Sound and Structural Vibration
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Lecture – 31
Numerical Example

Good morning and welcome to this following lecture on sound and structural vibration. We are looking at a panel vibrating with a backed cavity.

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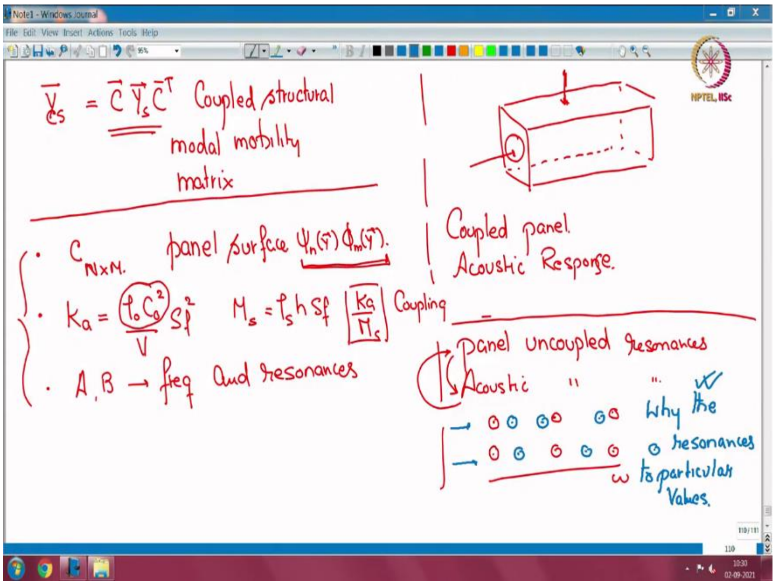

And last time I showed you this schematic here. So, we will look at it the problem that Kim and Brennan have solved. So, there is a box. The top cover is actually a vibrating plate with simple support conditions here and then it is back by this rigid cavity, the other 5 walls are rigid. So, one is you will have a point force excitation from the panel, so, we will look at vibration and sound.

Similarly, will excite it with an acoustic speaker and again look at vibration, sound and the paper describe theory and an experiment. So, I will now show you the paper.

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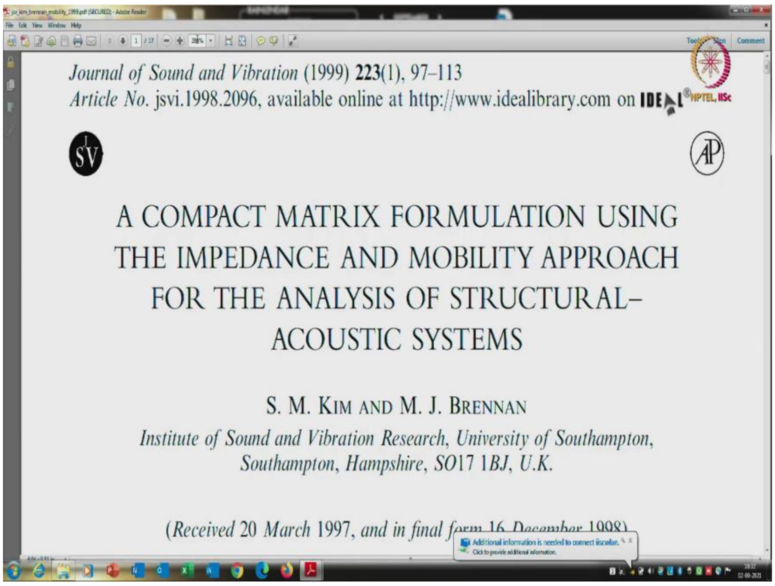

So, this is the paper here published in journal of sound and vibration 99 volume, 223 number 1 pages 97 to 113 and the full title compact matrix formulation using the impedance and mobility approach for the analysis of structural acoustic systems. So, let me move down so, this is the initial mobility and impedance part that I have described. We should be able to read the paper now then some initial definitions everything is here. I have described in the lecture, so, now they go to an example. This is the example.

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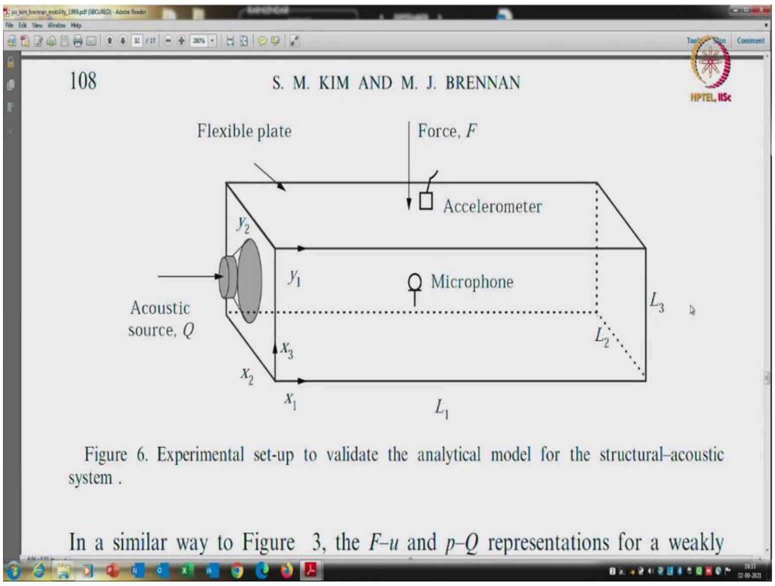

So, there is this box now, top cover is flexible, vibrates, other walls are rigid and it can be driven by a point force using an electromagnetic shaker and this wall can be; there is a speaker mounted that can drive the acoustic space. So, L_1, L_2 and L_3 are the dimensions.

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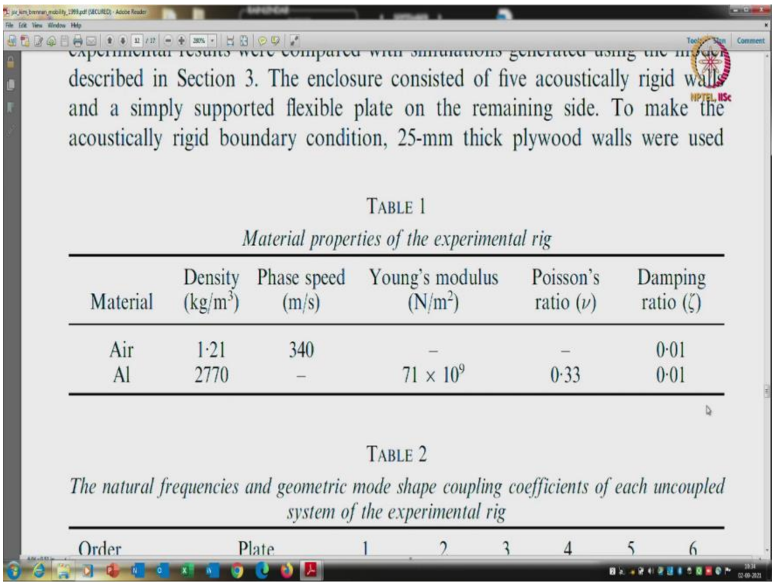

described in Section 3. The enclosure consisted of five acoustically rigid walls and a simply supported flexible plate on the remaining side. To make the acoustically rigid boundary condition, 25-mm thick plywood walls were used

TABLE 1

Material properties of the experimental rig

Material	Density (kg/m^3)	Phase speed (m/s)	Young's modulus (N/m^2)	Poisson's ratio (ν)	Damping ratio (ζ)
Air	1·21	340	–	–	0·01
Al	2770	–	71 × 10^9	0·33	0·01

TABLE 2

The natural frequencies and geometric mode shape coupling coefficients of each uncoupled system of the experimental rig

Order Plate 1 2 3 4 5 6

So, this describes the experimental setup and the parameter values, air density, phase, speed and the boxes aluminum or the vibrating panel is aluminum. The rest of the box is wooden, but the vibrating part is aluminum, and these are the values taken for the aluminum portion.

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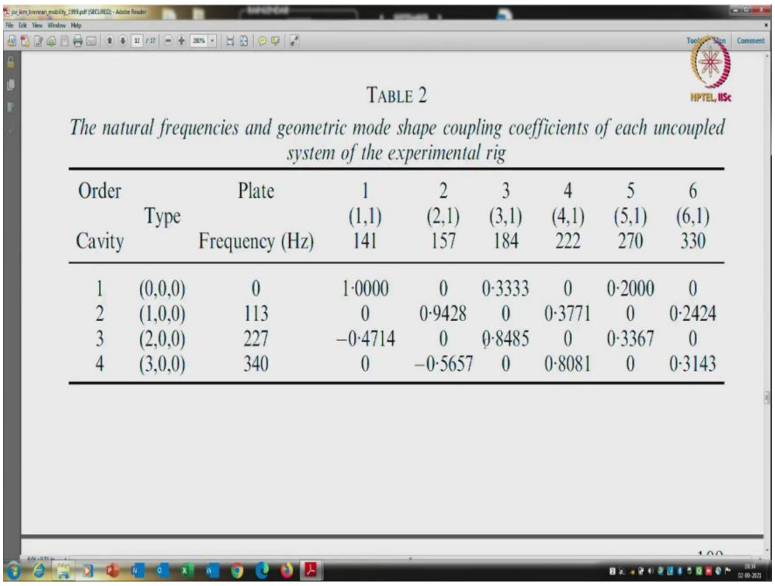

TABLE 2

The natural frequencies and geometric mode shape coupling coefficients of each uncoupled system of the experimental rig

Order (Cavity)	Type	Plate Frequency (Hz)	1 (1,1) 141	2 (2,1) 157	3 (3,1) 184	4 (4,1) 222	5 (5,1) 270	6 (6,1) 330
1	(0,0,0)	0	1·0000	0	0·3333	0	0·2000	0
2	(1,0,0)	113	0	0·9428	0	0·3771	0	0·2424
3	(2,0,0)	227	−0·4714	0	0·8485	0	0·3367	0
4	(3,0,0)	340	0	−0·5657	0	0·8081	0	0·3143

And these, of course, are the uncoupled resonances and the C n m values are very important. Ah Here are the plate, uncoupled resonances mode, 1 1, 2 1, 3 1, 4 1, 5 1, 6 1 and these are the uncoupled acoustic mode numbers and resonances. And these are the C_{nm} values I will speak more about this after this paper is done. These are very important.

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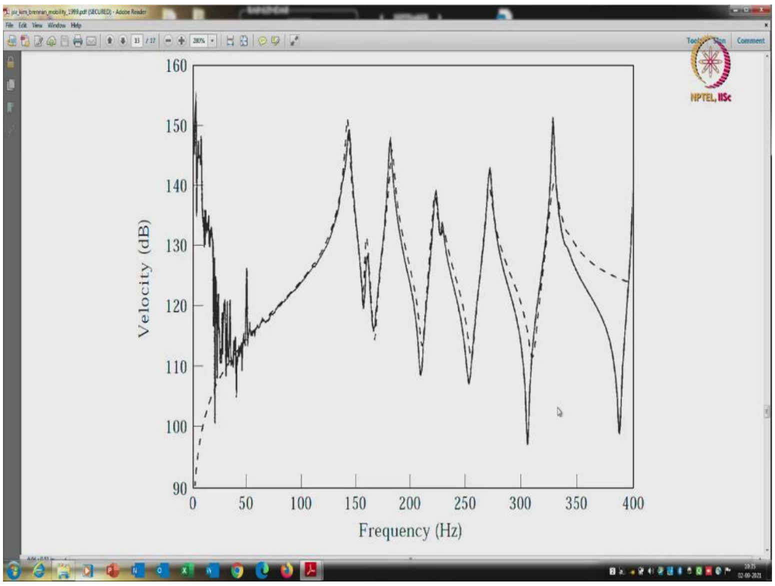

So, now, if you look at the velocity, this is the velocity response of the panel. So, one is one line is theory and the other line is experiment.

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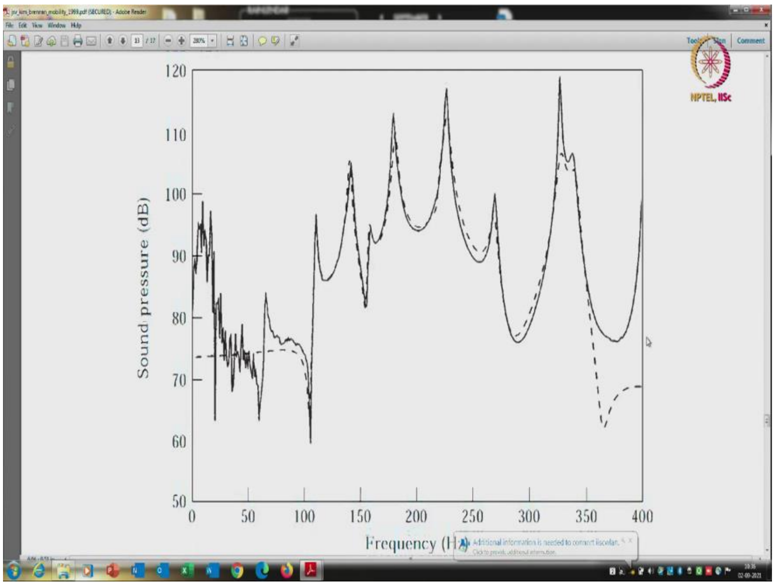

And this is the sound pressure. So, this is for point force excitation, there is a location where a point force is given, and these are the values, and you can more or less see that these peaks come where you have panel resonance or acoustic resonance. These are resonances of the system and because, actually it is air, it is very lightly loaded. So, the uncoupled resonances, more or less remain, they do not change very much. The next is due to acoustic excitation.

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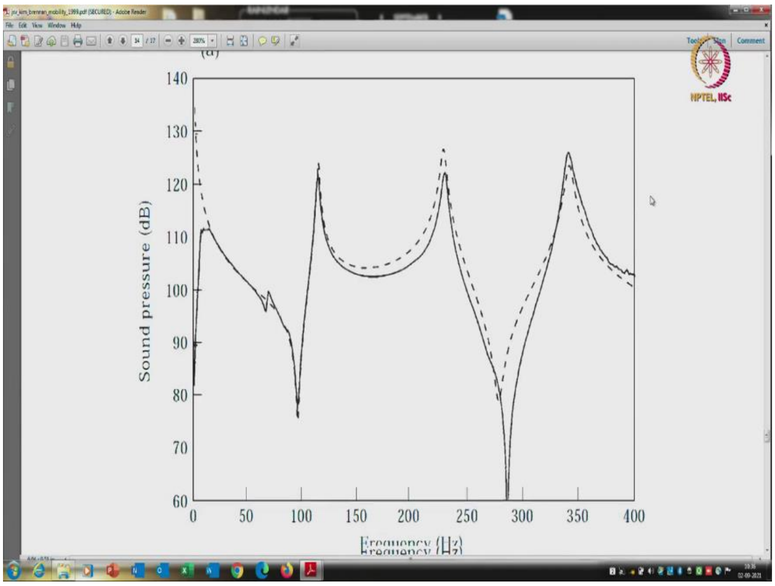

This is the sound pressure inside for experimental acoustic excitation a acoustic response with lymph masses placed on plate. A rigid, bald condition was assumed in the simulations b acoustic masses removed from the plate full coupled coupling was assumed in the simulations. So, this is the coupled response due to acoustic excitation and again these peaks, you see, are all the original resonances of the system.

So, air being light more or less, the uncoupled resonances hold, and these are the dimensions L 1, L 2, L 3 and the thickness of the plate is 5 millimeters and so, forth. So, using the derivation I have presented and this paper you should be able to get these results quite easily. Now I move back to my iPad and move forward with the next portion of the lecture. Now we will continue with the same problem and get some insights now. The insights proceed as follows.

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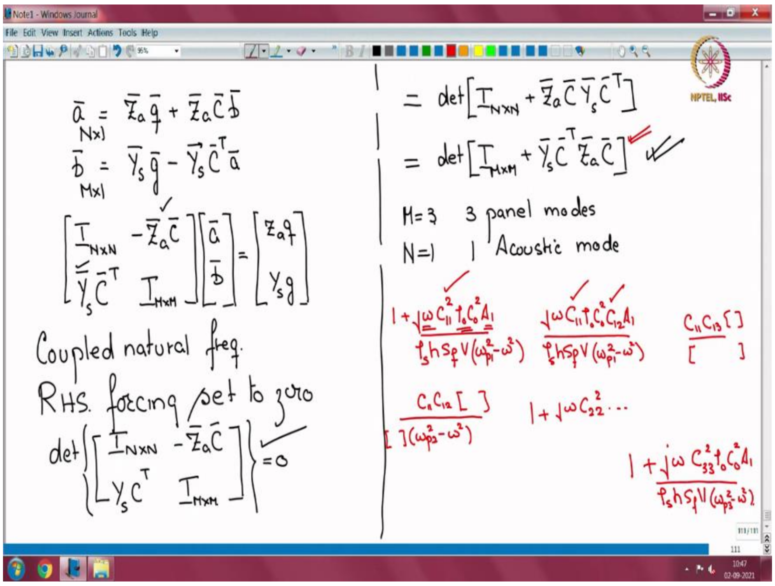

So, we have the acoustic amplitudes given by these set of equations. We can combine them in a matrix form. I get

$$\begin{bmatrix} I_{N\times N} & -\vec{Z}_a\vec{C} \\ \vec{Y}_s\vec{C}^T & I_{M\times M} \end{bmatrix}\begin{bmatrix} \vec{a} \\ \vec{b} \end{bmatrix} = \begin{bmatrix} \vec{Z}_a\vec{q} \\ \vec{Y}_s\vec{g} \end{bmatrix}.$$

So, $\vec{a}$ is N cross 1 and $\vec{b}$ is M cross 1. Now we want to find the coupled natural frequencies.

That means the plate is influenced by the acoustic space, so, its natural frequencies change, and the acoustic space is modified by the flexible wall and its natural frequency is changed. So, we want the coupled natural frequencies. And the way is we set the forcing to 0 the side forcing, external forcing functions set to 0 and we take the determinant of the remaining matrix. So, we take the determinant of the matrix.

As you know, in $\vec{Z}_a$ in $\vec{Y}_s$ you have the frequency term so, that generates a polynomial which will give you the frequencies. So, we have determinant of $\begin{bmatrix} I_{N\times N} & -\vec{Z}_a\vec{C} \\ \vec{Y}_s\vec{C}^T & I_{M\times M} \end{bmatrix}$. We take the determinant of this matrix and we set it to 0. That is what we do. Now very interestingly, it turns out that this determinant, this determinant is equal to, identically equal to the determinant of another matrix which is $\left[I_{N\times N} + \vec{Z}_a\vec{C}\vec{Y}_s\vec{C}^T\right]$.

It is interesting result also equal to the determinant of $\left[I_{M\times M} + \vec{Y}_s\vec{C}^T\vec{Z}_a\vec{C}\right]$. So now, that has to be set to 0. Obviously and we will take this form, will take the latter form, will take this separate this form and deal with it. Now so, suppose we consider M as equal to 3. That means 3 panel modes or 3 plate modes. It is an approximation, so, we can consider as many modes in acoustics or panel as we want. Because you deal with the modal sum you truncate somewhere.

So, you can consider as many as you want as a study to begin with. So, we will take now N equal to 1 which is 1 acoustic mode. If we do that, then I am going to write this in here. So, this determinant looks like this. It looks a little I mean that matrix first of all looks like this. It looks like

$$1 + \frac{j\omega {C_{11}}^2 \rho_0 {c_0}^2 A_1}{\rho_s h S_f V\left({\omega_{p1}}^2 - \omega^2\right)}$$

ω_{p1} is the first panel uncoupled resonance value. The 1,2 term in the matrix will be $\frac{j\omega C_{11}\rho_0 c_0{}^2 C_{12} A_1}{\rho_s h S_f V(\omega_{p1}{}^2-\omega^2)}$. So, you can see that this was $C_{11}{}^2$. This is $C_{11}C_{12}$ everything else is same.

So, you will have $C_{11}C_{13}$ and all else is same and in here the 1 will not be there. So, you will have $C_{11}C_{12}$ and all other constants and $\left(\omega_{p2}{}^2 - \omega^2\right)$ and all other constraints in the numerator $j\omega\rho_0 c_0{}^2 A_1$ etcetera. And here again you will have 1 + $j\omega C_{12}{}^2$ etcetera. In here you will have $1 + \frac{j\omega C_{13}{}^2\ \rho_0 c_0{}^2 A_1}{\rho_s h S_f V(\omega_{p3}{}^2-\omega^2)}$.

So, this is a matrix. So, there will be element here, similarly, there will be element here. Similarly, so, this is the matrix. Now, just remember the form of this matrix.

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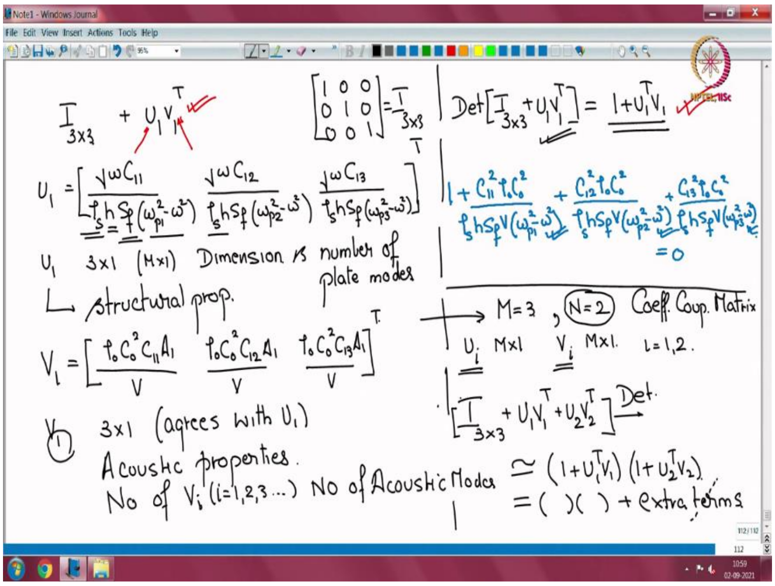

This can be given by or written down as $I_{3\times3} + U_1 V_1{}^T$. So, it can be written so simply and what is U_1? U_1 is given by

$$\left[\frac{j\omega C_{11}}{\rho_s h S_f V\left(\omega_{p1}{}^2 - \omega^2\right)} \quad \frac{j\omega C_{12}}{\rho_s h S_f V\left(\omega_{p2}{}^2 - \omega^2\right)} \quad \frac{j\omega C_{13}}{\rho_s h S_f V\left(\omega_{p3}{}^2 - \omega^2\right)}\right]^T.$$

So, U_1 is 3 cross 1 or rather it is M cross 1. So, the dimension is equal to number of panel modes plate modes that we are considering, and U_1 is related to structural properties. Plate properties, structural properties, plate density, plate thickness, plate, surface area, plate resonances. So, you can see that plate this related to structure. Now let us look at V_1. V_1 is given by

$$V_1 = \left[\frac{\rho_0 {c_0}^2 C_{11} A_1}{V} \quad \frac{\rho_0 {c_0}^2 C_{12} A_1}{V} \quad \frac{\rho_0 {c_0}^2 C_{13} A_1}{V}\right]^T.$$

So, V_1 is also 3 cross 1. That means it agrees with U_1. But it has all the acoustic properties and the number of V's. How many V's will be there? The number of V vectors that is V_i, whether 1 or 2 or 3, depends on number of acoustic modes considered that is the structure. That means what that complicated looking matrix first of all, can be represented very easily like this and U_1 has structural properties.

V_1 carries acoustic cavity properties. Now, even more interestingly now, if we are looking, at the determinant of this. So, the determinant of now $I_{3\times3}$ is a 3 by 3 identity matrix. It looks like this $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ that is $I_{3\times3}$. So, we are looking for the

$$Det\left[I_{3\times3} + U_1 {V_1}^T\right] = 1 + {U_1}^T V_1.$$

So, let me write that also. So, how does it look? How does this look? This looks like this, it looks like

$$1 + \frac{{C_{11}}^2 \rho_0 {c_0}^2}{\rho_s h S_f V\left({\omega_{p1}}^2 - \omega^2\right)} + \frac{{C_{12}}^2 \rho_0 {c_0}^2}{\rho_s h S_f V\left({\omega_{p2}}^2 - \omega^2\right)} + \frac{{C_{13}}^2 \rho_0 {c_0}^2}{\rho_s h S_f V\left({\omega_{p3}}^2 - \omega^2\right)},$$

that is the determinant.

We will set this to 0, to figure out my new solutions of ω later. Now, what would be the structure if now we choose M equal to 3? That means 3 panel modes and we choose N equal to 2 that is 2 acoustic modes. So, use there will be U_i and the U_i will have M cross 1 dimension and therefore the V's, V_i is also we have will have M cross 1 dimension nut now how many U's and how many V's will be there. That is decided by the number of acoustic modes.

So, i will run from 1 to 2 and I call it the coupling matrix. Let us give it a name. We will call it the coefficient, so, this matrix here that we have will give it a name. We will call it the coefficient coupling matrix coefficient coupling matrix or coupling matrix. So, whenever I refer to that, so, how does this coupling matrix look like for this situation of 3 panel modes and 2 acoustic modes? It can be written as $\left[I_{3\times3} + U_1 {V_1}^T + U_2 {V_2}^T\right]$. That is how the matrix looks like very simple.

Now, the next level is I want the determinant of this. We have to compute the determinant of this matrix and how does the determinant look? It approximately not exactly this was exact, till here it was exact but here how does the determinant look? It looks approximately like $\left(1 + {U_1}^T V_1\right)\left(1 + {U_2}^T V_2\right)$. But what is it exactly? It is this product plus some extra terms.

There are some extra terms, these extra terms become negligible, provided the fluid loading is low. That means fluid density is small compared to the plate density. So, this this approximation becomes better and better as the fluid density relative to plate density goes down. Right now, the time is up. So, we will continue this discussion in the next class. Thanks.