Sound and Structural Vibration
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Lecture - 27
Impedance and mobility

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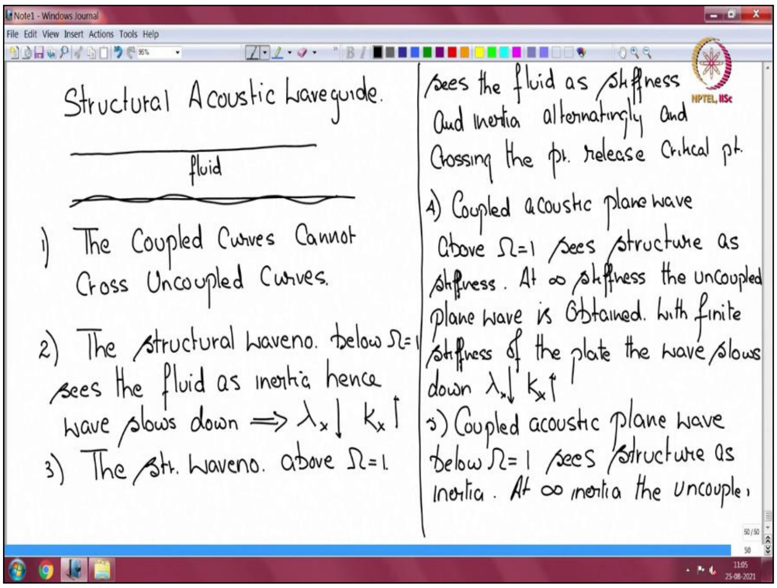

Good morning and welcome to this next lecture on sound and structural vibration. So, today I am going to summarize the entire sound in a structural acoustic waveguide problem, so, structural acoustic waveguide. So, this was the problem, so we had a waveguide with the top rigid plate and a bottom flexing plate with an acoustic fluid. Now I will just summarize one is the coupled curves cannot cross uncoupled curves except at certain critical points.

Two the structural wavenumber below coincidence below coincidence below Ω equal to 1 sees the fluid as inertia hence wave slows down which means the λ_x will drop which means k_x will go up that is what happens. Next the structural wavenumber above coincidence above Ω equal to 1 sees the fluid as stiffness and inertia alternatingly.

And crossing the pressure release point the pressure release critical point where briefly structure and fluid are decoupled. Next the coupled acoustic wavenumber coupled acoustic plane wave above Ω equal to 1 sees structure as stiffness which means at infinite stiffness. Let us say infinite stiffness the coupled the uncoupled plane wave is obtained with finite stiffness of the plate the wave slows down that means λ_x goes down.

And therefore, k_x goes up below coupled acoustic plane wave below Ω equal to 1 sees structure as inertia.

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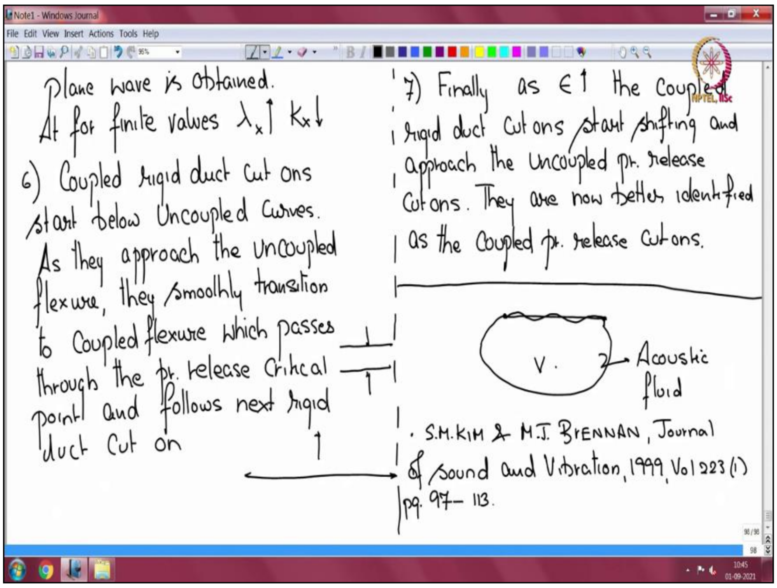

At infinite inertia the uncoupled plane wave is obtained couple plane wave is obtained and for finite values λ_x has to go up reduction of inertia and therefore k_x will start going down. I think point number six. The coupled rigid duct cut ons start below uncoupled curves as they approach the uncoupled flexure, they smoothly transition to coupled flexure which passes through the pressure release point pressure release critical point and follows next rigid duct cut on.

And finally, as ϵ goes up the rigid ducts the let us say coupled rigid duct cut ons start shifting and approach the pressure release cut ons the uncoupled pressure release cut ons they are better identified as the coupled pressure release cut ons so, that is the story of the structural acoustic waveguide. We are now going to start looking at the next problem which is a cavity of acoustic fluid interacting with a panel.

So, this is a flat panel which can vibrate, and this is a cavity with volume V having an acoustic fluid. So, now you can note the progression we did an infinite domain problem an infinite plate an infinite half space then we looked at a structural acoustic waveguide where you know one to one dimension it was infinite in the other dimension it was finite. Now it is a finite realistic system where you have a cavity interacting with a vibrating panel.

So, each will influence the other that is the idea. Now with regard to this I will follow a particular journal paper it is by the following authors S. M. Kim and M. J. Brennan it is a Journal of Sound and Vibration paper year is 1999 the volume is 223 part 1 pages 97-2113 and it starts with a such as a long title.

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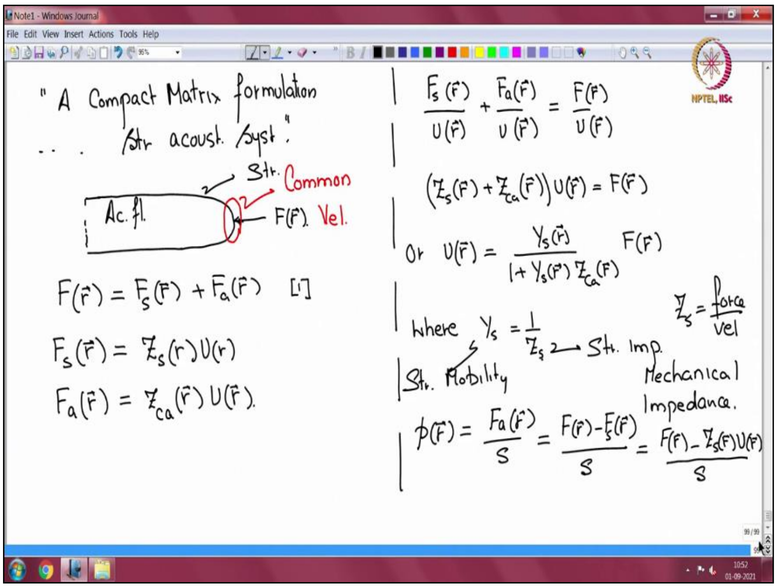

So, it has starts with this title let me put it A compact matrix formulation etcetera formulation with mobility etcetera and structural acoustic system. So, I will follow this particular document almost closely with notation etcetera. Now to begin with suppose I have a system like this, and I apply. So, this is a structure let me see what this is a structure and there is acoustic fluid inside.

So, now suppose I apply a force at some location these are very general ideas. So, nothing is very fixed. So, these are schematics. So, apply a force to the structure. So, what happens is that force will be partly taken up by the structure which I say $F_s(\vec{r})$ and partly by the fluid acoustic fluid $F_a(\vec{r})$. Now the structure deforms the structure deforms through its impedance which I call $Z_s(r)U(r)$.

$$F(\vec{r}) = F_s(\vec{r}) + F_a(\vec{r}),$$

$$F_s(\vec{r}) = Z_s(r)U(r).$$

Now this is a common velocity junction it is a common velocity junction and therefore the force will split, and the velocity will be common. So, that means $U(r)$ is common to the structure and the fluid. So, the force taken up by the fluid is given by the fluid acoustical impedance into the same velocity.

$$F_a(\vec{r}) = Z_{ca}(\vec{r})U(\vec{r}).$$

Now if I use this equation let us call it maybe [1] and I divide both sides by the common velocity.

So, divided by the common velocity

$$\frac{F_s(\vec{r})}{U(\vec{r})} + \frac{F_a(\vec{r})}{U(\vec{r})} = \frac{F(\vec{r})}{U(\vec{r})}.$$

So, now what happens over here so this is equivalent to saying

$$\big(Z_s(\vec{r}) + Z_{ca}(\vec{r})\big)U(\vec{r}) = F(\vec{r}),$$

or alternatively

$$U(\vec{r}) = \frac{Y_s(\vec{r})}{1 + Y_s(\vec{r})Z_{ca}(\vec{r})}F(\vec{r}),$$

where $Y_s = \frac{1}{Z_s}$, Y_s is called structural mobility and Z_s is called structural impedance and it has force over velocity units.

So, Z_s has force over velocity units which mean it has mechanical impedance units mechanical impedance units. Now the applied force gives rise to a pressure also. So, that pressure in the fluid is given by the force taken up by the fluid which is r by some area some notional area. So, this becomes equal to the total force minus what is taken up by the structure divided by the area.

$$p(\vec{r}) = \frac{F_a(\vec{r})}{S} = \frac{F(\vec{r}) - F_s(\vec{r})}{S} = \frac{F(\vec{r}) - Z_s(\vec{r})U(\vec{r})}{S}.$$

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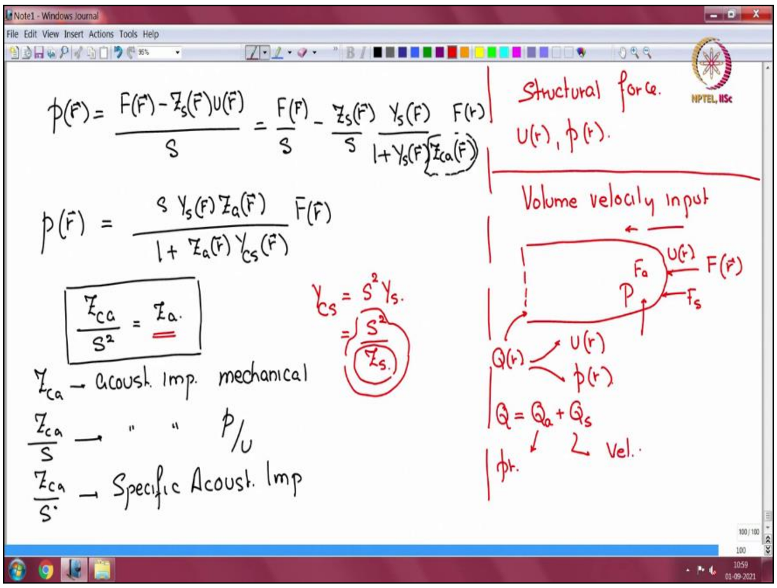

Now rewriting

$$p(\vec{r}) = \frac{F(\vec{r}) - Z_s(\vec{r})U(\vec{r})}{s} = \frac{F(\vec{r})}{s} - \frac{Z_s(\vec{r})}{s}\frac{Y_s(\vec{r})}{1 + Y_s(\vec{r})Z_{ca}(\vec{r})}F(\vec{r}).$$

I should just mention that typically when we say acoustic impedance its pressure over velocity. So, I am kind of using a sort of incorrect language this is acoustical impedance in the units of mechanical impedance, mechanical impedance related to acoustics.

So, then what happens to this, this becomes finally equal to

$$p(\vec{r}) = \frac{sY_s(\vec{r})Z_a(\vec{r})}{1 + Z_a(\vec{r})Y_{cs}(\vec{r})}F(\vec{r})$$

So, what is Z_{ca}? $\frac{Z_{ca}}{s^2} = Z_a$. So, Z_{ca} is acoustical impedance in mechanical impedance units force over velocity units.

So, Z_{ca} divided by a single s is actually the acoustic impedance which is pressure or velocity which is pressure by velocity then $\frac{Z_{ca}}{s^2}$ is what we call specific acoustic impedance force over area square units. So, that is what Z_a is. So, Z_a the acoustical part is in proper acoustical units. Similarly let me write here Y_{cs} what is Y_{cs} that is s^2Y_s or $\frac{s^2}{Z_s}$.

So, Z_s is mechanical units. So, $\frac{Z_s}{s^2}$ is like your specific acoustic impedance related to structure. So, Y_{cs} is inverse of that. So, Z_s and Y_s are proper structural mechanical impedance units where as Z_a is the appropriate specific acoustic impedance. So, for a structural force for a structure rule force applied we have found the velocity common velocity and the pressure inside the fluid we have done that so far.

Now if you see what happens if there is a volume velocity input volume velocity input if I draw the picture again. So, I have domain let me keep an opening I applied a force. So, this direction is kind of positive you can see. So, there is a velocity in this direction. So, this is taken up by F_a and F_s and inside there is a pressure. Now if a volume velocity is given. Now if volume velocity is given effectively here a volume velocity is given.

So, that will also generate a velocity and that will also generate a pressure. Now what happens the Q also gets divided by a compressive part related to acoustics and a deformation part that the structure takes up. So, the Q_a gives rise to pressure and the Q_s part gives rise to velocity. So, how does that happen?

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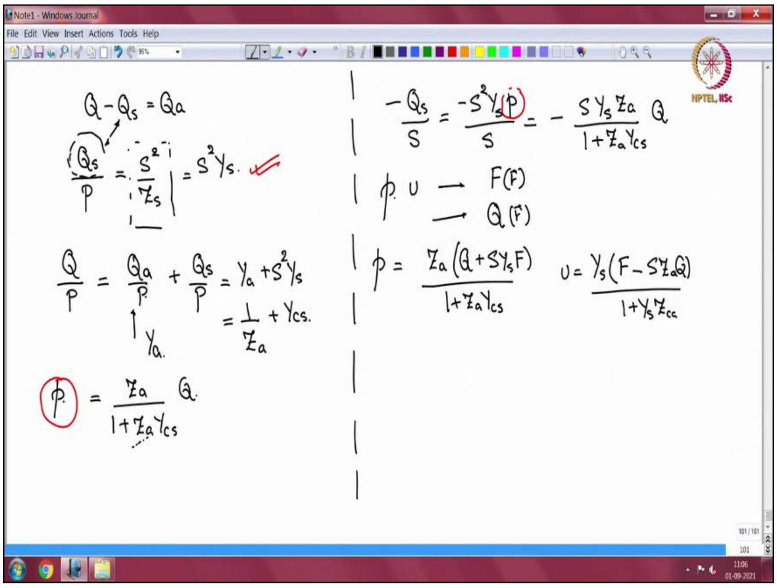

So, one thing you should note here is that a positive force here generates a velocity in one direction that causes an increase in pressure you can see from the directions and so forth whereas if a volume velocity is given and that causes a rise in pressure that generates a velocity it is going to be opposed to what the force generates that is actually the cracks there is a sign convention we have to observe that is the crux of the matter over here.

So, now let us see my $Q - Q_s = Q_a$ and Q_s is going to be related to the pressure Q_s is related to the positive pressure that is the pressure that is going to give take that deformation. So, that

$$\frac{Q_s}{p} = \frac{s^2}{Z_s} = s^2 Y_s.$$

So, the pressure that got generated deforms the structure and to this extent deforms the structure that is this Q_s.

Now if I have the total input volume velocity over pressure that is equal to

$$\frac{Q}{p} = \frac{Q_a}{p} + \frac{Q_s}{p} = Y_a + s^2 Y_s = \frac{1}{Z_a} + Y_{cs}.$$

So, what happens to now my p here,

$$p = \frac{Z_a}{1 + Z_a Y_{cs}} Q.$$

You can see here that if Y_{cs} is 0 that means the structure is infinitely stiff then p and Q are directly related through Z_a. Now here is the with Q_s the way Q_s deforms due to the pressure we have to bring in a sign over here because that velocity is in the opposite direction. So, this becomes equal to

$$\frac{-Q_s}{s} = \frac{-s^2 Y_s p}{s} = -\frac{s Y_s Z_a}{1 + Z_a Y_{cs}} Q.$$

This pressure found from this. So, finally what do we have we have pressure and velocity due to a force applied to the structure due to a volume velocity put in into the structure we have. So, my pressure is equal to

$$p = \frac{Z_a (Q + s Y_s F)}{1 + Z_a Y_{cs}},$$

$$U = \frac{Y_s (F - s Z_a Q)}{1 + Y_s Z_{ca}}.$$

Let us see, I will stop this lecture here we will continue next time, thanks.