Sound and Structural Vibration

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Lecture - 40
Modal Character Across the Frequency Range

Good morning and welcome to this next lecture on sound and structural vibration. We were looking at how a panel vibrating in a single mode changes its nature with respect to the frequency at which it is vibrated.

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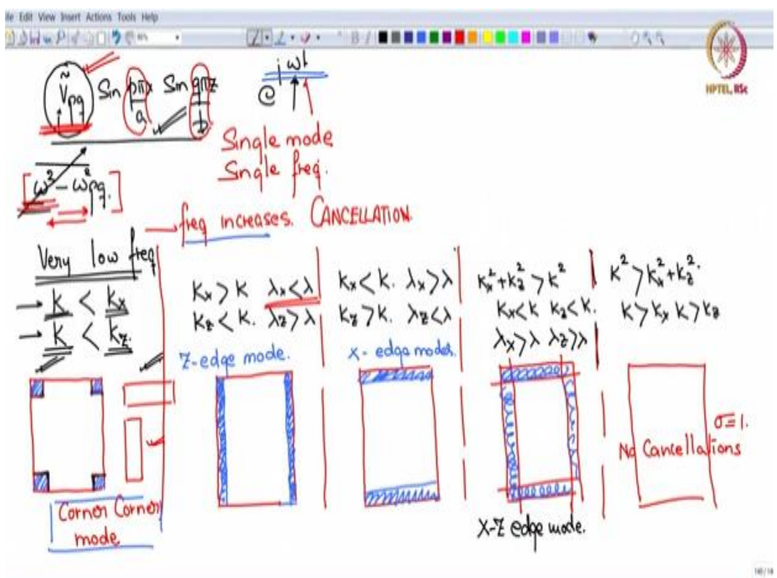

So, a mode of this form, $\tilde{V}_{pq}\sin\left(\frac{p\pi x}{a}\right)\sin\left(\frac{q\pi z}{b}\right)$ is driven at this ω. So, we are looking at single mode vibrations at single frequencies and very specifically we keep the amplitude a constant. For those who are aware of vibrations of plates and shells, the amplitude has this sort of a term in the denominator, $\omega^2 - {\omega_{pq}}^2$. And as ω the driving frequency changes and gets close to the resonance, you will have amplitude increase.

Or as it moves away you will have amplitude decrease as a function of frequency. But that is very non-linear. So, we do not want to bring that aspect in, and we just understand the cancellation effect that is what we are doing. And so, throughout this discussion $\tilde{V}_{pq}$ has been held a constant. Merely change in frequency is affording us cancellations or lack of cancellations. So, we started off with this plate over here case one at very low frequencies.

So, there is this panel which I have drawn, you can take it more rectangular; a is bigger than b or b bigger than a and at very low frequencies k_x overwhelms k, k_z overwhelms k, and therefore there is very heavy cancellation and only the corners remain uncancelled. So, it is a poor radiator. So, the same mode, mode remains the same, but due to the frequency of excitation. It is now a corner mode, only the corners radiate.

In the beginning we had seen a case, where the entire panel is equivalent to one cell. This is that case, four quarters make one cell. Now, as frequency increases, it is possible that k_x is greater than k, but k_z becomes less than k is possible. In which case lambda x is less than lambda the acoustic wavelength and lambda z is greater than lambda. So, then what happens, I will keep the same colours. So, then this panel has cancellations along the x direction.

So, what happens is? We end up with two regions strips actually along the z direction which remain uncancelled and this is called the Z edge mode. Now, it can also happen that k_x becomes less than k and k_z is above k, in which case lambda x is greater than acoustic wavelength, lambda z is less than acoustic wavelength can also happen based on dimensions. Based on the relative size $\frac{p\pi}{a}$ is x wave number $\frac{q\pi}{b}$ is z wave number.

So, one relative to the other based on dimensions and more number it could happen that we are presented with this situation as frequency increases. Then, what happens is that we have X edge modes, two strips of this sort remain uncancelled. These are X edge modes. Following this we can have the next situation where ${k_x}^2 + {k_z}^2$ is greater than k^2, but k_x is less than k and k_z is less than k. This is a unique situation.

So, lambda x is greater than lambda and lambda z is greater than lambda, but not totally. It is not a total, so what happens here? Here, we will get double edge modes. We get an edge here, we will get an edge here, we will get an edge here and we get an edge here. That are radiating, so these are called double edge modes or XZ edge modes. And lastly what happens is that k^2 is greater than ${k_x}^2 + {k_z}^2$.

And k is greater than k_x and k is greater than k_z, at which point the full plate is radiating, no cancellations now and radiation efficiency reaches one. One more way of looking at it.

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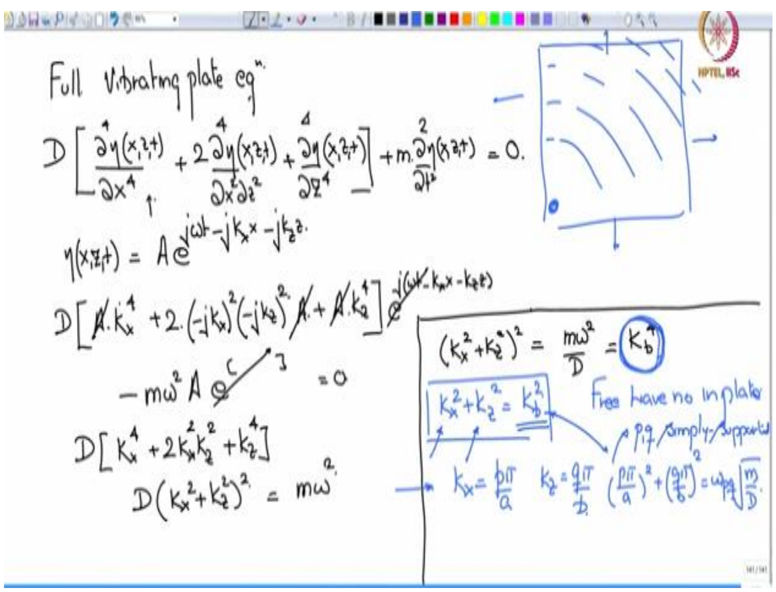

So, let me draw the picture first or let me do this derivation first. So, we have the full plate equation. The full vibrating plate equation is given by let us say,

$$D\left[\frac{\partial^4 \eta(x,z,t)}{\partial x^4} + 2\frac{\partial^4 \eta(x,z,t)}{\partial x^2 \partial z^2} + \frac{\partial^4 \eta(x,z,t)}{\partial z^4}\right] + m\frac{\partial^2 \eta(x,z,t)}{\partial t^2} = 0.$$

So, this can support the wave $\eta(x,z,t) = Ae^{j\omega t - jk_x x - jk_z z}$.

So, if we substitute this in here, I will get

$$D\left[A\,{k_x}^4+2(-jk_x)^2\,(-jk_z)^2A+A\,{k_z}^4\,\right]e^{j\omega t-jk_xx-jk_zz}-m\omega^2A\,e^{j\omega t-jk_xx-jk_zz}=0.$$

The propagators I am getting keeping outside $e^{j\omega t-jk_xx-jk_zz}$, I have kept it outside.

So, this propagator I can cancel from both sides, the amplitude I can cancel. So, this gives me

$$D\left[\,{k_x}^4+2{k_x}^2{k_Z}^2+\,{k_z}^4\right]=m\omega^2,$$

$$D\left(\,{k_x}^2+\,{k_z}^2\right)^2=m\omega^2.$$

$$\left(\,{k_x}^2+\,{k_z}^2\right)^2=\frac{m\omega^2}{D}.$$

And, if you recall this is actually ${k_b}^4$, the free wave number in the plate.

That means, what I have an infinite panel, I have an infinite plate that goes off to infinity. And somewhere locally far away I have given a local excitation, as a result of which wavenumbers are propagating and that at ω it is this ${k_b}^4$. That is the free wave number in the plate. Now, because it is a rectangular plate, you can have it in two components. So, you can have ${k_x}^2+\,{k_z}^2={k_b}^2$, k_x and k_b are unknown.

They can be any combination based on whatever boundary conditions etcetera. But this square sum is ${k_b}^2$. So, this is one bit of knowledge that we need. Then, it is also true, another equation is also true, that if my k_x is equal to say $\frac{p\pi}{a}$ and $k_z=\frac{q\pi}{b}$. Now, we are on to a finite panel, these are the modal wave numbers k_x and k_z. Then, $\left(\frac{p\pi}{a}\right)^2+\left(\frac{q\pi}{b}\right)^2=\omega_{pq}\sqrt{\frac{m}{D}}$ this is also true.

So, this is the equation for pq th mode resonance in a simply supported plate. So, we will see the meeting point of these two equations. So, let us see now.

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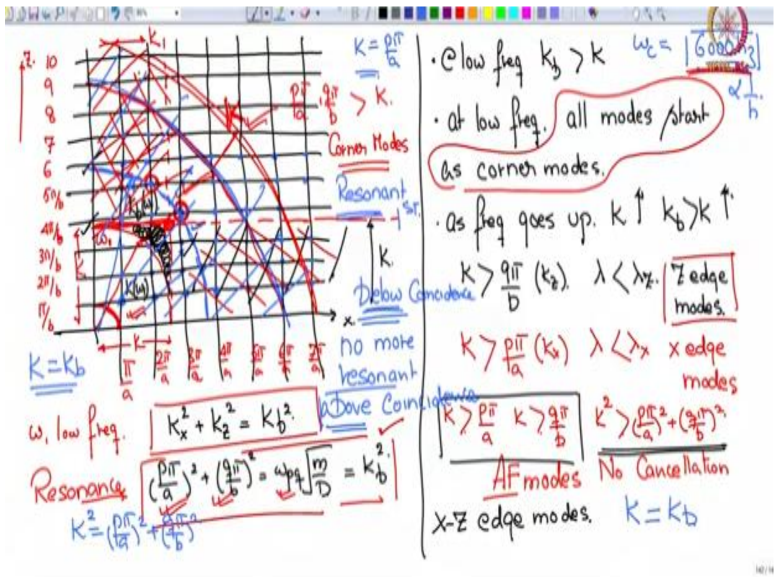

So, I have a wave number space. Let us say, let me make it here. So, this is x direction, along x direction, I will choose $\frac{\pi}{a}, \frac{2\pi}{a}, \frac{3\pi}{a}, \frac{4\pi}{a}, \frac{5\pi}{a}, \frac{6\pi}{a}, \frac{7\pi}{a}$ etcetera, these are x wave numbers. Then, let me choose, this little oblong, so I will choose here one line here, next line here, next line here, next line here, next line here. They are supposed to be equidistant and so forth.

What is this? This is $\frac{\pi}{b}$ here, $\frac{2\pi}{b}, \frac{3\pi}{b}, \frac{4\pi}{b}, \frac{5\pi}{b}$, 6, 7, 8, 9 etcetera. This is in the z direction. Now what do these represent? Each intersection represents a mode, this is wave number space. But each represents a mode, each of these blue dots is a mode, each of these is a mod. So, now what I will do is I will draw the acoustic circle. So, let me draw the acoustic circle, acoustic circle let us say goes like this, let it go like this. This is the acoustic circle.

So, let it go vertically and it turn horizontal. I would like it more representative. So, let us do this, take off vertically and comes off horizontally. This is the k circle, this is k, acoustic wave number k and let me draw these vertical lines. Now, let us say this is ω, corresponding to ω_1, let us say ω_1 and its low frequency ω_1 its low frequency. Now corresponding to that we will draw the free wavenumber line also.

What is that? Let this be the free wavenumber line. So, let us see this starts off like this and goes through this and it ends up here let us see. So, that is what, that is the free wavenumber line, k_b line, that is k_b circle. So, how to write it? This is k at ω_1, this is k_b at ω_1. Now, what did we say earlier? Earlier we said $k_x{}^2 + k_z{}^2 = k_b{}^2$. And we also said that $\left(\frac{p\pi}{a}\right)^2 + \left(\frac{q\pi}{b}\right)^2 = \omega_{pq}\sqrt{\frac{m}{D}}$.

And this is also $k_b{}^2$, is from the same equation. What do we mean? At low frequencies, so let us see, let us write it at low frequencies. Now, k_b is bigger than k, the acoustic wave number, so that is what is indicated over here. Now so at this frequency you can see, I wish I had more colours. Here what are these? These are $\frac{p\pi}{a}$ and $\frac{q\pi}{b}$ which are greater than k. So, these are what these are corner modes.

If you look at the previous picture, these are corner modes. At extremely low frequency, it would this red line would have been here. At the starting this would have been right here. That means what, let me just say, not at this frequency, at low frequency, all modes start as corner modes. Now, as frequency goes up, the k wavenumber goes up, also k_b which is already greater than k also goes up.

So, it is at an instant we are here, k circle is here, k_b circle is here. Now, look at this, what is this region? This region is where, this height is k. This region is where k is greater than $\frac{q\pi}{b}$ which is k_z which means what lambda is less than lambda z. What modes are those? Those are the Z edge modes. So, what happened? Whereas, when we started all the modes started as corner modes by the time we reach here, k reaches here, k_b reaches here.

Some modes have become Z edge modes. These were corner modes earlier, now they have become Z edge modes. Now, what about this set of modes? This set of modes is, where k is greater than $\frac{p\pi}{a}$, which is k_x. That means, what lambda is less than lambda x, these are what X edge modes. So, this region here above this line here, this is k. So, this point is k, this is k. So, these modes are X edge modes, these are Z edge modes, these here are corner modes.

So, now there is this region over here, where k is greater than $\frac{p\pi}{a}$, it is greater than $\frac{q\pi}{b}$, and also k^2 is greater than $\left(\frac{p\pi}{a}\right)^2 + \left(\frac{q\pi}{b}\right)^2$. That is what the circle implies. So, these are acoustically fast modes, there is no cancellation. Now, what about this small region here? There are hardly any modes now, but there is a small region here, what is this region?

This region is where these holds k is greater than $\frac{p\pi}{a}$, k is greater than $\frac{q\pi}{b}$, but k^2 is not greater than $\left(\frac{p\pi}{a}\right)^2 + \left(\frac{q\pi}{b}\right)^2$, it is not, it is less. And therefore, these are XZ edge modes. So, at any instant you will have all these modes, at any instant in frequency you will have all these modes. Now I will bring in the idea of a resonance. If you look at these two formulas, this of course comes from the free infinite plate.

Free bending wave number and this comes in from the finite plate. So, this k_b line which is the free wavenumber line. If it passes through a pq point that means ${k_b}^2$ is equal to this and ${k_b}^2$is equal to this and this is equal to this, so this is a resonance frequency. That means, when this blue line, in this blue line here, which is the k_b line, cuts or passes through or near one of these dots, that mode is resonant.

So, let me say it and I will write it in the next page, so you have modes becoming resonant first. Because k_b rises faster, k_b rises faster than k, k_b is above k. So, as k_b moves faster outward than k, it will cross these blue dotted lines, so modes get resonant first and are not below coincidence. So, they are not efficient radiators, but they are in resonance, so that does give you some resonant radiation.

And let me just tell you that the coincidence frequency for most thin panels is in the range of 6 kilohertz in that range. It is inversely proportional to thickness. So, thicker panels will go down, of course thicker panels are hard to excite and are poor radiators by themselves. So, we will take 6000 hertz as some kind of number for steel. So, it is mechanically it is very hard to excite that frequency.

You know mechanical excitations usually do not come up that high, unless you have cavitations or explosions or something like that. So, that means what most modes will be below coincidence, but they will pass through resonance. So, most radiation is that we will see is resonant radiation below coincidence. Now, what happens is that a little later as this k_b moves out, the acoustic wave number line moves in, so then they are no more resonant, but they are above coincidence.

Now, please do not confuse this coincidence with the coincidence between k and k_b. I hope I have made that clear; I will make it clear in a general lecture next. So, this is the plate material and acoustic speed related coincidence frequency. This is the absolute kind of coincidence frequency. But your k^2 can be equal to $\left(\frac{p\pi}{a}\right)^2 + \left(\frac{q\pi}{b}\right)^2$. That is when coincidence with the mode happens, in the 1D case we saw what and k becomes equal to $\frac{p\pi}{a}$, in the same sense over here.

So, now the mode was earlier resonant, because k_b pass through it, now it is above its own coincidence. So, it becomes an efficient model radiator, however it is no more in resonance. So, that is how it goes when the mode is resonant it is not above coincidence, when the mode is above coincidence, it is no more resonant. Now, as time goes by as frequency further increases, somewhere what will happen is; you will have the actual coincidence.

That means the k half circle or quarter circle will be equal lie on k_b quarter circle. I will show that as this. So, both of them come together in one place, that is actually the coincidence which I am speaking of here, which is also rare. Now, at that point what happens is? The modes through which k_b line moves are resonant, and they are also above coincidence. And beyond that what point, what happens is that you will have k going faster, the k starts to overwhelm, k circle starts to overwhelm.

So, those modes below the k circle are all above coincidence. So, all these modes will be above coincidence, so suppose k has moved here, and k_b the plate line plate quarter circle is somewhere over here. Then, all these modes are first of all above coincidence. And then, some of these modes are resonant also. So, this is how the modes behave as a result of change in frequency and we have brought in the idea of resonance also.

So, this is a story of panel radiation. Now, in the next class I will summarize this idea and we will look at.

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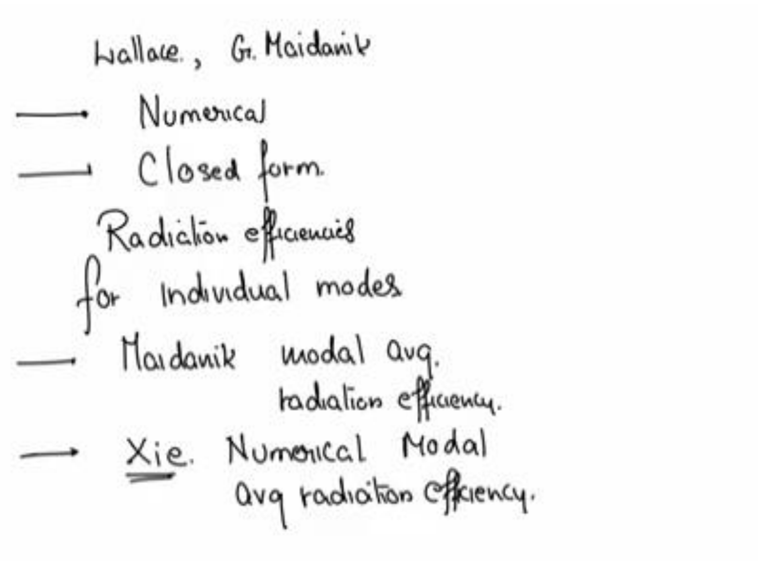

Start looking at two papers one by Wallace, the other by G. Maidanik who gave numerical versus closed form expressions for radiation efficiencies for individual modes. Of course, finally Maidanik gave an expression for modal averaged radiation efficiency, when all modes radiate together. I should mention one more paper which is by Xie, they also have given Numerics, they have given numerical calculations on modal average radiation efficiency. We will see this in the next class. I will stop here, thanks.