Robotics: Basics and Selected Advanced Concepts
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Lecture – 07
Introduction, Direct Kinematics of Serial Robots

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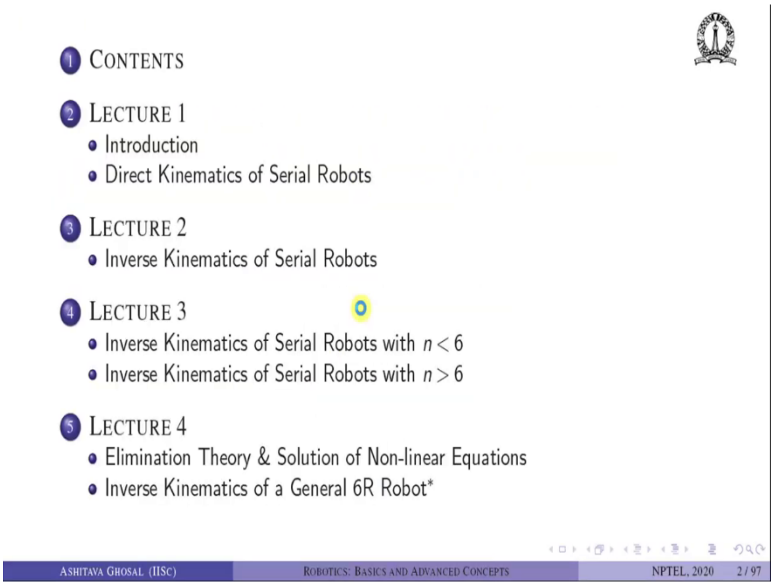

Welcome to this NPTEL lectures on Robotics: Basics and Advanced Concepts. In this week, we look at Kinematics of Serial Robots. Contents of this week are basically; we will introduce the concepts in kinematics, some important concepts in kinematics. Then in the rest of the lecture, I will look at direct kinematics of serial robots.

In the next lecture, we look at inverse kinematics of serial robots. In lecture 3 we look at inverse kinematics of serial robots in two special cases. So, when the number of links is less than 6 and when the number of links is greater than 6. And finally, in the last lecture in this week we will look at elimination theory and solution of non-linear equations.

So, basically we will see that we always have to solve non-linear equations. And I will present a method called elimination theory which can be used to obtain solutions to these non-linear equations. And, we will use this elimination theory to show you how the inverse kinematics of a general 6 degree of freedom 6R Robot can be solved.

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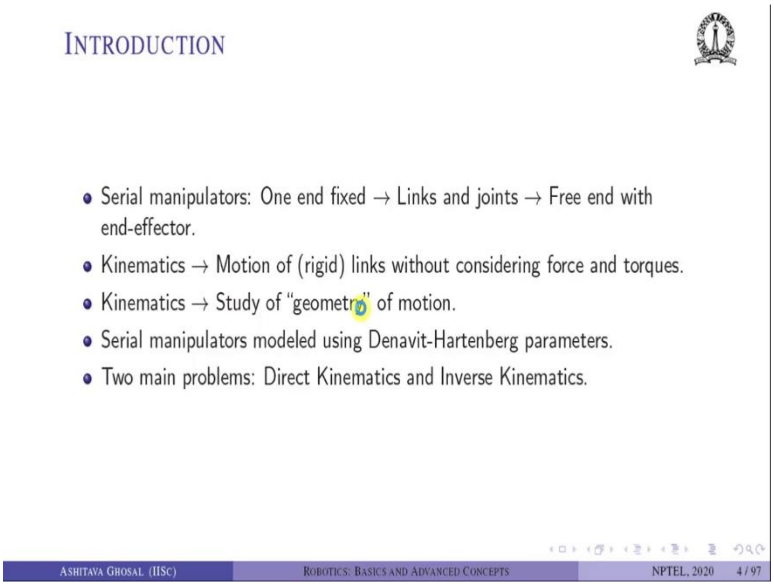

So, let us continue. Introduction - a serial manipulator is basically one where one end is fixed and then you have bunch of links and joints and then it ends up with a free end which is often called as the end effector. In kinematics basically we look at motion of rigid links without considering forces and torques. So, what do we study? In kinematics we studied the geometry of the motion ok. We will see what geometry of the motion really means.

In these lectures, we will assume that the serial manipulator has been modeled using Denavit-Hartenberg parameters. In the last week I showed you that the links of a robot can be modeled using four Denavit-Hartenberg parameters, namely; the twist angle, the link length, the joint rotation and the link offset.

We are going to use a Denavit-Hartenberg convention which we discussed and presented last week. There are two main problems in kinematics one is called as the direct kinematics problem and the other one is called the inverse kinematics problem.

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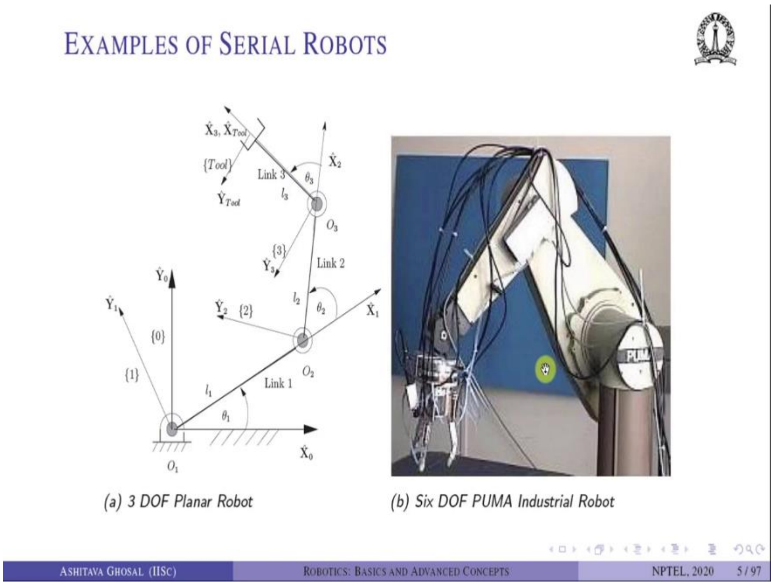

Let us quickly look at a few examples of serial robots we have seen some of these earlier. On the left we have a planar three degree of freedom robot - this is the simplest possible robot. The robot consists of three links. So, as I showed you with link length l_1, l_2, and l_3, there is also a fixed link. And then there are three joints which have marked at O_1, O_2, and O_3 ok. The whole motion of all the links and joints are in a plane. The joints are rotating about an axis which is perpendicular to this screen, but the links are all moving in this plane.

We can also have a six degree of freedom spatial robot and the one of the very well known ones which we also discussed last week is the PUMA six degree of freedom PUMA industrial robot. So, it has six joints and the end effector which is shown here can move in 3-D space. So, all these wires show how maybe some sensors are attached to the PUMA robot or it is taking some information from the sensors or it is powering some motors or some other actuators.

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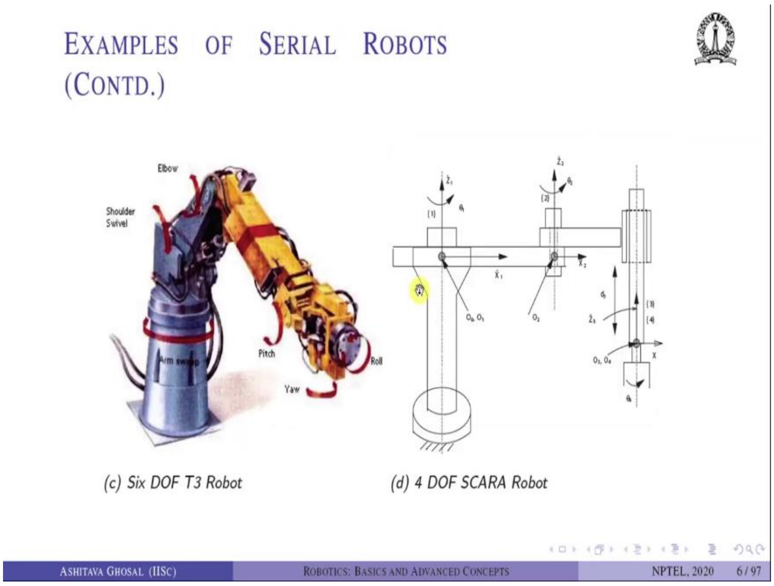

One of the sort of very well known classical six degrees of freedom robot is this six degree of freedom T3 Robot, ok. This was made by a very well-known American company called Cincinnati Milacron. Many of you may have heard of it, but nevertheless. So, it is a robot with six degrees of freedom. So, this is a arm sweep as shown by this red arrow here, there is a shoulder swivel, there is an elbow rotation and then there are these three rotations of roll, pitch and yaw, ok. So, they are about three perpendicular axis; one of the few industrial robots where the wrist has three rotations about three perpendicular axis. We can also have a 4 degree of freedom SCARA Robot this we had discussed last week. So, this is a robot with one rotation about Z_1, one rotation about Z_2, these two are parallel.

Then the last link can move up and down and the end effector can rotate. So, if you recall I had mentioned that this robot was invented for PCB assembling. Basically you can take a chip with pins and then take it to the place where it needs to be inserted into a PCB.

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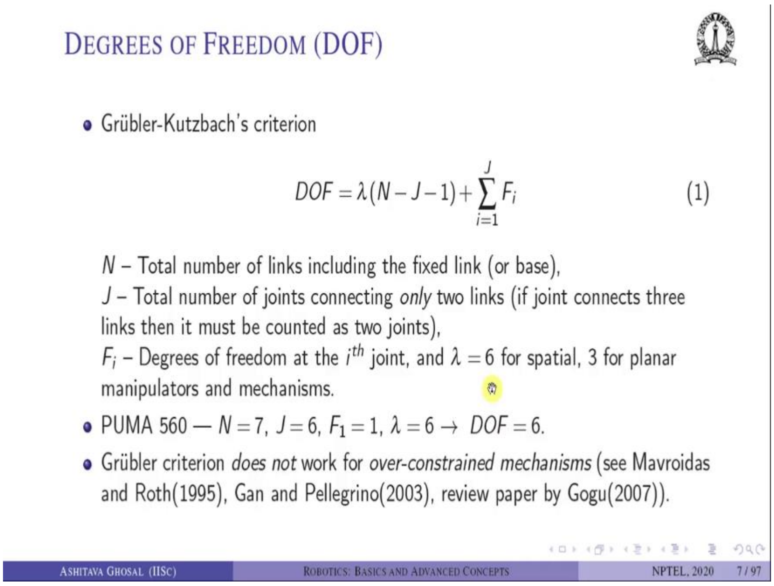

And important concept in kinematics is something called the degrees of freedom. So, the degree of freedom of a mechanism or a robot is given by this Grubler-Kutzbach's criterion. So, in this formula DOF stands for the degrees of freedom. And the right-hand side what you can see is there is something called N, which is the total number of links including the fixed base or the fixed link, J which is total number of joints connecting only two links.

So, if there is a joints, which connects more than two links it must be counted as two or more joints. There is also one term which is $\Sigma_{i=1}^{J} F_i$. So, this is F_i is nothing but the degree of freedom at the i'th joint, ok. So, recall last week we looked at if there are two rigid bodies in space, they have 6 plus 6 degrees of freedom. However, when it connect these two rigid bodies with a joint then the number of degrees of freedom reduces.

So, I had shown you the rotary joint introduces five constraints or it allows one relative degree of freedom. So, if this joint were a rotary joint F_i. would be 1. In this formula also there is something called as λ which is; if λ is equal to 6 then it is a spatial motion whereas, if λ is 3 than it is for planar manipulators and mechanism.

So, in the case of the PUMA the number of joints including the fixed joint is 7. So, number of links including the fixed link is 7 the number of joints is 6 and each of the joints are rotary joints. So, F_1, F_2 are all equal to 1 and then lambda is equal to 6 because it is moving in 3-D space. So, if you substitute all these things here you will get degree of freedom is equal to 6.

The Grubler criteria works very often in many mechanisms and robots; however, it does not work for something which are called over constrained mechanism ok. There is a lot of research going on over constrained mechanisms - has been done. So, basically in an over constrained mechanism there is certain geometry, which tells you that the degree of freedom according to Grubler's criteria is less than 1, but it will move. So, recall in this equation, in this Grubler Kutzbach's, criteria there is no mention of geometry or the length of the links and so on. However, in over-constrained mechanism you have some special dimensions, ok. Special arrangement of links which makes it move.

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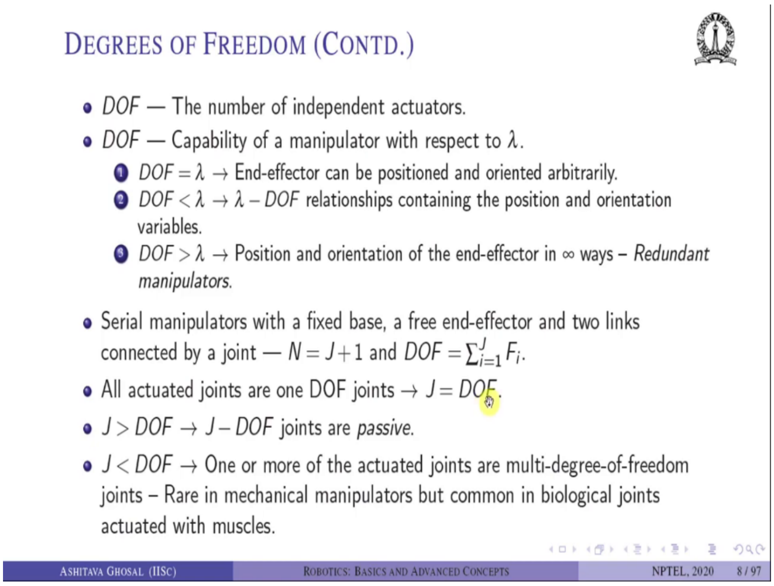

Let us continue a little bit what does the degree of freedom (*DOF*) that you calculate using Grubler Kutzbach's criteria indicate? The fact of basic level it tells you what are the number of independent actuators that you can have. So, if the degree of freedom of a mechanism is say two, I cannot put three independent actuators.

The degree of freedom also indicates the capability of the robot or the mechanism with respect to λ, ok. So, if *DOF* is equal to λ the end effector can be positioned and oriented arbitrarily in that space. If *DOF* is less than λ then λ – *DOF* relationships containing position and orientation variables, ok. So, for example, if you have a motion in plane, but so λ, but if the degree of freedom is 1, let us say then I cannot achieve all x, y and ϕ. So, we will see later; when we look at parallel mechanisms for example, in the four bar we cannot achieve the coupler or some point on this mechanism cannot give any x, y and ϕ.

There is some relationship between x, y and ϕ, ok. How many relationships? Three of them because degree of freedom is 1 and λ is 3, sorry, 2 of them 2 relationships, ok.

If the degree of freedom is greater than λ, then the position and orientation of the end effector can be obtained in infinitely many ways. This is a special class of robots and manipulators which are called redundant manipulators and we will look at them in more detail later on.

In a serial robot or a serial manipulator with fixed base, a free end-effector and two links connected by a joint only two links connected by a joint; you can easily see that N must be equal to $J + 1$ and the degree of freedom is nothing but the sum of the degrees of freedom at the joints. If all joints are actuated and I have one degree of freedom joints. So, J will be equal to degree of freedom, ok. So, for example, in the planar three degrees of freedom, there were three joints and the degree of freedom was three. In the PUMA case there were six joints and so J was six and the degree of freedom was six. If J is greater than degree of freedom, $J - DOF$ are passive - this is happens in parallel robots and mechanic closed loop mechanisms, ok.

So, for example, if you recall we looked at something called as a four-bar mechanism - the degree of freedom of a four bar mechanism was 1, but the number of joints were 4 ok. So, some three joints are not actuated - they are passive. If J is less than degree of freedom one or more of the actuated joints are multi degree of freedom joints, ok. So, you can think of the ball and socket joint or the shoulder joint in your arm, ok. So, between your body and the upper arm there is a shoulder joint, J is 1; however, the end of the upper arm can be positioned in 3D space. So, this case happens very quite often in biological joints which are actuated by muscles, ok. So, I can implement three degree of freedom shoulder joint with muscles, but not so easy to implement using purely mechanical components.

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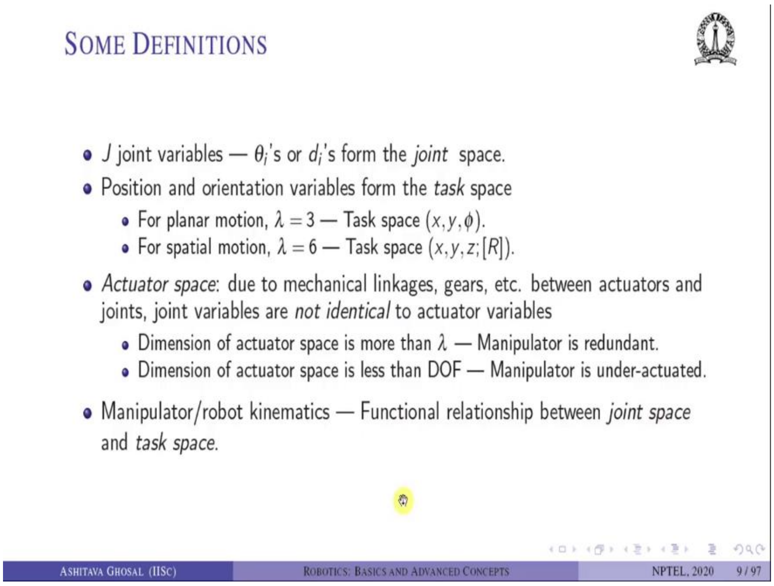

A few more definition - if *J* is the joint variables and they are typically in a serial robot θ*'s* with that the link rotations and *d* which are the translation of the links; so, all these θ*'s* and *d*'s form the joint space. The position and orientation variables form the task space. So, for planar motion λ is 3 and the task space is *x*, *y* and ϕ. For spatial motions λ is 6 and the task space as the three position variables *x*, *y*, *z* and the three parameters in a rotation matrix.

Often ,in robots we have something called as an actuator space. So, this is basically due to mechanical linkages gears between the actuators and the joint variables. So, the joint variable θ is not identical to the actuator variables; actuator here means the motor ok. So, the motor could be rotating much faster than the joint and there is a gearbox.

So, the dimension of the actuator space, if it is more than λ then the manipulator is redundant in a quite obvious. So, if you have 3 if you are moving in a plane, λ is 3, but I have more number of actuators let us say 4 of them so then the manipulator is redundant. If the dimension of the actuator space is less than *DOF* the manipulator is under actuated. So, if I have a planar three degree of freedom 3R robot, but I put purposely two actuators then it is under actuated.

So, what is manipulator or robot kinematics - what exactly do we study? We look at the function and relationships between joint space and task space variables. We want to look at what is the relationship between θ*'s* or *d*'s and the task space variable *x*, *y*, *z* etcetera.

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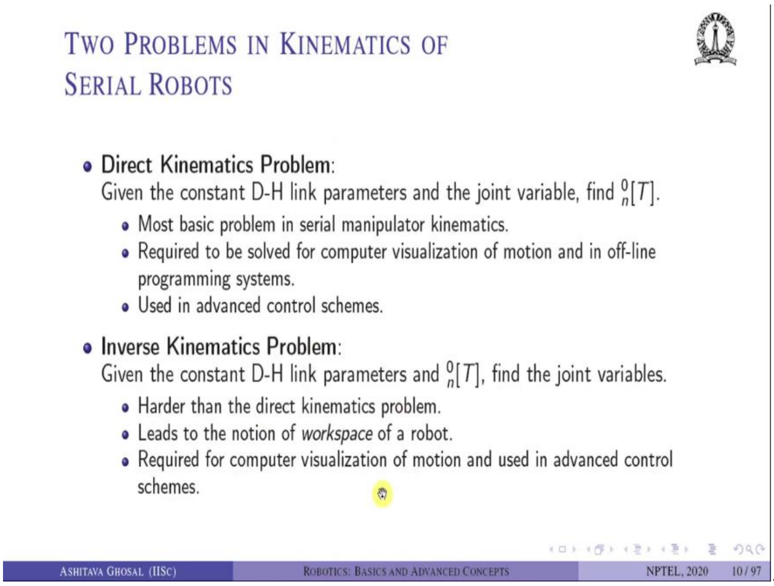

So, as I mentioned there are two main problems in kinematics of serial robots. First is the direct kinematics problem - the direct kinematics problem is given the constant D-H parameters and the joint variables find ${}^0_n[T]$. So, recall that ${}^0_n[T]$ - it means this is the 4 by 4 homogeneous transformation matrix which contains information about the orientation and position of the *n*th link with respect to the 0th link. So, top 3 by 3 matrix in the [T], ${}^0_n[T]$ is the rotation of the *n*th link with respect to 0th link. And the last column is the position of the last link with respect to the 0th link. This is the most basic problem in serial robots or serial manipulator kinematics.

So, given all the D-H parameters, the constant as well as the joint variables find the position and orientation of the last link or any other link for that matter. This problem needs to be solved for computer visualization of motion and in off-line programming systems. So, if I were to rotate or change this D-H link parameters and the joint variables, how does the end of the robot move?

If I want to show you how on a computer screen, I need to solve this direct kinematics problem. It is also used in advanced control schemes, which we will see later. The inverse kinematics problem on the other hand, is as the name implies, we are given only the constant D-H parameters and the 4 by 4 homogeneous transformation matrix of the *n*th link with respect to the 0th link. We have to find the joint variables.

So, this is much harder than the direct kinematics problem. I will show you that, but it leads to a notion of the workspace of a robot this is a very important concept of workspace. So, basically as the name implies, we need to find the region in 3D space where the robot can operate that is in some sense intuitively speaking what the word workspace means. And the inverse kinematics problem will show leads to this notion of this workspace. This problem also needs to be solved for computer visualization of motion and for using advanced control schemes. So, let us continue with rest of the lecture in we look at direct kinematics of serial robots.

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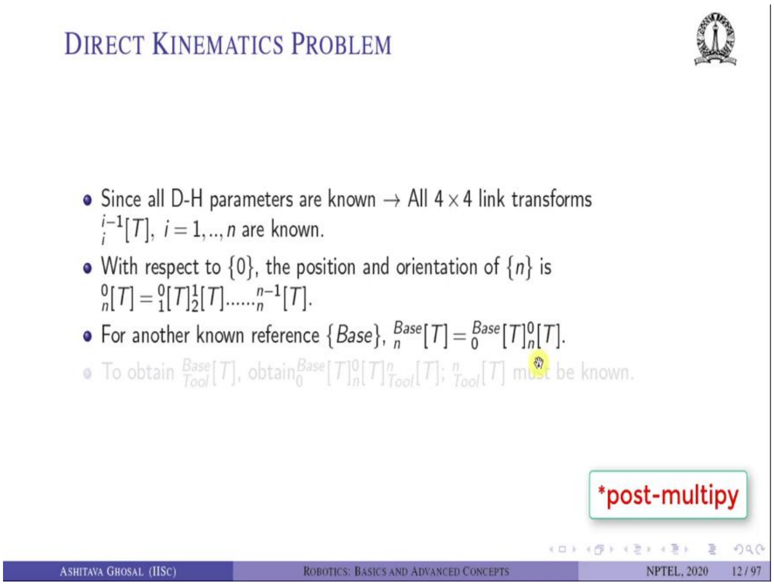

So, as the problem statement was already given - all the D-H parameters are known. So, α_{i-1}, a_{i-1}, d_i and θ_i - even the joint variable θ_i is known. So, if all the D-H parameters are known all the 4 by 4 link transformation matrices are known, ok.

So, remember the link transformation matrix i to with respect to i - 1 contains all the 4 D-H parameters. So, taking each row of the D-H table, I substitute in with the general formula. And I can find each of this link transformation matrix.

With respect to fixed base {0} coordinate system the position and orientation of link n is nothing but the product of these link transformation matrix. So, ${}^{0}_{n}[T]$ is nothing but ${}^{0}_{1}[T]$ ${}^{1}_{2}[T]$ .. ${}^{n-1}_{n}[T]$, ok. So, is the problem direct kinematics problem more or less solved? Yes, because all the D-H parameters are known all the right-hand side is known. So, we just multiply matrices one after another and we can find the position and orientation of the *n*th link with respect to the 0th link.

If you want to find the position and orientation of the last link with respect to some other coordinate system. So, for example, the {*Base*} so then we need to know a transformation matrix between the *Base* to *n* and then that is nothing but a product of two transformation matrix. So, we want to find *Base* to 0 and 0 to *n*.

So, this is sometimes useful problem because suppose I mount the robot on a moving platform ok. So, the origin or the fixed coordinate system for the robot is somewhere on the platform, but the platform itself is moving. So, if you tell me where the platform is with respect to let us say the corner of the room. So, that is {Base} to {0} we can find out and then we can pre-post- pre multiply with ${}^{0}_{n}[T]$. There is also an important thing called the {*Base*} to {*Tool*}.

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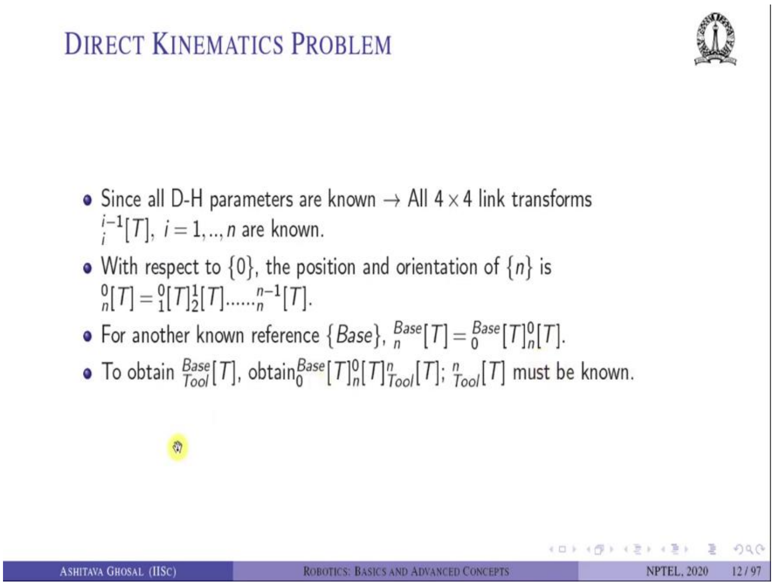

So, remember the {*Tool*} coordinate system is coming after the *n*th joint ok - the origin in our D-H convention is on the joint axis. So, and the link is asked for the joint in a robot that last link, the end-effector can be changed. So, we can have different end-effectors for the same robot.

So, we need a transformation matrix called ${}^{n}_{Tool}[T]$. So, the tool tip with respect to the *n*th coordinate system and if I want to find out where is the end of the tool with respect to the say the corner of the room. Then we multiply we need to know what is ${}^{Base}_{Tool}[T]$ this transformation matrix and all we need to do is we pre multiply ${}^{Base}_{0}[T]\ {}^{0}_{n}[T]\ {}^{n}_{Tool}[T]$.

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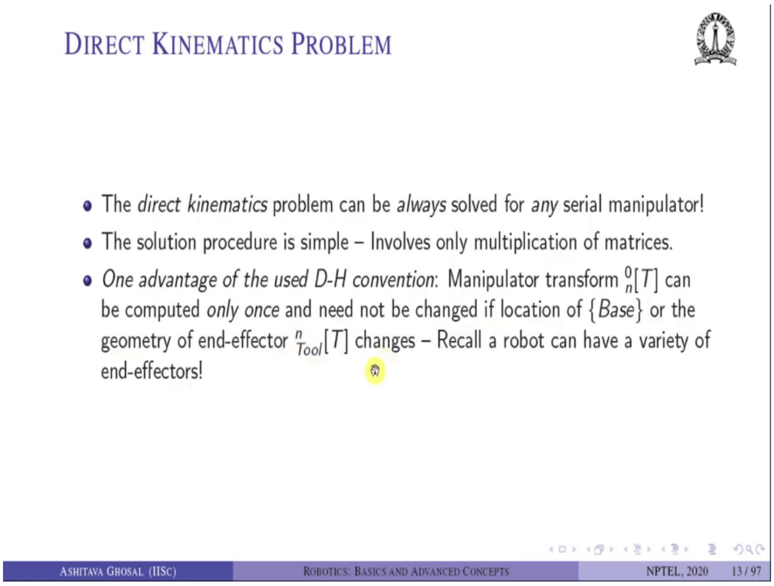

Let us continue the direct kinematics problem can be always solved for any serial robot ok; why? Because it just involves multiplication of matrices. So, there is no reason why we cannot multiply 4 by 4 homogeneous transformation matrices. This matrix multiplication is completely well defined.

So, the solution procedure is very simple it involves only multiplication of matrices. One advantage of the D-H convention that we have used is that the manipulator transformation matrix ${}^{0}_{n}[T]$ can be computed only once and need not be changed if the location of the base or the geometry of the end-effector and tool changes, ok.

This is a very useful reason or very big advantage of the D-H convention that we have used, ok. The other D-H conventions where the origin is not on the joint axis and the link is not after the joint will have some problems. There are advantages of this other conventions, but this is the main advantage. So, I do not need to worry about what is the transformation matrix between $\{Tool\}$ and $\{n\}$. I could use different tools and a robot will actually supposed to use a variety of end-effectors.

So, all we need to do is this transformation matrix ${}^{0}_{n}[T]$. I can pre-program and then whether I mount this robot on a mobile platform or hang it from the roof, which basically means that this base is changing or whether I choose a small tool or a big tool or a different tool altogether only this last portion is changing. The main manipulator transformation matrix ${}^0_n[T]$ need not change, ok.

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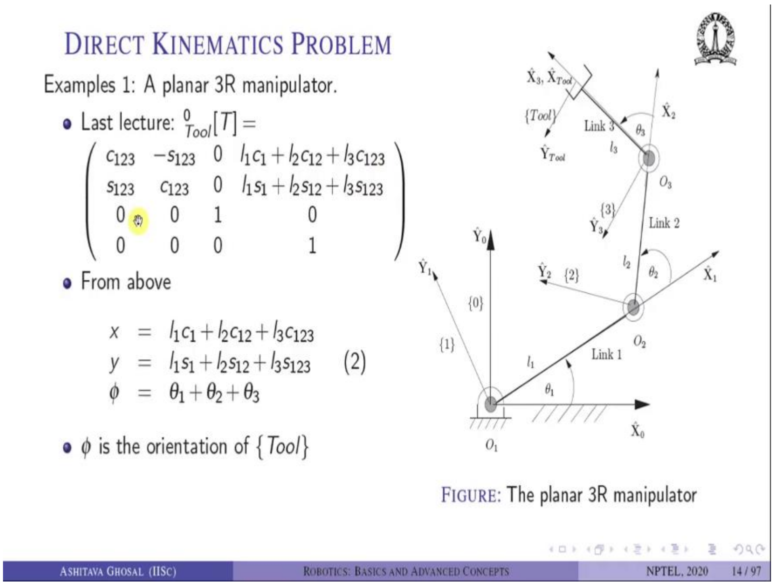

A few quick examples. So the direct kinematics problem for a planar three degree of freedom robot; so, what did we have? We have three joints at O_1, O_2 and O_3. And in the last lecture, I showed you what is this transformation matrix between the {*Tool*} coordinate system which is kept on this schematic parallel jaw gripper, middle of the parallel jaw gripper.

Recall, this tool coordinate system is parallel to the third coordinate system, but translated by a distance l_3, So, ${}^0_{Tool}[T]$ this transformation there is a 3 by 3 rotation matrix and the last column is the position of this point in at the middle of this parallel jaw gripper. And the position was given by l_1 c_1, l_2 c_{12}, l_3 c_{123} that is the X coordinate c_1 here means cosine θ_1, c_{123} here means cosine ($\theta_1 + \theta_2 + \theta_3$) and so on.

So, from the above you can easily see that the x and y are nothing but this first one four element and the two four element and ϕ the orientation of the last link is nothing but $\theta_1 + \theta_2 + \theta_3$. So, the orientation matrix here 3 by 3 rotation matrix gives you ϕ and ϕ is the orientation of the {*Tool*}. So, if I give you θ_1, θ_2, θ_3 and the D-H parameters which are l_1, l_2, l_3 in this case, I can easily find the end effector position and orientation.

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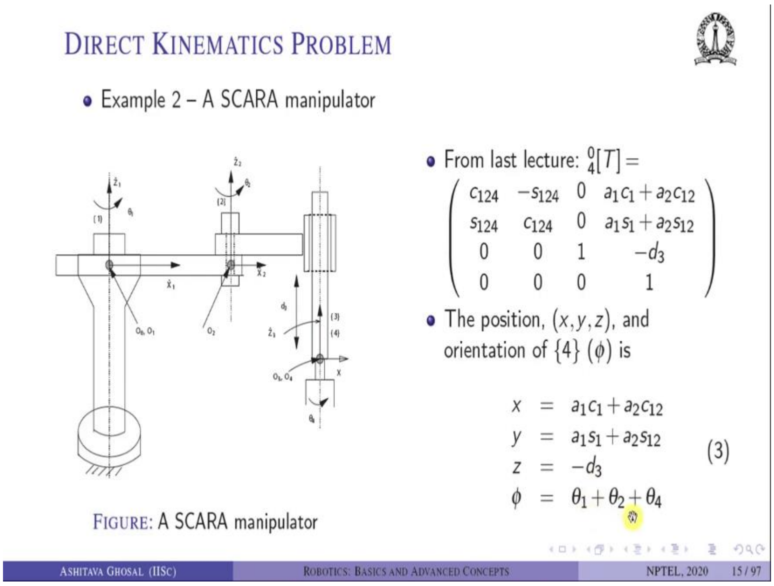

The direct kinematics problem for a SCARA robot can also be solved. As I showed you the SCARA robot consist of four joints along Z_1, Z_2, Z_3 and a final translation and a rotation. So, from the last lecture ${}^0{}_4$[T] was again given by cosine ($\theta_1 + \theta_2 + \theta_4$) and the position of this end effector point x y z was a_1 c_1 + a_2 c_{12} and a - d_3 for the Z coordinate. So, in this case the position of the end effector has x y and z and if the orientation is denoted by ϕ of the end effector we can easily see that the x y and z are given by a_1 c_1 + a_2 c_{12}, y is given by a_1 s_1 + a_2 s_{12} and z is - d_3 and ϕ is $\theta_1 + \theta_2 + \theta_4$.

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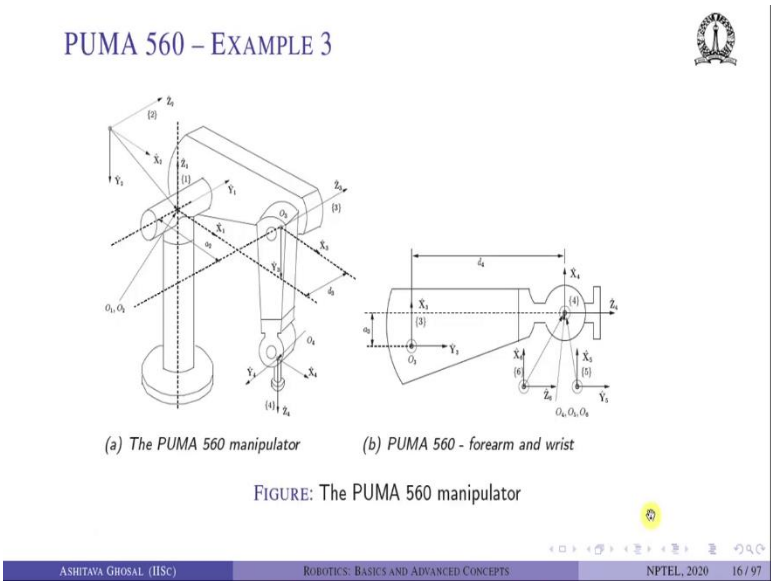

In the case of the PUMA, it is a six degree of freedom robot, spatial robot. And as I have mentioned earlier there is one joint which is vertical apart Z_1, one joint which is perpendicular to Z_1, then another joint which is parallel to Z_2 and then there are these three joints which are intersecting at the wrist point.

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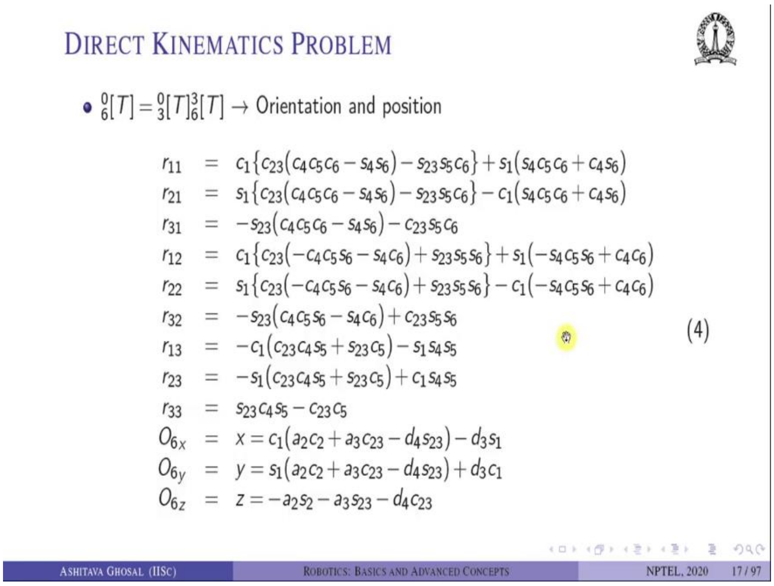

We had again derived the transformation matrix ${}^0_6[T]$ which is nothing but the product of two transformation matrices ${}^0_3[T]$ and ${}^3_6[T]$ - that is from last week. And then if you expand all these things and say that the rotation matrix is r_{11}, r_{21}, r_{31} all the way till r_{33}. You can see that it is a function of θ_1, θ_2, θ_3, θ_4 and θ_5 and θ_6. So, the rotation matrix contains cosine θ_1, cosine θ_4, sine θ_5, sine θ_6; all kinds of sine and cosine of all the six angles.

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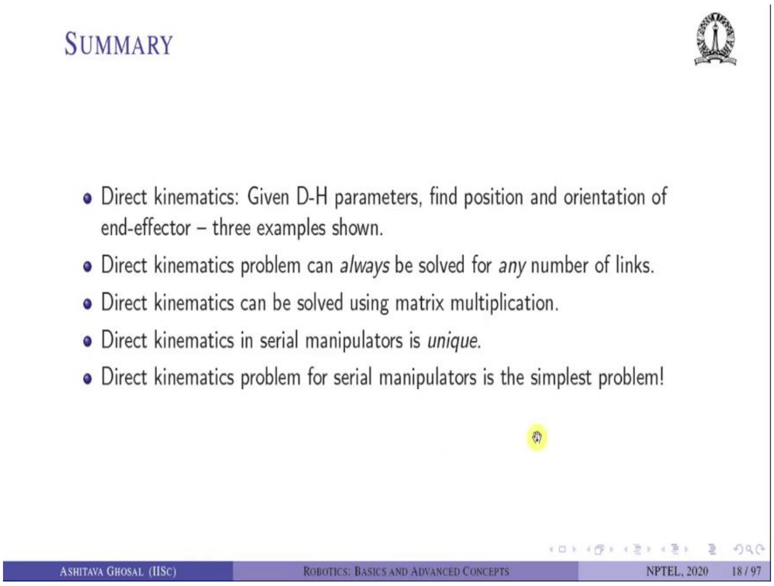

The position vector of the wrist point, which is this point here O_{6x}, O_{6y}, O_{6z} , turns out to be a function of only θ_1, θ_2 and θ_3. And we can show that it is related to the link lengths a_2, a_3, link offset d_4 and the angles θ_1, θ_2 and θ_3 alone. This is an important observation in the case of the PUMA robot.

So, in summary the direct kinematics problem is stated as following: given the D-H parameters, find the position and orientation of the end-effector. And I have shown you with three examples how we can obtain this. The direct kinematics problem can always be solved for any number of links. Why? Because it is nothing but simple matrix multiplication. So, if I give you 10 links, I need to find 10 transformation matrices, link transformation matrices, and I just multiply these matrices.

The direct kinematics in a serial manipulator is unique. If I give you the D-H parameters, then we can find the transformation matrix and the final position and orientation of the last link with respect to the 0th link is unique - that is it is not possible to find more than one solutions. And it turns out that the direct kinematics problem for serial manipulator is the simplest problem. In the next lecture, we will look at inverse kinematics of serial robots.