Sound and Structural Vibration
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Module No # 09
Lecture No # 43
Physics of Panel Radiation using Monopole Model

Good morning and welcome to this next lecture on sound and structural vibration we were modeling a radiating panel using monopoles. And we are giving suggestions for further refinements if a single-phase cell is inadequate, we could divide it into sub cells. And further because the mode itself has varying velocity over the panel we could allot appropriate amplitudes to the sub cells. So, these 2 aspects give a huge improvement in numeric.

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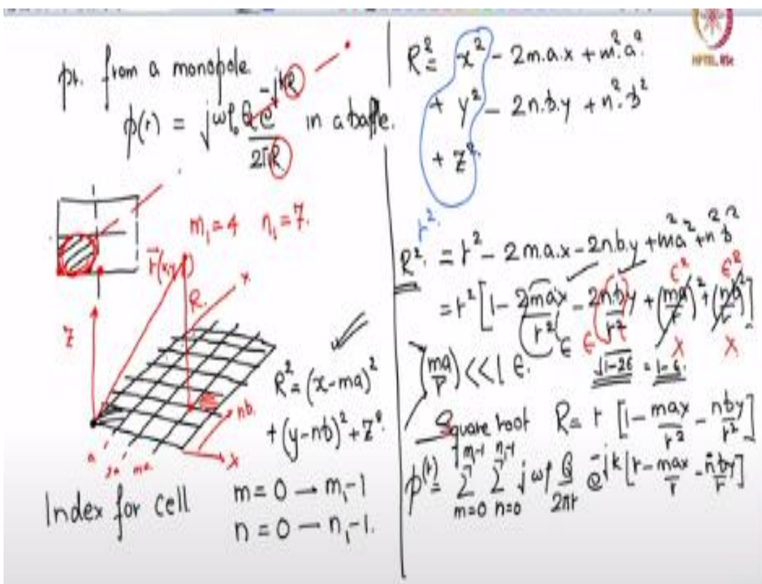

Now let us start looking at some derivation so we have pressure from a monopole this I expect you to know from acoustics. The p and some r

$$p(r) = j\omega\rho_0 Q \frac{e^{-jkR}}{2\pi R}.$$

Because the monopole in a baffle placed on a wall by itself is a doubling of pressure. Now we have to place we have to deal with now distances.

Source distance to receiver distance the source is the cell and monopole, and receiver is the receiver point. So, we have to locate although it is a certain volume velocity had to locate it in some place, so I am going to locate this at the corner. So, the whole dimension is placed at the corner and it will not cause too much of an error because we are going to locate the distance from the source to the receiver which is very far it is a far field.

So, from that distance spatial extent of the monopole is largely irrelevant so I will find a convenient point which is the left lower corner to place the monopole. So now if we are going to still look at m_1 is 4 and n_1= 7 they are looking at that let me put my coordinate system, so I have a panel here 1, 2, 3, 4, 5, 6, 7. And in this direction let us see is 4 7; 1, 2, 3, 4, 5, 6, 4 7. So let us say if am looking at a receiver point somewhere here far away of course this is my x that is my y.

And then I have this is a , $2a$, and somewhere I have ma, and this is my z axis. Then let us call this my r, it is some x, y, z. And I am looking at some location so this cell is located here so let us see that distance this distance is my R transmitter to receiver that distance is R. So, in this direction we have $1b, 2b$ and nb so now what is R we will need R right what is R? So R let us say

$$R^2 = (x - ma)^2 + (y - nb)^2 + z^2.$$

So now we need an index for the cell which also represents the distance right ma will be the x distance nb will be the y distance. So, the first cell location is at the left-hand corner so it is right here so my m and n shall begin from 0. So, m will go from 0 to $m_1 - 1$ and n will go from 0 to $n_1 - 1$, ma and nb are the distances of that monopole from the origin. So, this happens to be

$$R^2 = x^2 - 2\,m\,a\,x + m^2a^2 + y^2 - 2\,n\,b\,y + n^2b^2 + z^2.$$

$$R^2 = r^2 - 2\,m\,a\,x - 2\,n\,b\,y + m^2a^2 + n^2b^2,$$

$$= r^2\left[1 - \frac{2\,m\,a\,x}{r^2} - \frac{2\,n\,b\,y}{r^2} + \left(\frac{ma}{r}\right)^2 + \left(\frac{nb}{r}\right)^2\right]$$

Now what I am going to say is that $\frac{ma}{r}$ the distance from the origin of the cell over distance from the origin of the receiver is a very small quantity epsilon.

So, then this these terms carry epsilon square so these are much smaller, and we will drop them these. So, I get now this is

$$R = r\left[1 - \frac{m\,a\,x}{r^2} - \frac{n\,b\,y}{r^2}\right]$$

Where what have I done $\sqrt{1-2\epsilon} = 1-\epsilon$ I have done that binomial expansion I remove the square root and solved this. I drop these terms then I am left with r^2 took a square root these are small terms twice epsilon terms, so I made them $1-\epsilon$ I have received reach this point.

So now the pressure is a sound at that point r let us say is a sum so we are keeping the time term out by now you know right. So,

$$p(r) = \sum_{m=0}^{m_1-1} \sum_{n=0}^{n_1-1} j\omega\rho \frac{Q}{2\pi r} e^{-jk\left[r - \frac{m\,a\,x}{r} - \frac{n\,b\,y}{r}\right]},$$

so that is the pressure summed over all the monopoles.

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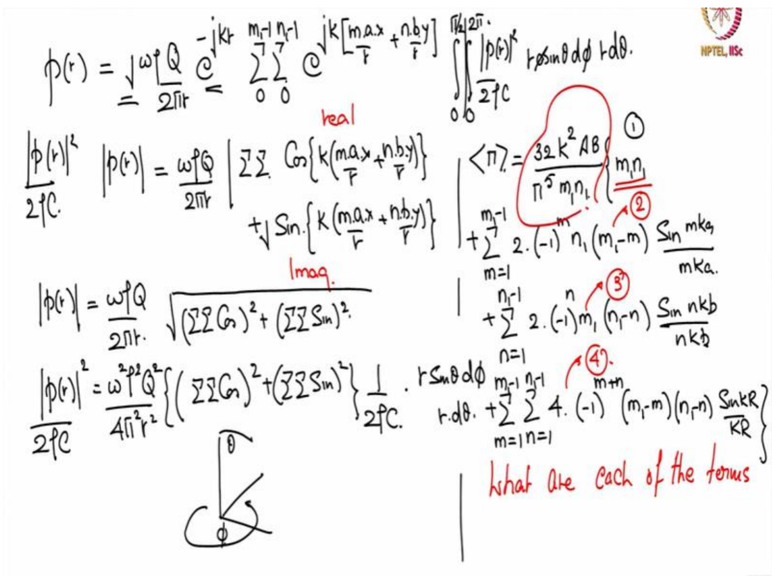

Now we get pressure at the receiver point given by

$$p(r) = j\omega\rho \frac{Q}{2\pi r} e^{-jkr} \sum_{m=0}^{m_1-1} \sum_{n=0}^{n_1-1} e^{jk\left[\frac{m\,a\,x}{r} + \frac{n\,b\,y}{r}\right]}.$$

So, we have to compute the intensity and we have to integrate over the hemisphere somewhere some integration will be there which is simple enough, but we will compute intensity for field intensity which is magnitude $p(r)$ square over $2\rho c$.

So first we need magnitude of pressure so that does what j goes away exponent goes away so I have

$$|p(r)| = \frac{\omega\rho Q}{2\pi r}\sum\sum \cos\left\{k\left(\frac{m\,a\,x}{r} + \frac{n\,b\,y}{r}\right)\right\} + j\sin\left\{k\left(\frac{m\,a\,x}{r} + \frac{n\,b\,y}{r}\right)\right\}.$$

So, this is the real term this is the imaginary term, and we are trying to get the magnitude of this thing.

So now magnitude of this thing is equal to

$$= \frac{\omega\rho Q}{2\pi r}\sqrt{\left(\sum\sum \cos[\,]\right)^2 + \left(\sum\sum \sin[\,]\right)^2}.$$

So real squared plus imaginary; square, square root that is the amplitude. So, then

$$|p(r)|^2 = \frac{\omega^2\rho^2Q^2}{4\pi^2r^2}\left\{\left(\sum\sum \cos[\,]\right)^2 + \left(\sum\sum \sin[\,]\right)^2\right\},$$

so, divide by $2\rho c$ so I will that is my intensity.

And we have to multiply by $r\ \sin\theta\ d\phi\ r\ d\theta$ in spherical coordinates. Where θ is the elevation angle measured this way and ϕ is the azimuthal angle measured this way.

$$\frac{|p(r)|^2}{2\rho c} = \frac{\omega^2\rho^2Q^2}{4\pi^2r^2}\left\{\left(\sum\sum \cos[\,]\right)^2 + \left(\sum\sum \sin[\,]\right)^2\right\}\frac{1}{2\rho c}\ r\ \sin\theta\ d\phi\ r\ d\theta.$$

Now you integrate $\frac{|p(r)|^2}{2\rho c}$ you integrate. With what θ limits 0 to π by 2, ϕ limits 0 to 2π so $r\ \sin\theta\ d\phi\ r\ d\theta$ integrator and it is relatively easily integrable.

Once you integrate that this power let me give it this symbol here the power is given by

$$\langle\pi\rangle = \frac{32\,k^2\,AB}{\pi^5 m_1 n_1}\left\{m_1 n_1 + \sum_{m=0}^{m_1-1} 2(-1)^m\, n_1(m_1 - m)\frac{\sin mka}{mka}\right.$$
$$+ \sum_{n=0}^{n_1-1} 2(-1)^n\, m_1(n_1 - n)\frac{\sin nkb}{nkb}$$
$$\left.+ \sum_{m=0}^{m_1-1}\sum_{n=0}^{n_1-1} 4(-1)^{m+n}\,(m_1 - m)(n_1 - n)\frac{\sin kR}{kR}\right\}$$

So, there are 4 terms this is the second term, this is the third term, this is the fourth term there are fourth terms and this is general and demonstrating with respect to m_1 is 4, n_1 is 7 but this is the general expression. So now what is each of these each of the terms what do they mean physical? So, remember $m_1 n_1$ and this term in front so let us see.

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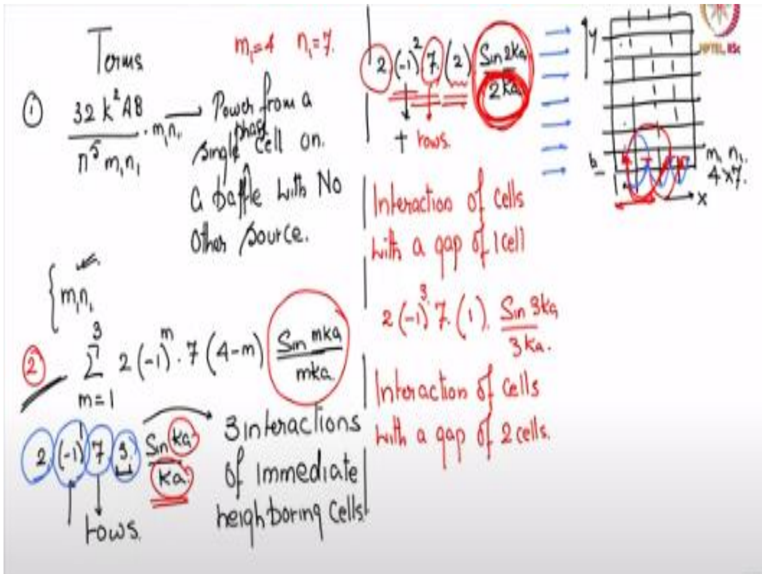

So, we have the terms let us call it terms the first term given by

$\frac{32\,k^2\,AB}{\pi^5 m_1 n_1}$ is power from a single cell on a baffle single phase cell on a baffle with no other source. So inside we have what $m_1 n_1$ so this multiplied by $m_1 n_1$ is the power from all the monopoles there are m_1 and n_1 phase cells so for all the phase cells that is the first step, so this is the first step.

So, we are now demonstrating for, m_1 = 4, n_1 = 7 I am making it specific now. So, the second term how does it look like term 2 it looks like

$$\sum_{m=1}^{3} 2(-1)^m 7\ (4-m)\frac{\sin mka}{mka}.$$

I need the schematic for this so let me go here let me say here so again let us see $1, 2, 3, 4, 5, 6$, very good so I have 4 by 7 so this is y direction this is x direction $m_1 n_1$.

So now let us see if I open this term out, I get for, $m =$ 1, I get twice into $-$ 1 to the power 1, 7, 4 – 1, 3 and $\sin ka$ by ka this short distance is a is b. So, I get a negative value I get 7 of these represent 7 rows and I get three of something what is 3 of something? I get three interactions of cells of neighboring cells of immediate neighboring cells across the. So, I get an interaction of this cell with that cell immediate neighbor that is 1 is get an interaction of this with this 2 this with this 3 that is the 3.

And this is a $+$ suppose this is $-$ and this is $+$ is a $-$ so there is a phase change of -1 across the cell that is this phase change. And I get 7 of these because there are 7 rows $1, 2, 3, 4, 5, 6, 7$ rows and I get 2 because there are 2 cells involved interaction of this with this interaction of this with these 2 cells involved that is the meaning of the first term for $m = 1$. So, when $m = 2$ now within this summation we are looking at the term 2 so when $m = 2$, I get twice -1 whole squared so positive now again 7 but this time $4 - 2$ which is 2 into sin $2ka$ by $2ka$.

So, in a sense this is actually the actual interaction term so you can see that as m increases m was 1 so ka over ka this ka diminishes reduces because sin has a peak value of 1 so now here it is $2ka$ so the distance is now more so interaction effect is less. So let us see the meaning of these terms so now the negative is gone this positive. There are 7 of these again to describe the rows and there are 2 of this, what are the 2 of this?

The interaction of cells with a gap of one cell so this cell interacts with that cell and this cell interacts with that cell. There are 2 such interactions in a row there are 7 rows the interaction is without a sign change because this is positive and that is positive, or this is negative and that is negative. And 2 because there are 2 cells this interaction with this and this interaction this there are 2 cells so that is the term.

And again, as I said this describes the interaction effect so as the distance this distance increases now its $2ka$, the effect is reduced. So now the next term we have or $m = 3$ which is the last term for the third sorry $m = 3$ within the same term 2 is we get again 2 this time again -1 to the power 3 the 7 and then $4 - 3$ which is 1 and sin $3ka$ over $3ka$. So, this is interaction of cells with a gap of 2 cells I am running out of time I will close here continue in the next class.