third row is 0, a_2, d_3, θ_3.

Likewise, for link number 4, you can see it is $-\pi/2$, a_3, d_4. This is the last one we will see carefully, the others you can see yourself ok. So, link 4, what is α_{i-1}? It is the angle between Z_3 and Z_4. So, Z_3 is going into the page here, Z_4 is going into the on the right side here along this axis and we have to find the angle between Z_3 which is into the page and Z_4 which is like this. So, again you can see that we have to rotate by 270 degrees or $-\pi/2$ ok. So, this is $-\pi/2$.

What is the distance? The distance is between Z_3 and Z_4 so, this is a_3. And finally, we have d_4 which is the distance between X_3 and X_4 which is this d_4. So, the fourth row is $-\pi/2$, a_3, d_4, θ_4. The fifth row you can see yourself that angle twist angle is $\pi/2$, this is 0, 0, θ_5. This, these two are 0 simply because we are intersecting.

Finally, the sixth row, the angle twist angle is well-defined it is $-\pi/2$, link length is well-defined because it is also 0, this d_i is according to convention, this is the last link d_i will be what? This will be the distance between X_6 and X_7, there is no such thing as X_7 for this ok, there is no 7th joint. So, by convention, we put this as 0 ok.

So, as you can see for this PUMA robot which is one of the original robots. We now have a D-H table which consists of six rows -- because there are six joints and as you can see, there are quite a lot of special cases ok. So, although we looked at special cases in D-H parameters after the general case, but most of the time a robot is built by a designer with joint axis which are parallel or intersecting. So, for example, many of these cases here, this joint and this joint is parallel, Z_1 and Z_2 are intersecting and so on.

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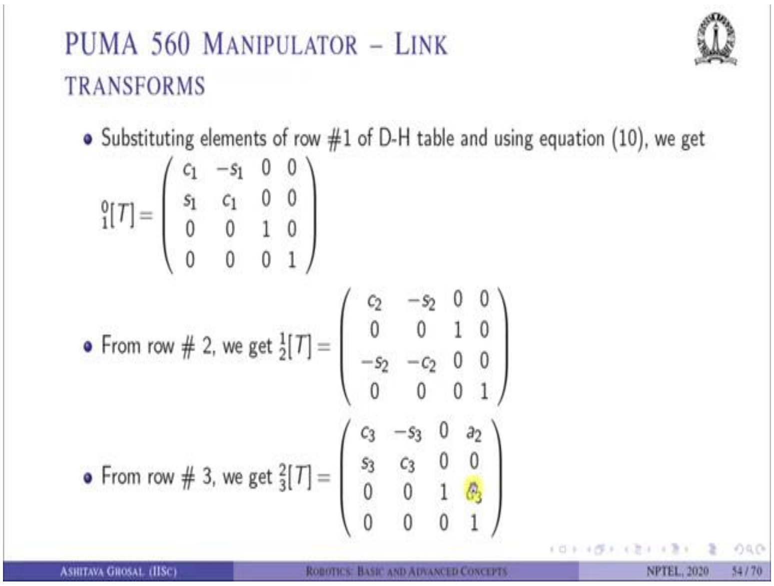

So, once we have this D-H table, we can easily get all the link transformation matrices. So, if you go and substitute 0, 0, 0, θ_1 in the 1st row of that transformation matrix, you will get this $^0{}_1[T]$. For row number 2, we can get $^1{}_2[T]$ and we will get this transformation matrix. So, it will be a function of θ_2. $^0{}_1[T]$ will be a function of θ_1 ok. For $^2{}_3[T]$ it will be a function of θ_3 and this link lengths a_2 and d_3. So, you can work it out and you can show - all we need to do is take each row and put it into the general form of the expression for the link transformation matrix.

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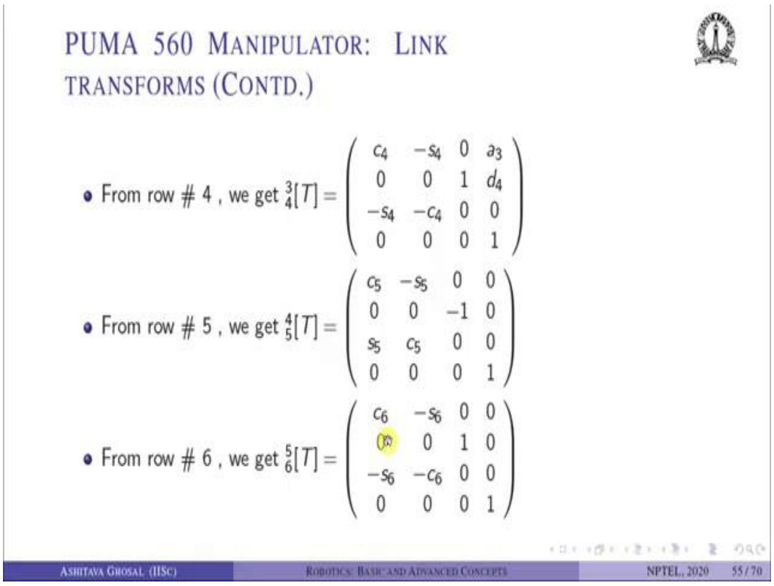

For row 4, we get ${}^{3}_{4}[T]$. It is the function of θ_4 and also this link length a_3 and this link offset d_4 will show up. And for 5, we have θ_5 and there is no link length or an offset and finally, for 6, it is a function of θ_6. We can easily find each linked transformation matrix or another way of saying, the position and orientation of a link with respect to the previous link.

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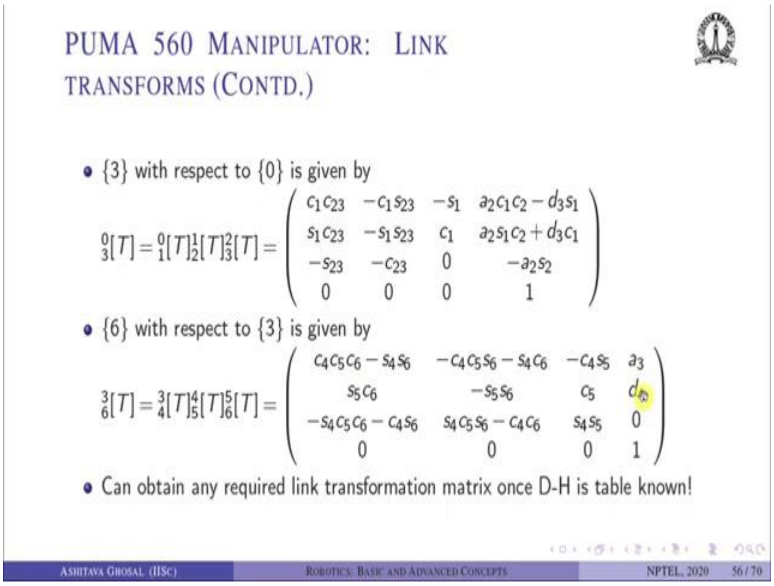

If you want to find, just as an example, where what is the position and orientation of link 3 with respect to 0? So, we can do ${}^{0}_{3}[T]$ which will give you ${}^{0}_{1}[T]$ ${}^{1}_{2}[T]$ ${}^{2}_{3}[T]$ and if you multiply it out, it will be a function of θ_1, θ_2, θ_3 and of course, this a_2 and d_3 ok. c_{23} here corresponds to cosine of $(\theta_2 + \theta_3)$.

If you want to find 6 with respect to 3 - the coordinate system 6 with respect to the 3rd coordinate system - the position and orientation, again all we need to do is multiply relevant transformation matrices. So, ${}^{3}_{6}[T]$ is nothing, but ${}^{3}_{4}[T]$ ${}^{4}_{5}[T]$ ${}^{5}_{6}[T]$ and again, we have a rotation matrix which contains θ_4, θ_5 and θ_6 and the position vector which contains a_3 and d_4. And as I have said, we can obtain any required link transformation matrix once the D-H table is known. I have just given you two examples in this slide.

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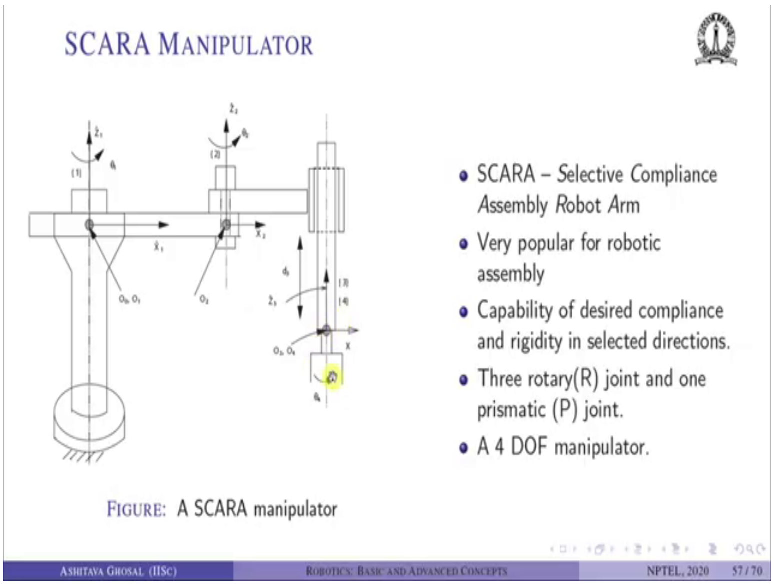

Let us look at one more very commonly used robot which is called as a SCARA manipulator. So, SCARA stands for Selective Compliance Assembly Robot Arm ok. This manipulator is very very commonly used in electronic assembly. So, basically it is a four-degree of freedom robot and the four-degrees of freedom are there is one rotation here which is about Z_1, θ_1, there is another rotation which is parallel to the first one which is θ_2, then this last link can go up and down with the variable d_3 and finally, the end effector, the last link can rotate also about θ_4. So, it has three rotary joints and one prismatic joint.

It is a 4 degree of freedom robot and it was invented by the Japanese to do assembly of electronic components. So, basically the robot can pick up an electronic assembly, take it to a certain place and push it down ok, the pins can be aligned with the holes of the PCB and then, you can push it down. The reason why it is called selective compliance is because we can by using controller gains properly, we can make sure that it is reasonably stiff while it is pushing down. However, if there is any small misalignment between the pins and the holes in the PCB, the θ_1 and θ_2 can change a little bit so, there can be some compliance in the rotation axis θ_1 and θ_2 and θ_4 ok. So, you can think of it that this robot is picking up a pin, IC with pins and then, it is taken to the PCB however, it will never be exactly aligned ok.

So, while you are trying to push it down, this θ_1 and θ_2 and θ_4 will slightly adjust slightly means very small amount and then, it can be pushed down by using θ_4 yeah sorry d_3.

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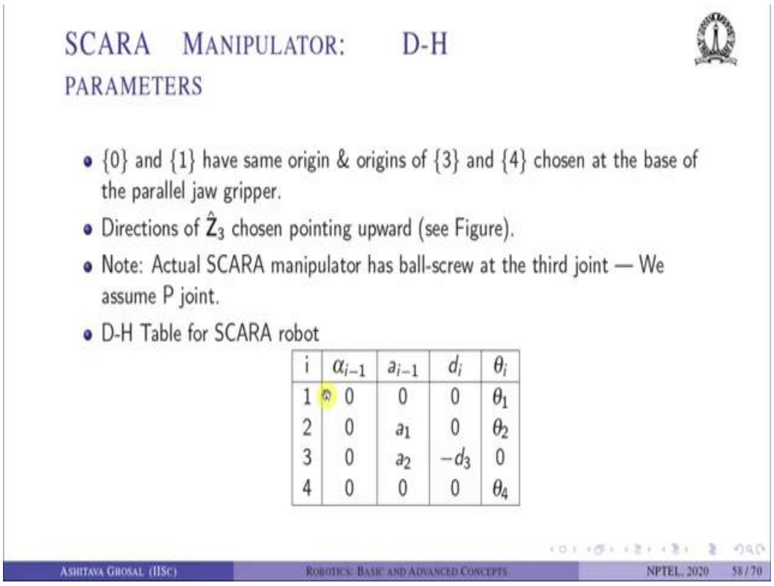

So, how about the D-H parameters of this robot? So, the 0th and the 1st coordinate system have the same origin by convention. The origins of {3} and {4} are chosen at the base of the parallel jaw gripper ok. So, {0} and {1} is at the same place here, then {3} and {4} are chosen at the base of the parallel jaw gripper. Direction of Z_3 is chosen pointing downwards. In this example, Z_3 is this way ok. So, the reason why we are saying Z_3 and Z_4 and all these things are chosen this way because they determine the D-H table -- oh sorry Z_3 is chosen pointing upwards not downwards. In the actual SCARA robot has a ball and screw joint at the third joint. However, we assume that the third joint is a prismatic joint.

So, the D-H table for the SCARA robot this first row is 0, 0, 0, θ_1. This is first joint is rotary and this is a special case. For link number 2, it is 0, a_1, 0, θ_2 ok. Why is it 0, a_1, 0 θ_2? So, this is the distance from here to here is a_1, the distance from here to here is a_2 and this motion here is d_3. So, second row is 0, a_1, 0, θ_2.

Third row is 0, a_2, - d_3, 0 - why - d_3? Because the Z_3 axis is chosen pointing upwards ok. So, if the Z_3 axis was chosen in the opposite direction, then d_3 would be positive. And the last row is also a special case it is 0, 0, 0, θ_4.

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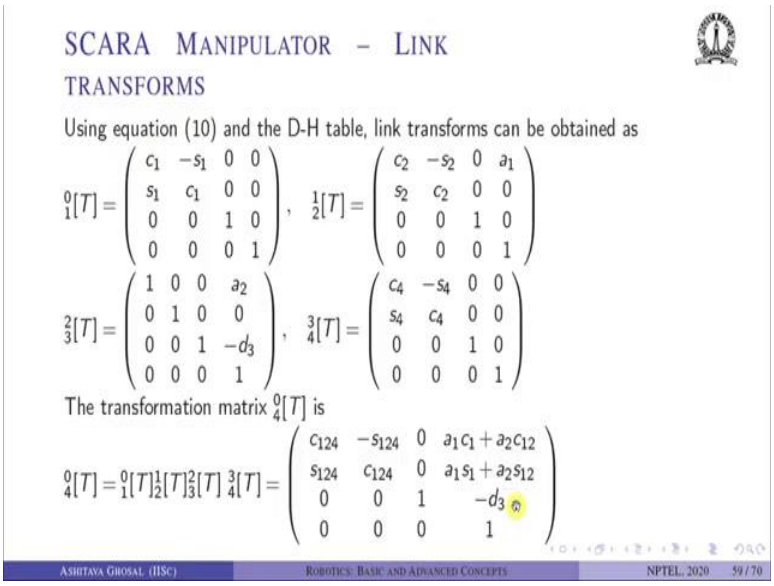

And again, we can easily find each of these link transformation matrices so, for example, $^0{}_1[T]$ is nothing, but cos θ_1, - sin θ_1 and last row 0, 0, 0, 1 and so on. $^1{}_2[T]$ is like this, it is c_2, - s_2 so, all we do is you take the elements of each row and substitute in the general link transformation matrix formula. So, a_1 will show up here, a_2 and – d_3 will show up here. So, you can see {2} and {3} coordinate systems are parallel to each other.

Likewise, θ_4 so, {2} and {3} are they parallel to each other? Yes. So, this is {2} and {3}, they are parallel to each other, there is no rotation happening at this third joint, third joint is a prismatic joint. So, we can find the D-H table for the SCARA robot.

And once we know the D-H table, we can go and find each of these link transformation matrices and as I have said if I want, for example, the position and orientation of the 4th link with respect to the 0th link, I multiply $^0{}_1[T]$ $^1{}_2[T]$ $^2{}_3[T]$ and $^3{}_4[T]$ and you will end up with a rotation matrix containing θ_1, θ_2 and θ_4 and then, the translation components, X component will be a a_1 c_1 + a_2 c_{12}, Y component this and d_3 is the Z. So, Z motion is d_3 and the orientation is θ_1 + θ_2 + θ_4. Third joint is prismatic; it does not contain any orientation or rotation ok.

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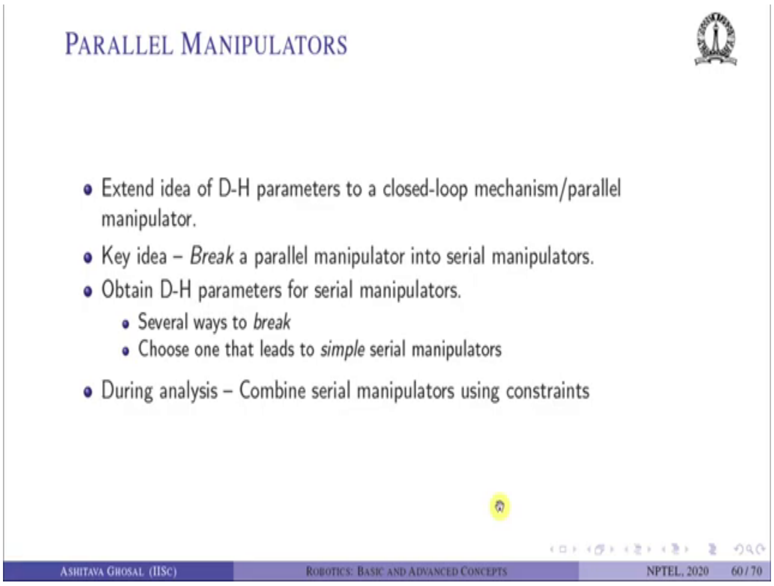

Such that after these two examples, I want to extend this notion of D-H parameters to parallel robots or to close loop mechanism. The key idea is that we break a parallel robot or a closed loop mechanism into serial robots that is the most important thing. So, once you break it ok, so then we can have a parallel robot as broken into one or more serial robots and for each serial robot, we can easily find the D-H parameters ok.

What is the complication? The complication is there are several ways to break a parallel robot or a closed loop mechanism into serial robots. So, if you break it in one way, you will get one set of D-H table. If you break it in some other way, we will break, we will get another set of D-H parameters.

So, the obvious thing is to choose way to break such that you get simple serial manipulators. So, what is simple, the D-H table is simple that is the whole idea. And finally, when we want to do analysis, we want to do kinematic analysis, we can combine the serial manipulators using constraints.

So, I will show you, I can break off closed loop mechanism in several ways to obtain serial robots, I can find the D-H table of each one of the serial robots, but later on if I want to do analysis, I have to put it back at the place where I have broken which basically means you have to reintroduce some constraints.

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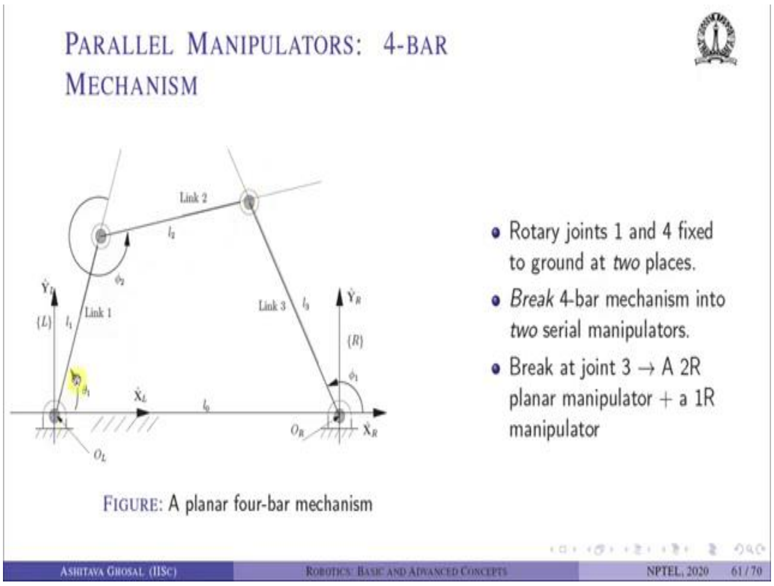

The simplest possible parallel robot is actually a 4-bar mechanism. It is a one-degree of freedom mechanism. So, basically, I have one link here, link 2, link 3 and then there is a fixed link. It consists of four rotary joints 1, 2, 3 and 4. So, it is a closed loop mechanism.

So, we need to have different, two different fixed coordinate systems, these are Y_L and X_L and O_L ok. So, this define the left coordinate system, fixed coordinate system likewise, we have a right coordinate system consisting of X_R, Y_R and origin O_R ok. So, this is a choice which makes the life simple, which makes the analysis simple.

Likewise, you can find the distance between O_L and this two consecutive revolute joints this is l_1, this distance is l_2, this distance is l_3 and the location of this $\{R\}$ coordinate system, fixed coordinate system, with respect to the $\{L\}$ coordinate system is l_0. So, remember in a 4-bar mechanism, we have four links ok; one of them is the fixed link that is the way the links are counted in a 4-bar mechanism.

I can break the 4-bar mechanism into two serial manipulators ok. So, for example, if I break at the 3rd joint so, this is joint 1, joint 2, joint 3 so, if I break it here, so, what do I have? I have a 2R manipulator and a 1R manipulator. 1R means a single link manipulated with 1 joint ok, if I break it here.

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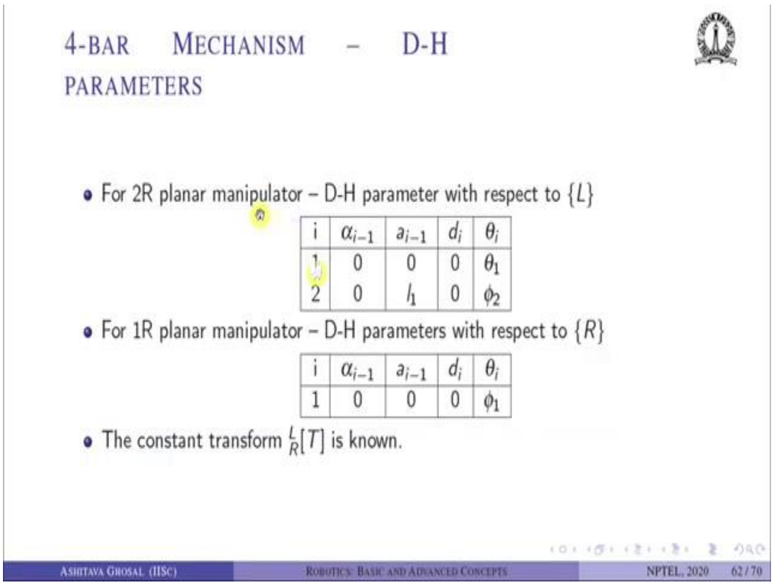

So, if I break it there, for the 2R manipulator which we have looked at in the case of the planar 3R, we have two D-H tables ok.

So, for the 2R with respect to {L}, we have 0, 0, θ_1, 0, l_1, 0, ϕ_2. Remember we are not, we are calling this angle as ϕ_2, this is θ_1 and this is ϕ_1 and for the 1R manipulator with respect to the R coordinate system, we have 0, 0, 0, ϕ_1 and there is a constant transformation matrix between {L} and {R} ok. They are parallel to each other, but they are separated by a distance l_0.

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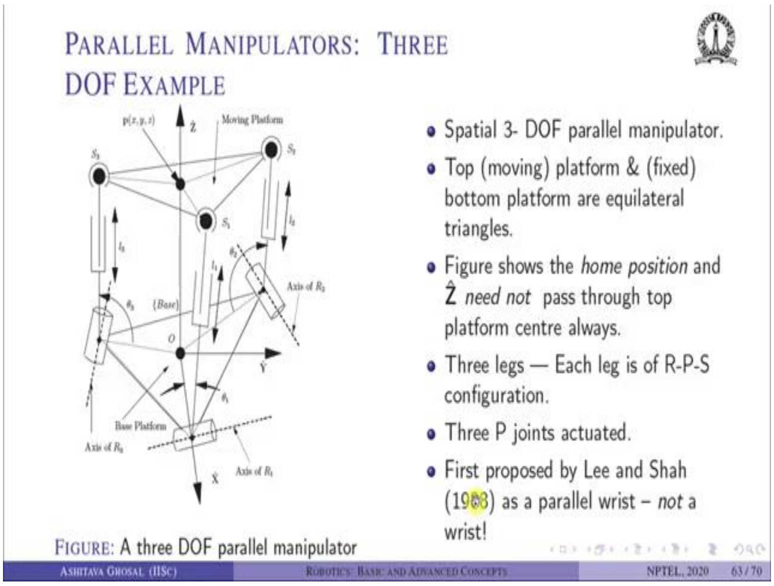

Let us look at a more complicated three-degree of freedom spatial parallel manipulator ok. So, this is called as a 3 R-P-S parallel manipulator - it is a spatial 3-degree of freedom parallel manipulator. It consists of three rotary joints in a fixed base and three spherical joints in a top moving platform and the spherical joint to the rotary joint, there is a prismatic joint ok.

So, the orientation of this link is given by θ_3, the translation along the prismatic joint is l_3 and then this is S_3. Likewise, we have R_1, l_1 and S_1 and then, we have R_2, then one prismatic joint which is determined by l_2 and this is S_2 and the rotation is θ_1 here, this is θ_1 for the first chain, this is θ_2 for the second chain and θ_3 for this third chain.

So, in this example, as I have mentioned, this is a parallel robot with passive joints meaning what? Only the 3 prismatic joints l_1, l_2, l_3 are actuated, this rotary joint θ_1, θ_2, θ_3, R_1, R_2, R_3 they are passive, and we can also have three spherical joints which are passive so, S_1, S_2, S_3. This is called 3 R-P-S because this is R, P, S and there are three of them ok.

So, we have a top moving platform and a fixed base. So, if I change the prismatic joint variables l_1, l_2, l_3, this top platform can move and you can show later or we have seen if you want to apply some formula, we can show that it has 3 degrees of freedom. In this figure, it is shown at the home position; the Z axis of the fixed coordinate system X, Y, Z and this 0 need not pass through the point center of the top platform.

Each leg is R-P-S configuration. There are three prismatic joints which are actuated. This was first proposed by Lee and Shah, two researchers in 1988, as a parallel wrist - meaning parallel wrist means we want to orient the platform - it is done in the PUMA robot with the last three joints ok, the last three joint is orient the object which it is gripping. Actually, it is not a parallel wrist the three-degrees of freedom are not the three rotations ok.

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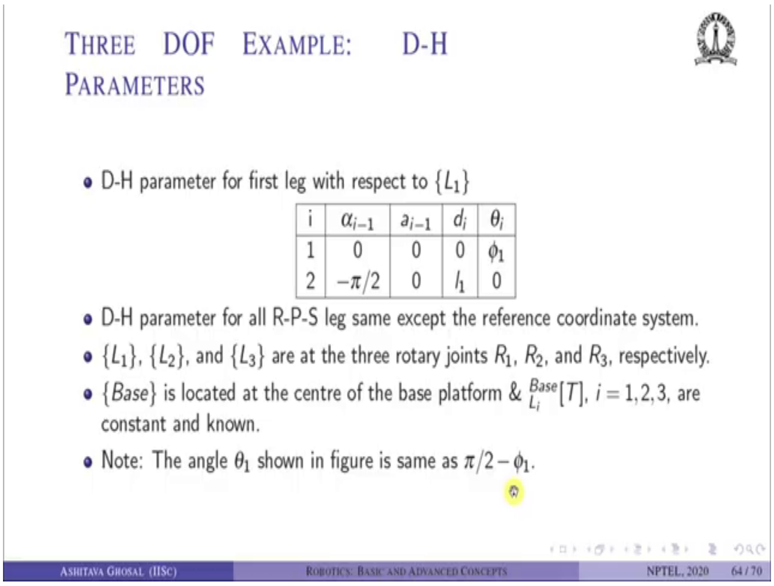

So, the D-H parameters for the first leg with respect to $\{L_1\}$ so, we fix one coordinate system $\{L_1\}$, another one $\{L_1\}$ here, $\{L_2\}$ here, $\{L_3\}$ here fixed coordinate system in the base can be shown to be nothing but 0, 0, ϕ_1. What is ϕ_1? It is some rotation angle of this first one, it is not the same as θ_1, but they are related ok. What will be ϕ_1? Actually, ϕ_1 will be between X and the Z axis, but the angle here is shown as between this line and this line so, it is basically π - ϕ_1 ok. So, the α_{i-1} angle, which is nothing, but the angle between the rotary joint axis and this prismatic joint axis is $\pi/2$ is $-\pi/2$ actually and the joint variables is l_1 ok.

So, the D-H parameters for all the three legs is same except the reference coordinate systems are So, inherently what have we done here? What we have done is we have broken up this parallel robot into three serial robots. So, R, P and it is broken here, it is broken here, and it is broken at the S joints. So, basically each serial link has two joints, a rotary joint and a prismatic joint.

So, this D-H table is quite simple, it contains only two joints and hence and two links so, i is equal to 1 and 2 and we get this one joint variable which is ϕ_1 and one joint variable which is l_1. For the second one, it will be labeled as ϕ_2 and l_2 and so on.

So, the $\{Base\}$ is located at the center of the base platform. So, we can have a coordinate system between $\{Base\}$ and $\{L_1\}$. So, we can have a transformation matrix between this base coordinate system which is X, Y, Z and this origin and the $\{L_1\}$ coordinate system which is here. So, this example or in this figure, we can see that the translation from the base origin to the origin is along some along the X axis whereas for this, it is not along the X axis, it is at some angle to the X axis and for the third one, it is some other angle to the X axis. This can be found out. So, we can find the transformation matrix between the {*Base*} and the three, {L_1}, {L_2}, {L_3} and note the angle θ_1 shown in the figure is same as $\pi/2 - \phi_1$.

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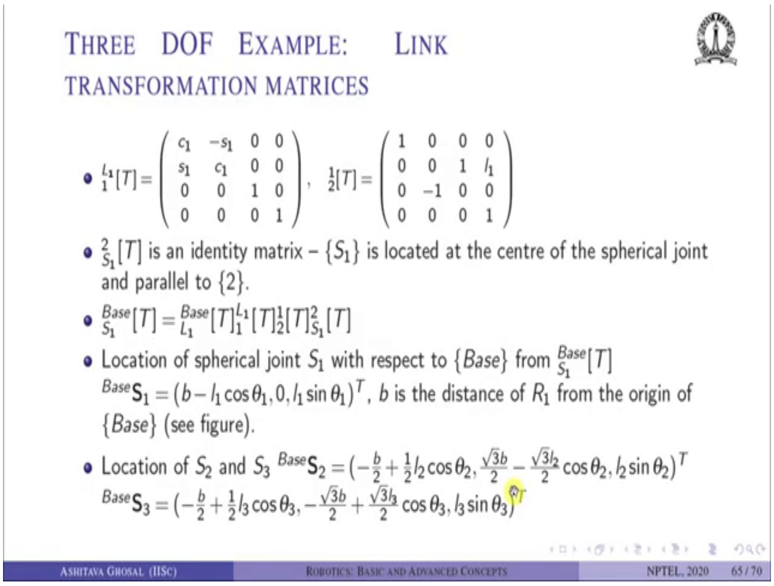

Now we can find three transformation matrices for each of these legs. So, between {L_1} and {1}, we have this transformation matrix, which is given by c_1, - s_1, s_1, c_1 and so on. Between {1} and {2}, I have a transformation matrix which contains l_1 ok.

The transformation matrix between {2} and the spherical joint is identity matrix because S_1 is located at the center of the spherical joint and is parallel to {2}. So, how do I find what is the location of the 1st spherical joint with respect to the {*Base*}? We go from {*Base*} to {L_1} which is known, {L_1} and {1} which is a function of θ_1, {1} to {2} which is the function of the translation and between {2} and S_1 which is identity - it is some distance if maybe.

Likewise, we can multiply this out and what you can show is, so the {*Base*} to S_1, this position vector, so from here to this vector, will be nothing, but we go along this, then we go along this, did go to this ok. So, you can see it will be some distance from O to this point along the X axis, then some something which will vary with the angle θ_1 and something with which will vary with respect to l_1. It turns out that you will get the position vector of the first spherical joint is b - l_1 cos θ_1, Y coordinate will be 0 and the Z coordinate will be l_1 sin θ_1. Does this make sense? More or less ok, you can go back and see the picture.

So, this distance is b so, it is b and then, this angle is θ_1 so, l_1 cos θ_1 so, it will be b - l_1 cos θ_1 so, this three points lie in a plane, the Y axis is perpendicular to that plane. So, the Y coordinate will be 0 and the Z coordinate will be l_1 sin θ_1. So, that is what is written here. So, b is the distance of R_1 from the origin of the {*Base*}.

Likewise, the location of S_2 and S_3, the position vector locating the spherical joints can be determined by – b/2 (1/2) l_2 cos θ_2 and so on. So, we have assumed that these three points are 120 degrees apart. So, that is why we have we get this √3/2 and so on ok.

So, these three vectors can be obtained. Again, just go back to the D-H table, find out the transformation matrices, link transformation matrices, multiply the link transformation matrices and then pick the last column of the 4 by 4 homogeneous transformation matrix.

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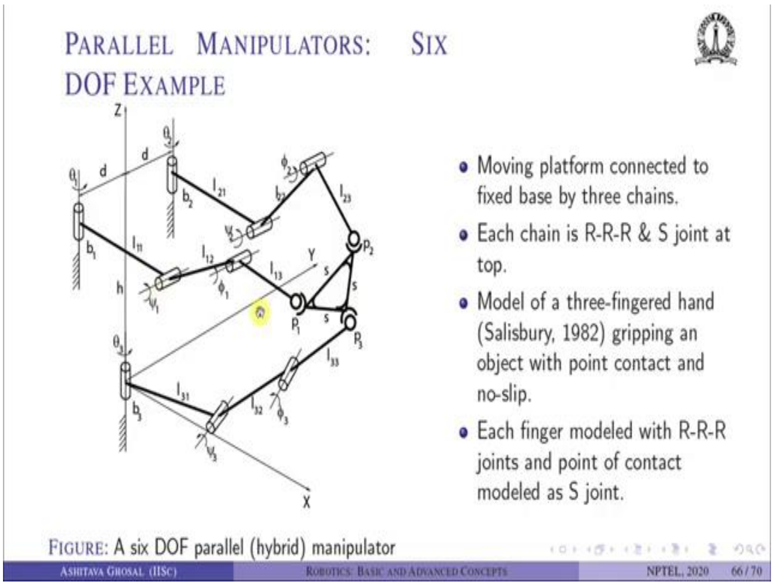

This is another example of a six-degree of freedom parallel robot ok. So, this is basically a hybrid robot why? Because there are these chains which are serial robots and then, there is this loops. So, I can go from this first joint, I can go like this to one spherical joint, then go to another spherical joint and come back to 3 likewise, I can go from one to one spherical joint then come back to another the second fixed rotary joint ok.

So, each chain is R-R-R and S to the top. So, this is an interesting robot. It has three chains, and it is also a model of a three-fingered hand and which was proposed by Salisbury in 1982. If you have these three-fingers and each finger is modeled as three rotary joints so, you can see that the first rotary joint is what is happening at the one point, then you have these two joints which are parallel to each other. So, this is let us say the index finger, this is the middle finger, and this is the thumb ok. So, this is a model of a three-fingered hand which is gripping an object. So, when you grip an object, there are various kinds of contact between the fingers and the object and one such model is a spherical joint, it is called point contact with friction.

So, the point of contact between the finger and the object is modeled as a spherical joint and we have each finger is modeled with three rotary joints. So, we want to model this parallel robot. So, first step again is to find out the D-H table and then, maybe we can find out where, what is this point of contact here with respect to the fixed coordinate system and so on.

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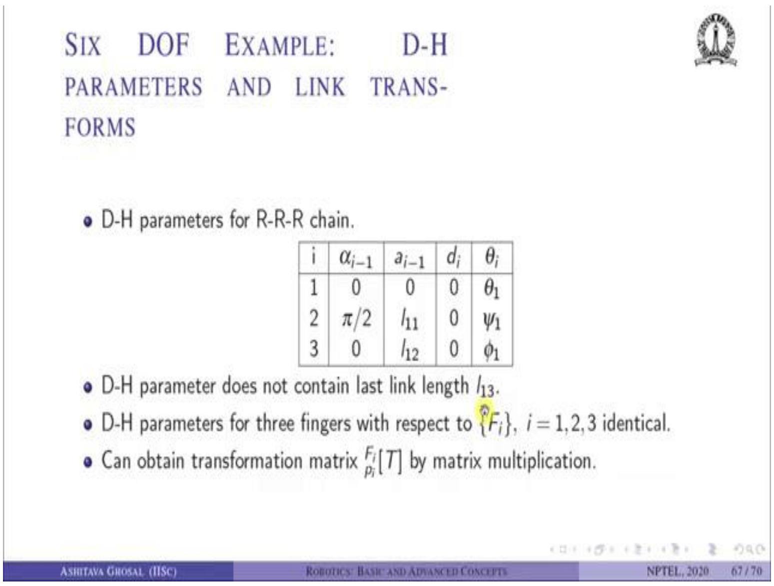

So, if you have this R-R-R chain so, I have one rotary joint, another rotary joint, another rotary joint. So, what you can see is that the first rotary joint is along the Z, the second rotary joint is perpendicular to the first rotary joint and the third rotary joint is parallel to the second rotary joint. So, the D-H table here 1st one is 0, 0, 0, θ_1 because this is convention, this is the first link. The 2nd rotary joint is rotated 90 degrees with respect to the 1st rotary joint and hence, you have π/2 and then, you have these two link lengths l_{11} and l_{12} ok.

So, what is l_{11} ? l_{11} is the distance from this point to this point and l_{12} is the distance of this to this point, l_{13} does not appear why? Because the D-H parameter is starting from the first joint to the origin of the last link. So, it does not know what is this l_{13}. So, these are the two D-H parameters for the first R-R-R chain.

The D-H parameter does not contain the last link l_{13} ok. So, the D-H parameters for each of the three fingers are identical. You can see except so, this one, this one and this one, they are all identical except they are with respect to three different coordinate systems ok. So, this is with respect to this coordinate system, this is with respect to this coordinate system, I have located at this fixed place and this is the third one ok.

So, if you call the fixed points of the R-R-R chain as F_i so, with respect to {F_i } they are identical, and we can find the transformation matrix ${}^{F_i}_{P_i}$[T]. What is P_i? P_i is this points, P_1, P_2, P_3. So, I can find the transformation matrix of ${}^{F_i}_{P_i}$[T] - P_i to F_i, P_1 to F_1, P_2 to F_2 and so on. How do I do that --it is just by matrix multiplication again.

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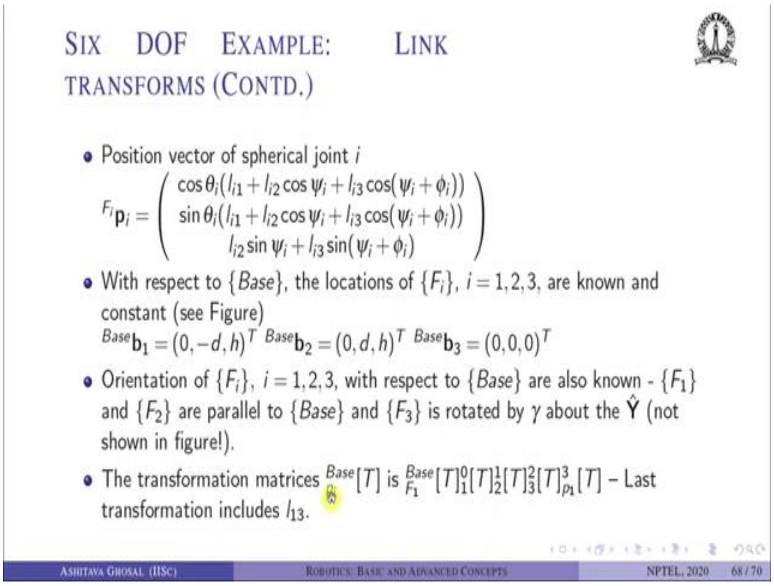

So, it turns out that in this case, so, p_1 to F_i, they are all same, it is determined by cos θ_i, cos ψ_i and ϕ_i. So, these are the three rotations of each finger θ_1, ψ_1, ϕ_1. So, as if you can see in this figure, θ_i is rotation at the base here, ψ_1 is the rotation of the second joint, ϕ_1 is the rotation at the third joint and likewise, for all the three fingers. So, for each one of these fingers, we can locate the position or the contact point with the object in terms of θ, ψ, and ϕ and the link lengths l's ok.

So, with respect to a {*Base*}, first thing is we need to find the location of the fingers, the coordinates of the fingers, starting point of the fingers with respect to the {*Base*}. So that is known and so, we can find out that the {*Base*} with respect to the first finger is 0, -*d*, *h*, 0, *d*, *h* and 0, 0, 0 is that correct? Yes, you can go back and check the figure.

So, we can know the orientation of {F_i} with respect to the {*Base*} and so on. So, {F_1} and {F_2} are parallel to the {*Base*} coordinate system, {F_3 } is rotated by an angle γ about Y which is not shown in the figure, but you can think of your thumb. So, the first joint of the thumb is not parallel to the first joint of your say the index finger ok, it is rotated by an angle.

So, we can find the position vector of the point p_i with respect to the {*Base*} by multiplying first {*Base*} to {F_1}, then ${}^0_1[T]$, ${}^1_2[T]$, ${}^2_3[T]$, ${}^3_{p1}[T]$ and the last transformation will include this l_{13}.

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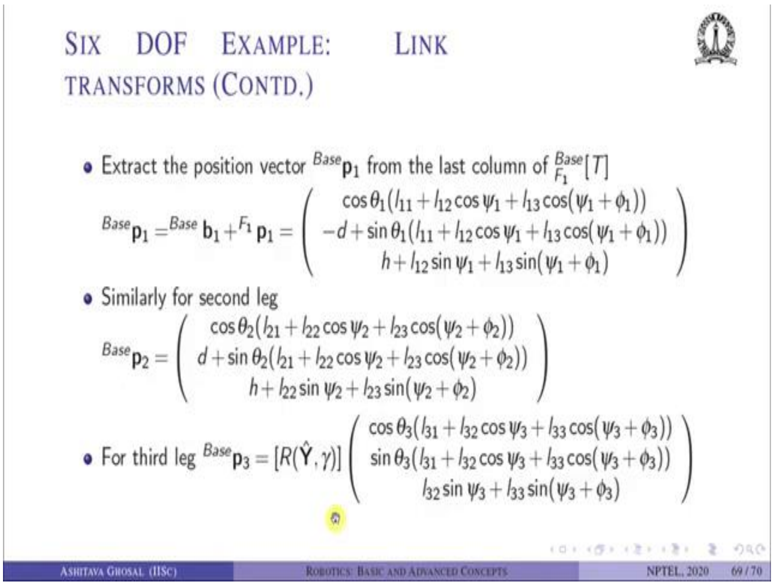

So, we can extract the last column, so, $^{Base}p_1$ is given by $^{Base}b_1 + {}^{F}{}_{1}p_1$ and it turns out that we have this reasonably complicated expression, but basically it contains θ_1, ψ_1, ϕ_1 and this linked length l_{11}, l_{12}, l_{13} o. So, X; X component contains cos, Y components contains sin and then this Z component does not contain θ_1.

Similarly, for second length, I can find out $^{Base}p_2$ and for third length, we can find out $^{Base}p_3$, but as I mentioned, in the actual model of the three fingered hand, the first coordinate is rotated by a angle γ about Y axis so, we have to pre-multiply whatever we get in all of this with another rotation matrix with a constant γ.

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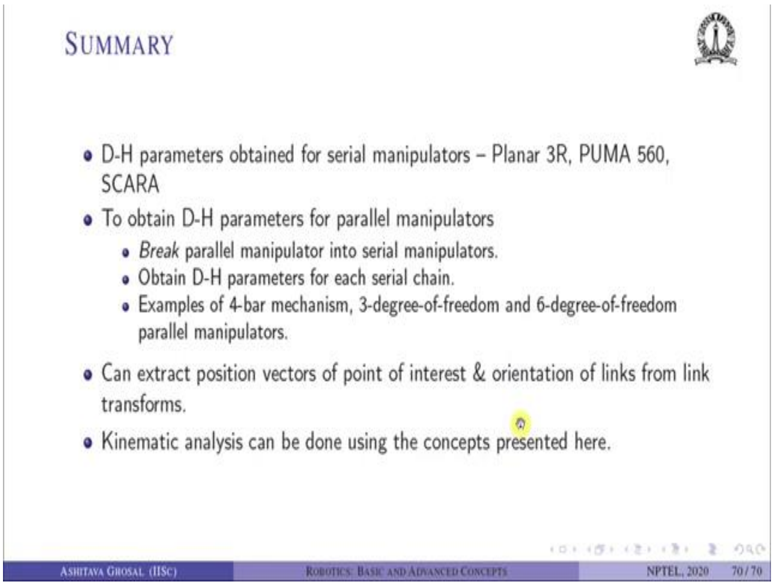

So, in summary, I have shown you D-H parameters for serial robots, a planar 3R manipulator, a PUMA 560 manipulator and a SCARA robot. The first one is three-degrees of freedom, the second one is spatial six-degrees of freedom and the SCARA is a four-degree of freedom robot.

So, to obtain D-H parameters for parallel manipulators, we basically break the parallel manipulator into serial manipulators. We obtain the D-H parameters for each serial chain and as I showed you examples of a 4-bar mechanism, a 3 R-P-S 3-degree of freedom spatial mechanism and the model of a 3-fingered hand which was a 6-degree of freedom parallel manipulator.

So, once I have the D-H parameters, I can find the orientation and position of any link with respect to some fixed link or any other link and then, from that we can extract the position vectors of point of interest or the orientation of links from the link transformation matrices and then, we can do kinematic analysis ok.

And we will see in the next week, how we can do not next week that week after the next week, how we can do kinematic analysis for both serial and parallel robots based on whatever we have learnt and discussed here.

Thank you very much.