Sound and Structural Vibration

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Lecture - 39
Physics of the Vibration Spectrum in 2-D

Good morning and welcome to this next lecture on sound and structural vibration. Last time we looked at the frequency domain explanation for coincidence phenomenon.

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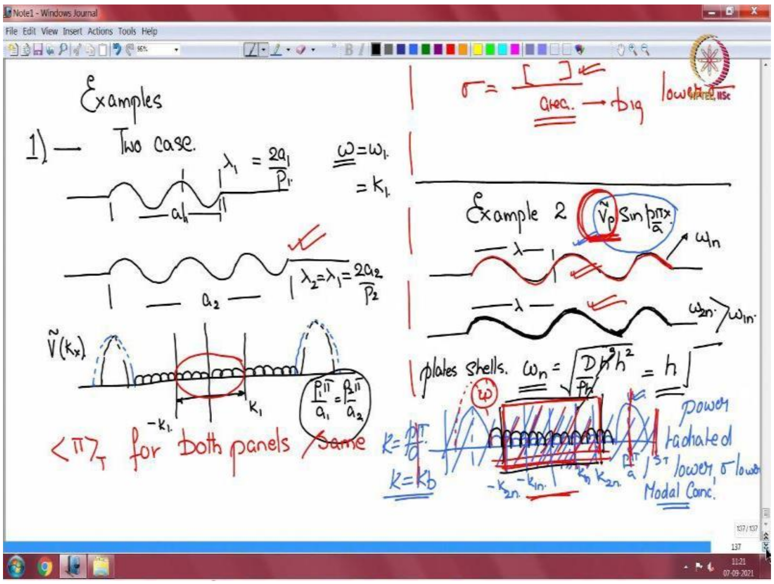

This time we are going to take a second example to better understand that so the second example. Example 2 is here we have 2 panels vibrating in a certain mode. So, $\tilde{V}_p \sin\left(\frac{p\pi x}{a}\right)$ and there is a second panel which is also vibrating in the same mode. But the second panel is thicker, and they are vibrating at resonance, their resonances. So, this panel is vibrating at its own resonance $1n$ in this panel at its own presence to $2n$.

However, the wavelengths are same these lambdas are same for both or $\frac{p\pi}{a}$ is same for both. Now, if you recall your vibrations of plates and shells, your natural frequency comes out as a

$\sqrt{\frac{D}{\rho h}}$ and D carries your $\frac{Eh^3}{1-\nu^2}$ etcetera so it carries an h^3 in it. So, this h and that h^3 becomes h^2 and once you bring it out of the square root this h comes out and other terms remain.

So, natural frequency is proportional to the thickness of the panel. So, the same lambda ω_{2n} is bigger than ω_{1n} . So, now what happens is you have a situation were based on this alone both have $\tilde{V}_p \sin\left(\frac{p\pi x}{a}\right)$ by just that expression is same for both and therefore the wave number domain shape is the same for both. So, now the second plate ω_{2n} is bigger so the k_{2n} will be bigger.

This is k_{2n} , - k_{2n} for the first one k_{1n} is smaller. So, k_{1n} and you have - k_{1n} and therefore for the first plate you get a smaller range of wave numbers included. So, therefore the power radiated by the first is slower and hence sigma is lower, if radiation efficiency is lower. Now, there are cases here for example when k approaches $\frac{p\pi}{a}$ that is when coincidence occurs, that means coincidence with the mode occurs.

Let us call it modal coincidence. This is different from the coincidence of the full plate where k = k_b the free wave number is different. It is just here k = $\frac{p\pi}{a}$. So, when k approaches $\frac{p\pi}{a}$ that is the coincidence with the mode and once it crosses $\frac{p\pi}{a}$ then you get a very large range of wave numbers within radiation and the radiation efficiency is 1 is close to 1. Now, within that this panel could have a resonance below here it could have a resonance here or it could have its resonance beyond.

So, we will see those cases later. Here let us see the simple example which just illustrates that based on the ω you could include larger or fewer number of radiating wave numbers. This is a scatter, this is a scattering phenomenon because, you have a finite panel with a certain wavenumber in it that finite wavenumber scatters into all these extra wave numbers. So, we are just illustrating that even below coincidence these are all below coincidence now coincidence happens here.

So, below coincidence because of the finiteness of this panel there is radiation possible because of this all these scattered wave numbers. And within that being below coincidence within that range, we have taken examples to show how you could have more radiation or less radiation. And especially, if you have a resonance with increased range suppose that ω_n as it happened over here then this value will be bigger actually.

This amplitude will be bigger because $\tilde{V}_p$ will be bigger. So, you will get more radiation. Now this should also give you a hint as to why we have not allowed $\tilde{V}_p$ to change that has helped us in doing these comparisons $\tilde{V}_p$ also changes that complicates. But now I have added that line that you could have a resonance in this range and that would help you for that why because these wavenumbers will grow in their amplitude will have a higher amplitude $\tilde{V}_p$ will be higher.

Just with this last statement I have allowed $\tilde{V}_p$ to change with frequency. So, now as I said the extra cases which come up with coincidence versus resonance, resonance being above coincidence or below.

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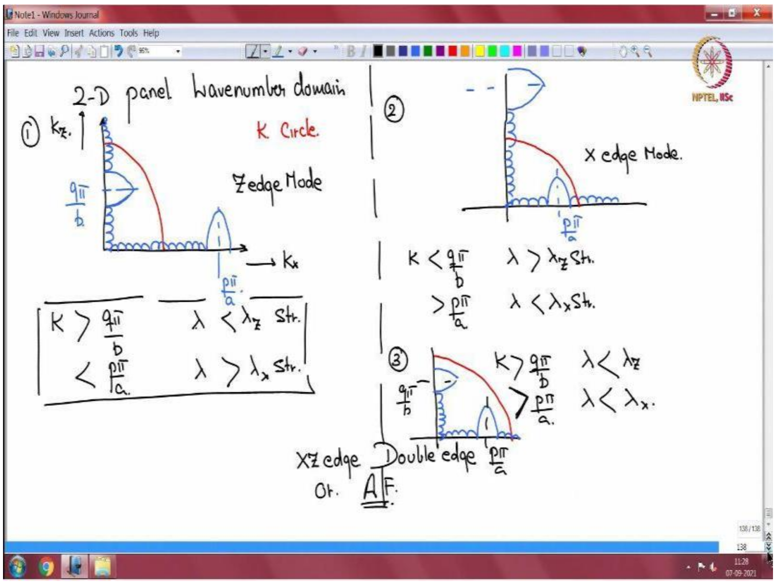

We will see those now, and before doing that let me do all the other cases for a 2D panel. So, this time from 1D we move to 2D, and we look at it in the wave number domain. So, let us see the cases in a 2D panel in the wavenumber domain. So, we have this is the k_x axis and this is the k_z axis and let us say that $\frac{p\pi x}{a}$ is here, $\frac{p\pi}{a}$ is here and the here is my $\frac{q\pi}{b}$. Now, let us draw on top of this the acoustic wave number circle k circle.

So, if we draw that and let us say we I draw it here this is where it is, and the remaining scattered wavenumbers look like this. So, this is a case where k happens to be greater than $\frac{q\pi}{b}$ but less than $\frac{p\pi}{a}$, which means lambda is less than lambda z structural wave number and lambda is greater than lambda x structural wave number. That is how the plot looks like, this is one case. So, let me give a name here let me give it a name Z edge mode.

This will make sense later let me give it a name Z edge mode. Now, the second case 2, first case one case 2 we have in this case $\frac{p\pi}{a}$ comes closer and $\frac{q\pi}{b}$ is big away and this is my k circle, now in this case k is less than $\frac{q\pi}{b}$. So, lambda is greater than lambda z structural and it is greater than $\frac{p\pi}{a}$ and lambda is less than lambda x structural. So, I will again give this a name, call it X edge mode.

Now, the third case will be where we have this situation. So, this is $\frac{p\pi}{a}$ this is $\frac{q\pi}{b}$. So, k happens to be greater than $\frac{q\pi}{b}$. It has to be greater than $\frac{p\pi}{a}$ and therefore lambda is less than lambda z, lambda is less than lambda x. So, we will give these two names we will call them double edge, or you call this double edge or XZ edge or will or I am just saying an or we will call it acoustically fast modes.

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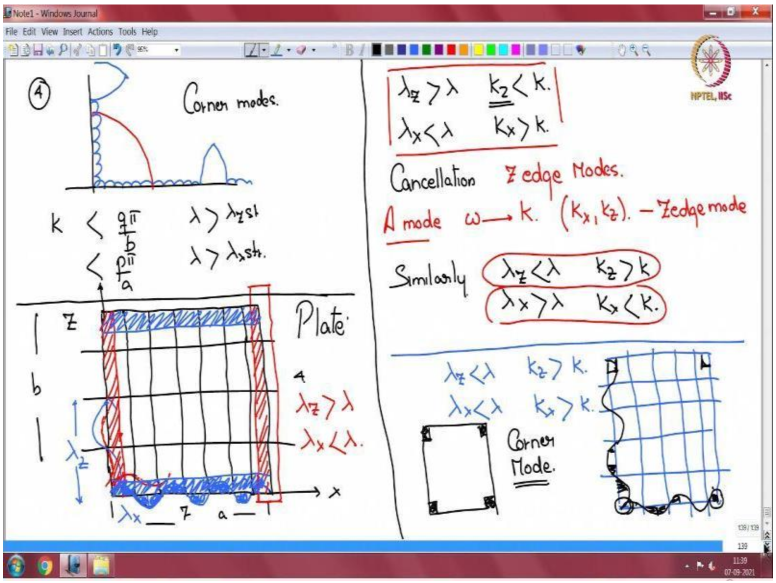

In the last case is here fourth case is here, where we have the acoustic circle and the $\frac{p\pi}{a}$ peak lies here, $\frac{q\pi}{b}$ peak lies here which means k is less than $\frac{q\pi}{b}$ is less than $\frac{p\pi}{a}$ which means lambda is greater than lambda z structure lambda is greater than lambda x structure. We will call these the corner modes these situations occur. Now I will show this on an actual panel. Let us see this on an actual panel.

So, we have a panel here 1 2 3 4 5 then 6 7 and we have 7 cells here we have 4 cells here. So, this is let us say my x direction let us say this is my z direction. So, there is a panel or a plate. This total is a this total is b. Now, this happens to be this happens to be the let me change colour this happens to be the lambda x. So, this now happens to be this let us say this happens to be lambda z, this happens to be lambda z.

And let us now assume that my lambda, the acoustic lambda, is slightly longer than lambda x and slightly shorter than lambda z. So, that means lambda z is bigger than lambda and lambda x is shorter than lambda. That means what if lambda z is bigger than lambda that means k_z is smaller than k. So, with respect to z model wave number we are above coincidence and lambda x is less than lambda means k_x is greater than k which means we are below coincidence with respect to x.

So, there is going to be now cancellation and how will the cancellation occur. So, let us see we have let us take blue colour. Now, this part will cancel with this part, this part will cancel with this quarter, this quarter cancels with that quarter, this quarter cancels with this quarter, this quarter with that quarter and a portion remains uncancelled. So, now this entire portion like that will remain uncancelled. This entire portion will remain uncancelled.

That means this region on the plate similarly on this side, on this side this entire region will remain uncancelled everything in between will cancel. So, this leads to Z edge modes. So, what am I saying a mode when vibrated at a certain frequency omega which gives you a certain acoustic wavenumber k, with fixed k_z or I mean k_x and k_z fixed, it ends up looking like a Z edge mode provided these relations hold.

And that is why I gave it the Z edge mode name. So, lambda here is less than lambda z so along z there is no cancellation we are above coincidence. But in the x direction the lambda is greater than lambda x, so we are below coincidence. So, cancellation occurs and how does it occur it occurs in these nice rectangular patterns. So, that mode becomes or behaves like a Z edge mode and these two edges will radiate.

Similarly, you could have a situation I will not redraw it again. So, similarly you could have a situation where lambda z is smaller and lambda x is greater and therefore k_z is greater than k_x is smaller. That means here we are below coincidence with respect to z modal wave number but with respect to x modal wave number we are above the coincidence. So, what will happen is? We will have X edge radiators.

Now, along the same lines you could have a situation where lambda z happens to be less than lambda. Lambda x also happens to be less than lambda which means k_z happens to be bigger than k, k_x happens to be bigger than k. So, then you will have cancellations happening in both the directions and what will happen is I will just show you again a schematic. Let us say $1\ 2\ 3\ 4, 1\ 2\ 3\ 4$ that it is it.

So, if we take now the mode behaviour in this direction these are supposed to be equal length scale cells but do not mind. What will happen is? Cancellations will occur here, each neighbouring quarter cell but 1 quarter cell will remain uncancelled here. Similarly, 1 quarter cell similarly everything else will cancel here, except 1 quarter cell which will be left and similarly 1 quarter cell here.

So, in this panel there will be a quarter cell here, quarter cell here, quarter cell here, and quarter cell here. So, this is the corner mode.

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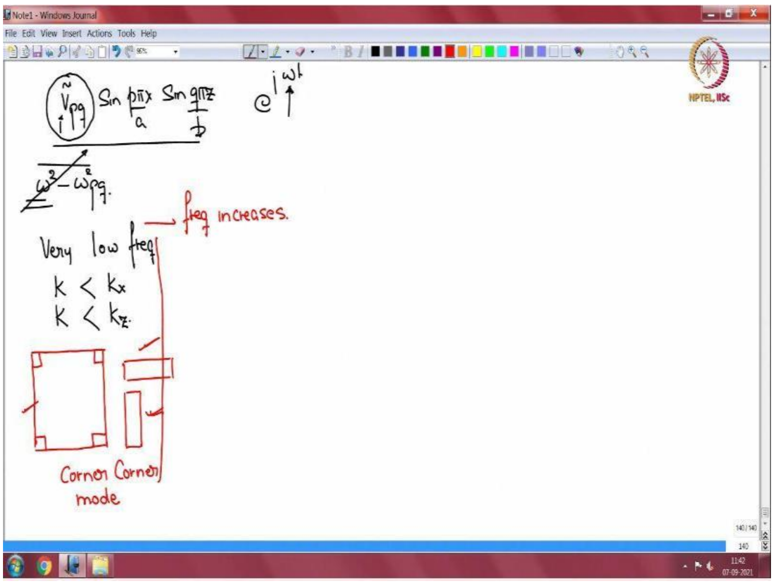

So, let us see now the progression, a mode suppose there is a mode, now let me write it in some way $\tilde{V}_{pq}\sin\left(\frac{p\pi x}{a}\right)\sin\left(\frac{q\pi z}{b}\right)$ and again we keep $\tilde{V}_{pq}$ constant by now you know why we do that, if that also starts to change then it is a very complicated situation you know from plates and shells that the denominator is of the form omega square minus omega square pq.

So, now as omega starts to shift it will get closer or distant from omega square pq and the $\tilde{V}_{pq}$ will start changing very nonlinearly. So, we avoid that by keeping this term for the moment out and so we keep $\tilde{V}_{pq}$ constant. So, now this mode now excited at a certain frequency which we will vary to see the effect. So, at very low frequencies k is I mean k is less than k_x , k is less than k_z .

Similarly, the lambdas are greater correspondingly so we will get this picture that mode is a corner, corner mode to start with it starts off as a corner, corner mode let me partition it. Now, I just done almost a square plate, but you could actually have it rectangular in one direction or the other, a could be bigger than b, b could be bigger than a. So, this is the more representative of an actual plate, but I just drawn it almost square do not worry about it.

Now, as frequency increases, I think that time is up. So, I will stop here we will continue in the next class. Thank you.