**Sound and Structural Vibration**
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**Lecture – 4**
**A Classical Problem in Sound-Structure Interaction**

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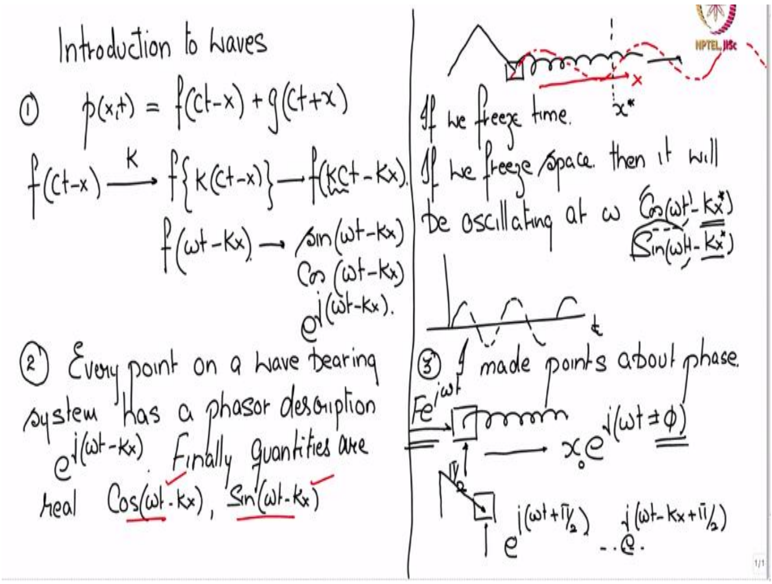


Good morning and welcome to this next lecture on sound and structural vibration. Last class we did a very basic introduction to waves and wave bearing systems. So, we looked at some basic nomenclatures. Now, before I move on to the actual problem, first problem we are going to look at I just want to add a few points that I missed in the introduction. So, first point is that we said that $\mathrm{p(x,t)} = \mathrm{f}(ct - x) + \mathrm{g}(ct + x)$. So, let us take $\mathrm{f}(ct - x)$.

Suppose I multiply within the argument a k, so this becomes $\mathrm{f}\{k(ct - x)\}$, k is a constant. So, in every derivative on the left or right of the wave equation k falls out and cancels out, it does not matter. Then what I have is $\mathrm{f}(kct - kx)$, k is the wave number in kc is ω. So, I get $\mathrm{f}(\omega t - kx)$. So, now if ω happens to be a single frequency then this has to be written either as a $\sin(\omega t - kx)$ or $\cos(\omega t - kx)$ or in a combined sense the phasor notation $e^{j(\omega t - kx)}$, so one point I wanted to make.

Then the second point is I said every point on a wave bearing system vibrates, sees the wave coming and going and has a phasor description that means it has its own $e^{j(\omega t - kx)}$, but I also said that everything is real. Finally, we are interested in finally quantities are real which means what? The expression has to be either a $\cos(\omega t - kx)$ or a $\sin(\omega t - kx)$.

So, in that crank and spring example if we freeze time that means in these expressions we freeze time and then we move in space then what the expression is a sinusoid, right. So, I will see a sine wave as I move in space, regions of compression, regions of rarefaction as a sinusoid. Similarly if we freeze space that means we remain in one place on the spring some point $x^*$, then it will be oscillating also at the same omega.

So, the descriptor will be either some $\cos(\omega t - kx^*)$ or $\sin(\omega t - kx^*)$, this is a constant now, right. So, it is a cosine in time, sine in time. A single point will be oscillating in time as a sinusoid or a cosinusoid. So, any way you look at it the descriptor is sinusoidal that is the second point I wanted to make. Then third point I said many things about phase. I made points about phase.

In actuality if the system that is mass and the spring is driven by a force, then of course the description, the response, let us say displacement will be; let us say displacement of the first point $x_0$ will be ωt plus or minus some phase. Now it is phase with respect to the force, but because I was doing a schematic, not an actual calculation, so I had my phasor starting at $\frac{\pi}{2}$ and the mass was hard tight with no flexibility to that crank.

So, I decided to give this mass also a phase of $\frac{\pi}{2}$, so we can do that, plus $\frac{\pi}{2}$, so that every next point will have a phase related to the starting mass. These are the points I wanted to make.

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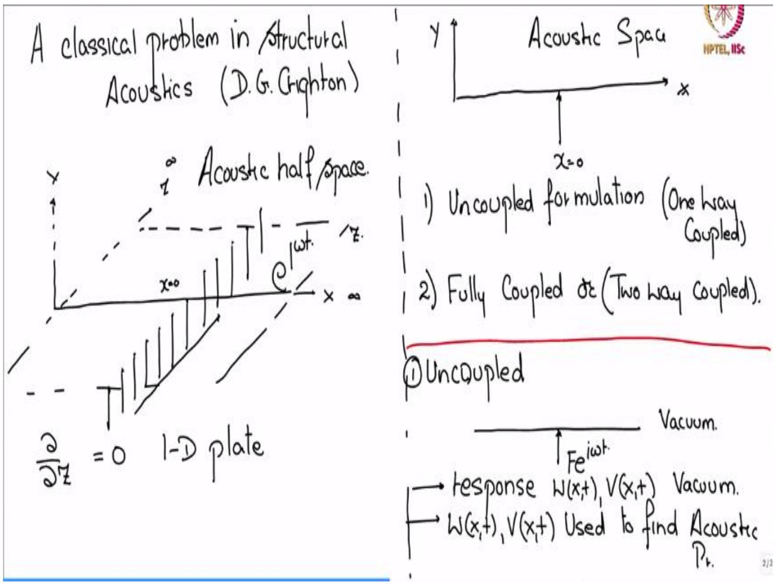


Now, we will begin a problem which I call as a classical problem in structural acoustics. This problem has been discussed extensively in the literature and mainly due to one author called D. G. Crighton. If you type this name you will get the papers on any of the Google Scholar type sites. So, what is this problem? It is a plate, a flexing plate that resides in the x z plane. It resides in the x z plane, this is x that is z and this vertical is y.

So, there is this plate, it is an infinite infinite plate, it extends to infinity in z and it extends to infinity in the x direction both directions and above you have an acoustic half space. And there is a line force applied at $x = 0$ along the entire z direction, there is a line force applied, a harmonic line force that means e to the j omega t line force applied to this infinite plate at $x = 0$ in the z direction.

So, by the very nature of forcings or derivatives, all dependencies of z will go to 0 and therefore this plate becomes a 1D plate. So, we can show the schematic in one dimension. So, now what do we have? We have a one dimensional plate with an acoustic half space with a line force acting at $x = 0$ that is y direction, this is x direction. So, the acoustic problem has become two dimensional and the wave problem has become one dimensional.

Now in sound structure interaction problems, there are two ways the problems are formulated; one is called the uncoupled formulation or also called one way coupling or it is called fully coupled or two way coupled. So, what is one way coupled? So, one way coupling or uncoupled one, formulation one is uncoupled which is the structure is first placed in vacuum and it is forced.

And the response that is displacement or velocity are found first with vacuum as the medium. Then the response is used to find acoustic pressures that is now the medium is brought in. So that means only one unknown at a time.

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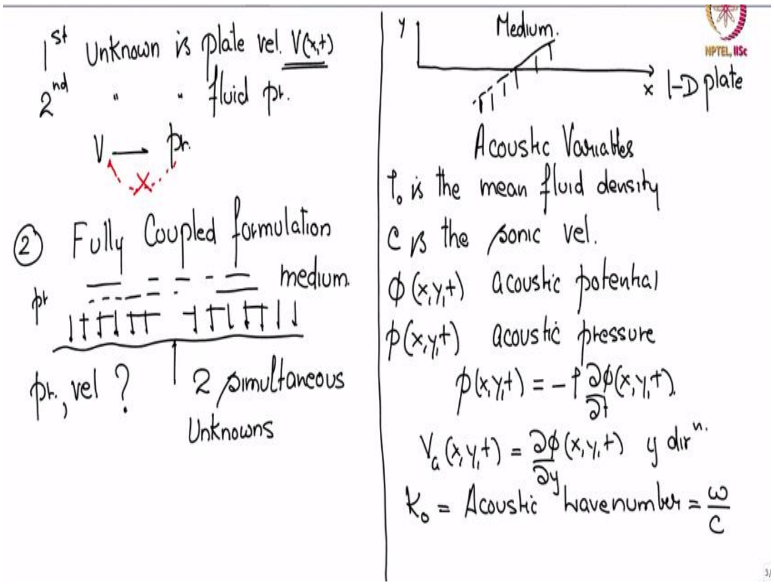


There is one unknown at a time. The first unknown is let us say plate velocity. Then after it is found, the second unknown is fluid pressure. So, this is one way coupled. So, why is it called one way coupled because in this case the velocity that you have found does not change anymore. The velocity decides the pressure. The pressure in turn does not or cannot decide or modify the original pressure, no that does not happen.

So, in contrast we have number two the fully coupled formulation or two way coupled, what is that? The plate is excited and the plate excitations create pressure ripples in the fluid. So, medium is already there, it is not vacuum anymore, so pressures are created. Now, these pressures are immediately applied to the plate. So, till a steady state is reached both pressure and velocity of the plate are unknown.

So we have two unknowns, two simultaneous unknowns, both have to be solved together. So, we will be looking at a two way coupled problem now. So, let us see. I have y direction this plate, it is a 1D plate. There is a line force medium. So, the acoustic side variables, the acoustic variables. So, $\rho_0$ is the mean fluid density; c is the sonic velocity, sound velocity; $\phi(x, y, t)$ is the acoustic potential; $p(x, y, t)$ is the acoustic pressure, perturbation pressure.

Then we have $p(x, y, t)$ given by $-\rho \frac{\partial \phi(x,y,t)}{\partial t}$ and acoustic particle velocity $v_a(x, y, t)$ is given by $\frac{\partial \phi(x,y,t)}{\partial y}$ in the y direction, we will be interested in the y direction velocities. Then $k_0$ is the acoustic wave number given by $\frac{\omega}{c}$.

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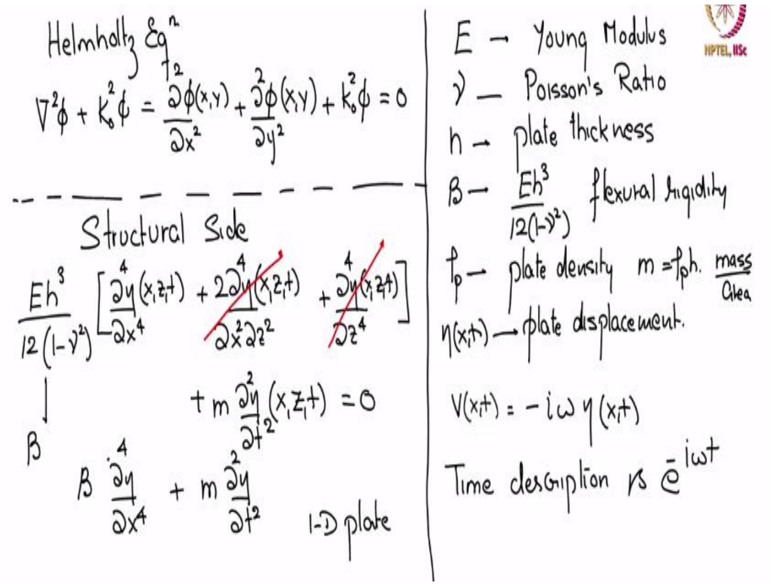


Now, the Helmholtz equation that means the acoustic wave equation for a harmonic case and the time removed is

$$\nabla^2\phi + {k_0}^2\phi = \frac{\partial^2\phi(x,y)}{\partial x^2} + \frac{\partial^2\phi(x,y)}{\partial y^2} + {k_0}^2\phi = 0.$$

The whole course is about harmonic excitations, not transients and therefore I will often keep time and remove time in some convenient manner, it is cumbersome to keep carrying the $e^{i\omega t}$ type terms.

So, this is the Helmholtz equation. Then on the structural side we have

$$\frac{Eh^3}{12(1-\nu^2)}\left[\frac{\partial^4\eta(x,z,t)}{\partial x^4} + 2\frac{\partial^4\eta(x,z,t)}{\partial x^2\,\partial z^2} + \frac{\partial^4\eta(x,z,t)}{\partial z^4}\right] + m\frac{\partial^2\eta(x,z,t)}{\partial t^2} = 0.$$

The configuration we said was such that all z dependencies will go, so this term goes away.

So, this term goes away, this term goes away because of the nature of the forcing and so we are going to be left with, I will give this a name called B so that I do not repeat it. So, B $\frac{\partial^4\eta}{\partial x^4}$ + m $\frac{\partial^2\eta}{\partial t^2}$ , this is my 1D plate. Now, what are the plate parameters? E is the Young modulus

modulus of the plate material, ν is the Poisson's ratio, h is the plate thickness, B is of course $\frac{Eh^3}{12(1-\nu^2)}$ is the flexural rigidity.

Then $\rho_p$ is the plate density, then m is the mass per unit area given by $\rho_p$ h. Then $\eta(x,t)$ we will take it as the plate displacement and $v(x,t)$ is $-i\omega\eta(x,t)$ because my time descriptor or the harmonic descriptor is $e^{-i\omega t}$. Let us see now.

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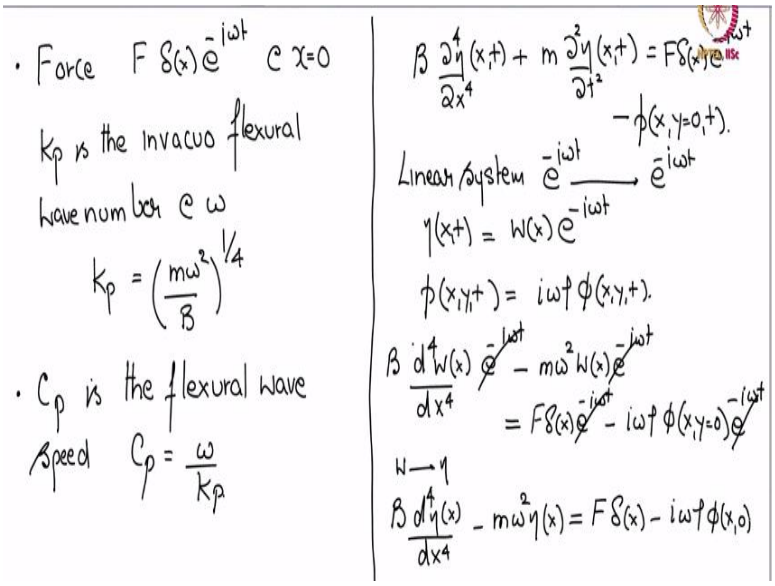


The force is given by $F\,\delta(x)e^{-i\omega t}$ as it says applied at $x = 0$ and $k_p$ is the invacuo flexural wave number and it is at a given frequency at ω given by $k_p = \left(\frac{m\omega^2}{B}\right)^{1/4}$. Then we have $c_p$ is the flexural wave speed given by $c_p = \frac{\omega}{k_p}$. And then we have plate equation

$$B\,\frac{\partial^4\eta(x,t)}{\partial x^4} + m\frac{\partial^2\eta(x,t)}{\partial t^2} = F\,\delta(x)e^{-i\omega t} - p(x, y = 0, t).$$

This is how the coupling happens. The pressure generated is applied back. Now, it is a linear system, so driven at $e^{-i\omega t}$ the response will be at $e^{-i\omega t}$. So, $\eta(x,t)$ I will take it as some $W(x)e^{-i\omega t}$ in my pressure $p(x,y,t) = i\omega\rho\phi(x,y,t)$.

So, what I now have is

$$B\frac{d^4W(x)}{dx^4}e^{-i\omega t} - m\omega^2 W(x)e^{-i\omega t} = F\,\delta(x)e^{-i\omega t} - i\omega\rho\phi(x, y = 0)e^{-i\omega t}.$$

Actually $\phi(x,y)$ and $\phi(x,y,t)$ are slightly different, but please I do not want to keep changing notation. So, these goes away. So, now I will replace W with η for my own convenience. So, we get

$$B\frac{d^4\eta(x)}{dx^4} - m\omega^2\eta(x) = F\,\delta(x) - i\omega\rho\phi(x, y = 0).$$

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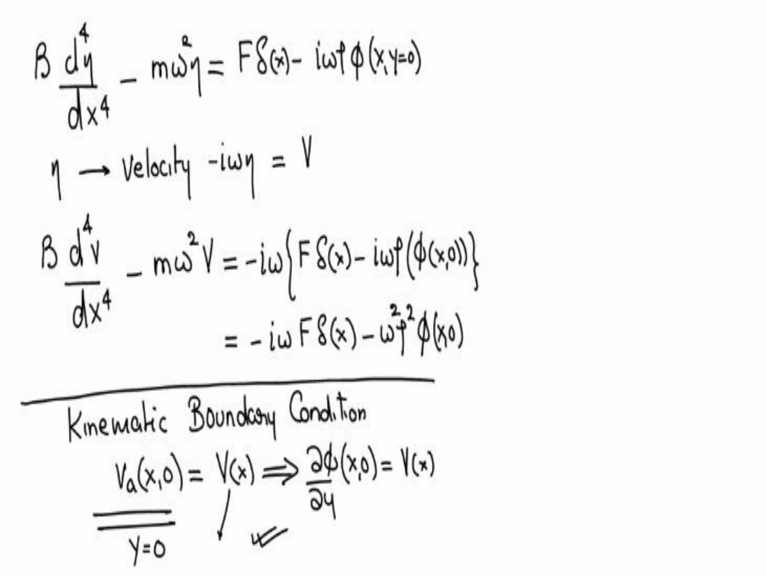


Now, we have

$$B\frac{d^4\eta}{dx^4} - m\omega^2\eta = F\,\delta(x) - i\omega\rho\phi(x, y = 0),$$

and we convert η to velocity $-i\omega\eta$ as velocity so that

$$B\frac{d^4V}{dx^4} - m\omega^2V = -i\omega\{F\,\delta(x) - i\omega\rho\phi(x,0)\},$$

$$= -i\omega F\,\delta(x) - \omega^2\rho^2\phi(x,0).$$

In the kinematic boundary condition is given by

$$v_a(x,0) = v(x) \Rightarrow \frac{\partial\phi(x,0)}{\partial y} = v(x).$$

This means at $y = 0$, this is plate velocity. Our time is up, so we will leave it here for today.