Sound and Structural Vibration
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Lecture - 29
Derivation of Vibro-Acoustic response continued

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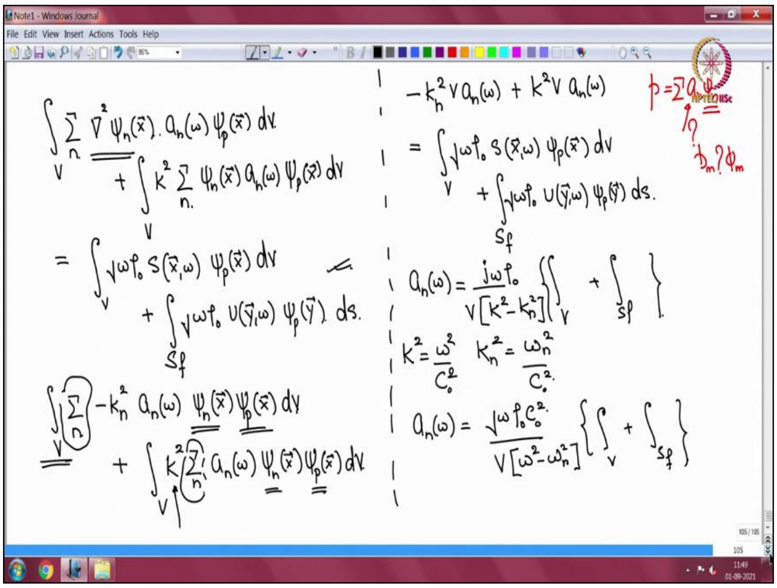

Good morning welcome to this next lecture on sound and structural vibration. See we ended up last time looking at this equation here I have

$$\int \sum_n \nabla^2 \psi_n(\vec{x})\, a_n(\omega)\, \psi_p(\vec{x})\, dV$$

$$+ \int k^2 \sum_n \psi_n(\vec{x}) a_n(\omega)\ \psi_p(\vec{x})\ dV = \int j\omega\rho_0 S(\vec{x},\omega)\, \psi_p(\vec{x})\, dV$$

$$+ \int j\omega\rho_0 U(\vec{y},\omega)\psi_p(\vec{y})\, dS.$$

So, now I am going to use what I showed last time the operation on del square gives me minus. So, I have

$$\int \sum_n -{k_n}^2 a_n(\omega)\psi_n(\vec{x})\, \psi_p(\vec{x})\, dV + \int k^2 \sum_n a_n(\omega)\psi_n(\vec{x})\, \psi_p(\vec{x})\, dV$$

The other side there is not much of a change. So, let us look at this part. So, the right-hand side does not much of change. So, let us look at this part. So, now I will get this integral over $\psi_n(\vec{x})$n and $\psi_p(\vec{x})$ if I use orthogonality, it will be 0 if n is not equal to p. So, out of the sum only one fellow is picked out same here this is 0 if n and p are not equal therefore again only one term is picked up.

So, that becomes equal to $-{k_n}^2 V a_n(\omega)$ integral gives me the volume back and then I get an $a_n(\omega)$ and this k is the forcing k. Now you should remember that this k is not the homogeneous k this k is the forcing k. So, this will be $k^2 V a_n(\omega)$ and the other side is what? Now let me just repeat it, it is $\int j\omega\rho_0 S(\vec{x},\omega)\,\psi_p(\vec{x})\,dV + \int j\omega\rho_0 U(\vec{y},\omega)\psi_p(\vec{y})\,dS$.

$$-{k_n}^2 V a_n(\omega) + k^2 V a_n(\omega) = \int j\omega\rho_0 S(\vec{x},\omega)\,\psi_p(\vec{x})\,dV + \int j\omega\rho_0 U(\vec{y},\omega)\psi_p(\vec{y})\,dS.$$

So, now we want a_n. So, we should not forget what the unknown pressure is, was what a summation over $a_n\psi_n$, ψ_n are the known mode shape shapes a_n is the unknown. So, we want a_n. Similarly, we want b_m because ϕ_m are known mode shapes we want b_m. So, now what do we have we have a_n which I want that is equal to $\frac{j\omega\rho_0}{V[k^2-{k_n}^2]}\{\int dV + \int dS\}$.

We will also do one more thing k^2 is of course $\frac{\omega^2}{{c_0}^2}$ the forcing frequency ω, ${k_n}^2$ we will write it as $\frac{{\omega_n}^2}{{c_0}^2}$ which is now the natural frequency of the nth mode by ${c_0}^2$, so, one more time

$$a_n(\omega) = \frac{j\omega\rho_0{c_0}^2}{V[\omega^2-{\omega_n}^2]}\left\{\int dV + \int dS\right\}.$$

So, there is a compact notation.

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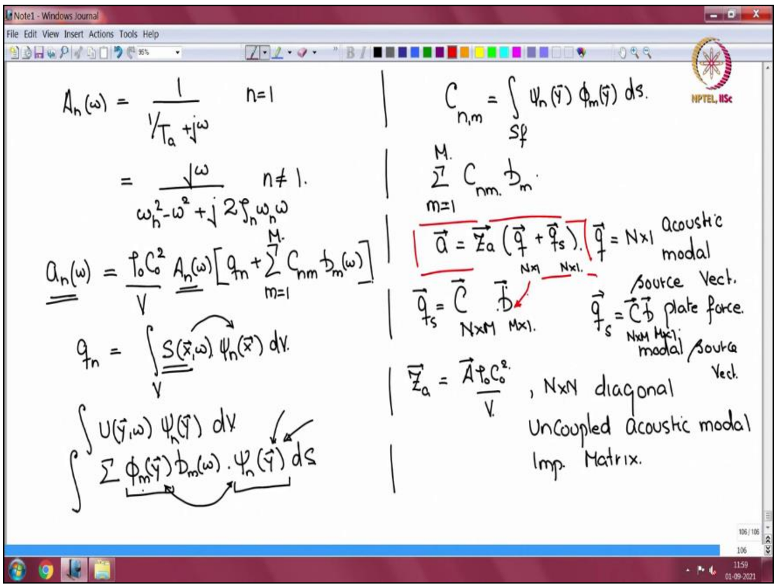

Now possible we will write

$$A_n(\omega) = \frac{1}{\frac{1}{T_a} + j\omega}.$$

This is n equal to one the rigid body mode it does not have a natural frequency, but it just has a relaxation time. Then we have the other as $\frac{j\omega}{{\omega_n}^2 - \omega^2 + j2\zeta_n\omega_n\omega}$, this is for n not equal to 1.

So, what does a_n becomes in some compact form

$$a_n(\omega) = \frac{\rho_0 {c_0}^2}{V} A_n(\omega) \left[q_n + \sum_{n=1}^{M} C_{nm} b_m(\omega) \right]$$

where what have we done what we have done is first of all q_n is of course the integral over the volume of the source with a particular motor shape there are n of this.

$$q_n = \int S(\vec{x}, \omega) \psi_n(\vec{x})\, dV.$$

So, it is kind of the modal forcing factor modal volume velocity factor may be how much of this source contributes to a particular mode on top of that we have the second integral was the integral of the velocity again multiplied by ψ at the panel surface.

So, this we replace with $\int \sum \phi_m(\vec{y}) b_m(\omega) \psi_n(\vec{y})\, ds$. So, it is now an integral of one type of mode shape with the acoustic mode shape at the plate surface. So, this now is there are m of these there are n of these. So, it is now in there are n into m elements. Now there are n into m elements. So, that is given the nomenclature C_{nm} is equal to $\int \psi_n(\vec{y}) \phi_m(\vec{y})\, ds$. So, it is now an n cross m matrix it is an n cross m matrix.

So, what is that term? Now so, this term ends up looking like a $\sum_{m=1}^{M} C_{nm} b_m$. So, here m is equal to 1 to capital M the single n mode coupling to several m modes. So, this is there are m of this. So, that is your $C_{nm} b_m$ in a further compact notation we will write it as

$$\vec{a} = \overrightarrow{Z_a}(\vec{q} + \overrightarrow{q_s}).$$

So, this $\vec{q}$ is an N cross one acoustic modal source vector and whereas $\overrightarrow{q_s}$ is the same given by $C_{N\times M} b_{M\times 1}$ panel or plate force modal source vector this is N cross M and this is b M cross 1.

So, it is easy to easy to understand and what is $\overrightarrow{Z_a}$? $\overrightarrow{Z_a}$ is $\frac{\vec{A}\rho_0 {c_0}^2}{V}$, $\overrightarrow{Z_a}$ is an N cross N is called the diagonal uncoupled acoustic modal impedance matrix. So, we have arrived it at the acoustic amplitudes in terms of the other forces and then the acoustic amplitude contains b which is also an unknown. So, we have to formulate a second equation what is the second equation?

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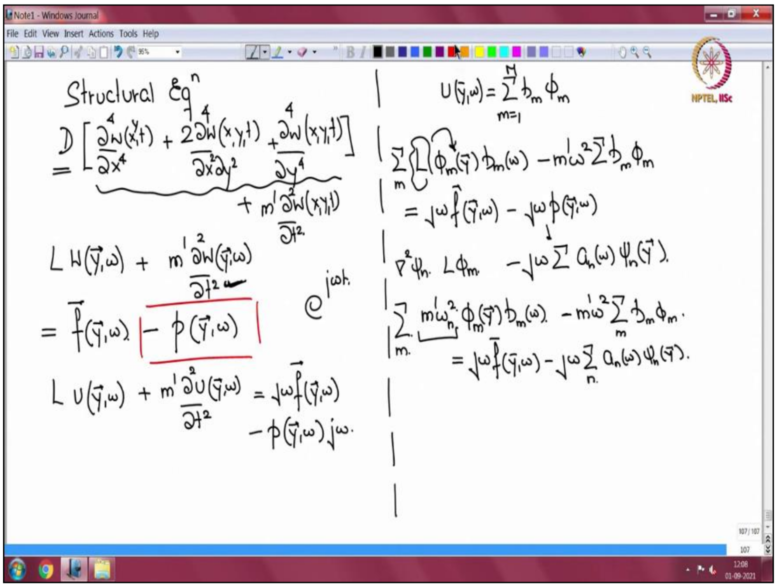

It is the structural equation the structural equation what is that structural equation to begin with there is. So, just for the plate suppose I show you what it is for a plate some

$$D\left[\frac{\partial^4 W(x,y,t)}{\partial x^4}+\frac{2\partial^4 W(x,y,t)}{\partial x^2\partial y^2}+\frac{\partial^4 W(x,y,t)}{\partial y^4}\right]+m'\frac{\partial^2 W(x,y,t)}{\partial t^2}.$$

So, we will say we have an operator that is acting on my plate $LW(\vec{y},\omega)+m'\frac{\partial^2 W(\vec{y},\omega)}{\partial t^2}$. So, now there is a force applied to it force applied to it over y and force is force per unit area units that you should know. Now and here is the crux we showed you I mean I showed you in the very beginning how the pressure due to a volume velocity can generate a velocity opposed to what the force generates.

So, here I choose minus the pressure from the cavity acting on the panel. This is the one of the issues that one should remember in structure sound structure interaction problems.

$$LW(\vec{y},\omega)+m'\frac{\partial^2 W(\vec{y},\omega)}{\partial t^2}=f(\vec{y},\omega)-p(\vec{y},\omega).$$

So, now I will we are doing everything in terms of velocity. So, we will do that. So, if we do that in velocities

$$LU(\vec{y},\omega)+m'\frac{\partial^2 U(\vec{y},\omega)}{\partial t^2}=j\omega f(\vec{y},\omega)-p(\vec{y},\omega)j\omega.$$

Now let us go back to using our modal sum the $U(\vec{y},\omega)$ is a model sum with m going from 1 to capital M this time.

$$U(\vec{y},\omega)=\sum_{m=1}^{M} b_m\phi_m.$$

So, now if you substitute is this, I have this

$$\sum_{m=1}^{M} L\,\phi_m(\vec{y})b_m(\omega) - m'\omega^2 \sum b_m\phi_m = j\omega f(\vec{y},\omega) - p(\vec{y},\omega)j\omega.$$

$$\sum_{m=1}^{M} L\,\phi_m(\vec{y})b_m(\omega) - m'\omega^2 \sum b_m\phi_m = j\omega f(\vec{y},\omega) - j\omega \sum a_n(\omega)\,\psi_n(\vec{y}).$$

So, now what happens we perform this L operator on ϕ_m and what that does is I should get

$$\sum_{m=1}^{M} m'{\omega_m}^2\phi_m(\vec{y})b_m(\omega) - m'\omega^2 \sum_{m=1}^{M} b_m\phi_m = j\omega f(\vec{y},\omega) - j\omega \sum_{n=1}^{N} a_n(\omega)\,\psi_n(\vec{y}).$$

Now we multiply this we are running out of time I will stop the lecture here we will continue from here the next class.