Sound and Structural Vibration

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Lecture – 46
Radiation Resistance Derivation From Maidanik's Work, Contd

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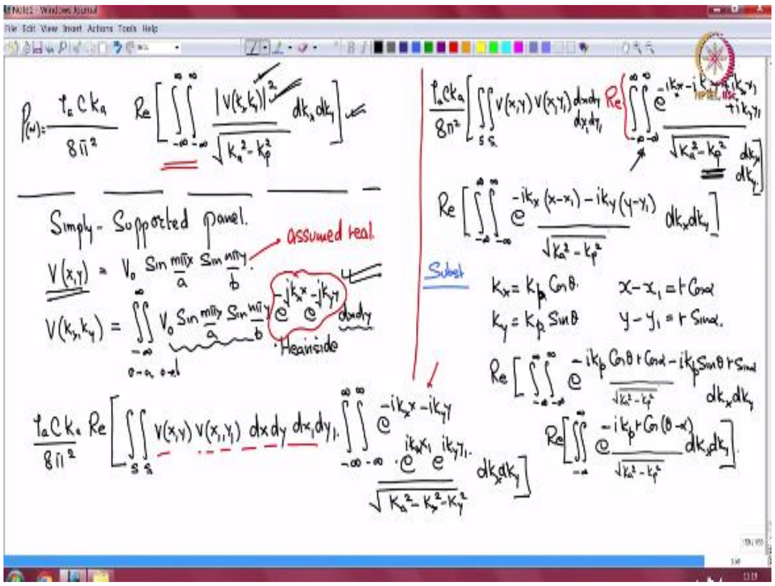

So, welcome to this next lecture let me continue from where I left off. So, let me close this integral here. So, we look at this part here. So, what I have is a

$$Re\left\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{e^{-ik_x(x-x_1)-ik_y(y-y_1)}}{\sqrt{{k_a}^2-{k_p}^2}}dk_x\,dk_y\right\}.$$

So, here we make some substitutions which is what I say. So, where substitution $k_x = k_p\cos\theta$, $k_y = k_p\sin\theta$.

$x - x_1 = r\ \cos\alpha$ and $y - y_1 = r\ \sin\alpha$. So, if we do this we get

$$Re\left\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{e^{-ik_p\cos\theta\, r\cos\alpha-ik_p\sin\theta\, r\ \sin\alpha}}{\sqrt{{k_a}^2-{k_p}^2}}dk_x\,dk_y\right\},$$

$$Re\left\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{e^{-ik_p\, r\ \cos(\theta-\alpha)}}{\sqrt{{k_a}^2-{k_p}^2}}dk_x\,dk_y\right\}.$$

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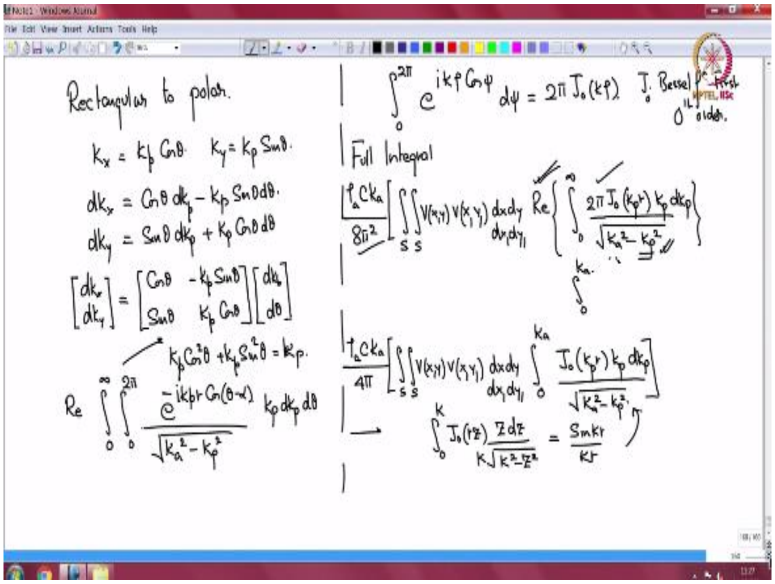

Then, so now what we will do is we will convert from rectangular to polar. So, my dk_x so, what do I have here? I have $k_x = k_p \cos\theta$ then I have $k_y = k_p \sin\theta$. So, I have dk_x given by $\cos\theta \; dk_p - \; k_p \; \sin\theta \; d\theta$ similarly, dk_y is given by $\sin\theta \; dk_p + \; k_p \; \cos\theta \; d\theta$. By put it in a matrix form I have

$$\begin{bmatrix} dk_x \\ dk_y \end{bmatrix} = \begin{bmatrix} \cos\theta & -\, k_p \; \sin\theta \\ \sin\theta & k_p \; \cos\theta \end{bmatrix} \begin{bmatrix} dk_p \\ d\theta \end{bmatrix}.$$

So, this is the matrix where the Jacobian comes from so, I get the determinant as $k_p \cos^2\theta + k_p \sin^2\theta$ which is k_p so, if I change the integral from rectangular to polar. So, what I will get is the

$$Re\left\{\int_0^{\infty}\int_0^{2\pi} \frac{e^{-ik_p\, r\, \cos(\theta-\alpha)}}{\sqrt{{k_a}^2 - {k_p}^2}} k_p \; dk_p \; d\theta\right\}.$$

Now, I will give you a result here

$$\int_0^{2\pi} e^{ik\rho\,\cos\psi} d\psi = 2\pi\, J_0(\,k\rho).$$

where J_0 is what the Bessel function first kind 0th order oscillating decaying sinusoid. So, now the full integral becomes

$$\frac{\rho_a c k_a}{8\,\pi^2}\left[\int_0^a\int_0^b V(x,y)V(x_1,y_1)\,dx\,dy\,dx_1\,dy_1\;\; Re\left\{\int_0^\infty \frac{2\pi\, J_0(k_p r)\,k_p}{\sqrt{{k_a}^2-{k_p}^2}}\,dk_p\right\}\right].$$

Now, this integral will remain real only as long as k_p is less than or equal to k_a. If k_p exceeds k_a this is becomes imaginary. So, what I will do is this removal of this real and the integral limits change from 0 to k_a. So now, if I use this to p and do some cancellation here then I can write

$$\frac{\rho_a c k_a}{4\,\pi}\left[\int_0^a\int_0^b V(x,y)V(x_1,y_1)\,dx\,dy\,dx_1\,dy_1\;\;\int_0^{k_a}\frac{J_0(k_p r)k_p}{\sqrt{{k_a}^2-{k_p}^2}}\,dk_p\right].$$

So, I use another result now, which is just a result from integral tables

$$\int_0^k J_0(rz)\frac{z\,dz}{k\sqrt{k^2-z^2}} = \frac{\sin kr}{kr},$$

we will use this result. So, we will use this result here. So, now what happens?

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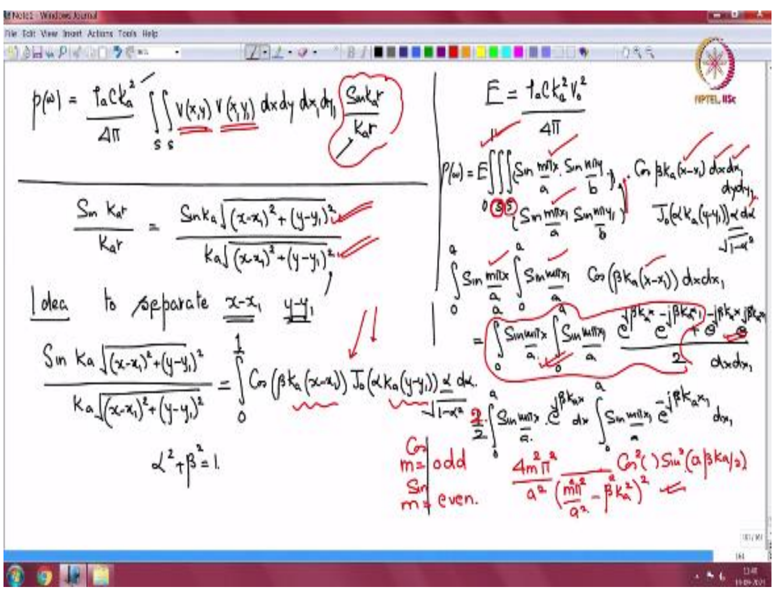

What that does is we get this power integral

$$p(\omega) = \frac{\rho_a c {k_a}^2}{4\,\pi}\int_0^a\int_0^b V(x,y)V(x_1,y_1)\,dx\,dy\,dx_1\,dy_1\;\;\frac{\sin k_a r}{k_a r}.$$

Now we have $\frac{\sin k_a r}{k_a r}$ is written as if you recall the r definition is $\sqrt{(x-x_1)^2+\left(y-y_1\right)^2}$,

$$\frac{\sin k_a r}{k_a r} = \frac{\sin k_a\sqrt{(x-x_1)^2+\left(y-y_1\right)^2}}{k_a\sqrt{(x-x_1)^2+\left(y-y_1\right)^2}}.$$

So, how do we use this? See, Maidanik's main idea here at this stage was to separate the $x-x_1$ type term from $y-y_1$ type term there to both of them caught up under the square root. You wanted to separate the two. So, how does it happen? We have a very nice relation

$$\frac{\sin k_a\sqrt{(x-x_1)^2+\left(y-y_1\right)^2}}{k_a\sqrt{(x-x_1)^2+\left(y-y_1\right)^2}} = \int_0^1 \cos\left(\beta k_a(x-x_1)\right) J_0\left(\alpha k_a\left(y-y_1\right)\right) \frac{\alpha}{\sqrt{1-\alpha^2}}\, d\alpha,$$

α and β are arbitrary numbers.

$\alpha^2+\beta^2=1$. So, now, this is going to be replaced by this why is that because this is a simply supported plate, so, x function and y are separated. However, it becomes a problem here because $x-x_1$ and $y-y_1$ are peculiarly related here, but you can see if I replace this with this $x-x_1$ and $y-y_1$ becomes separate and outside of the square root. So that is a great advantage. So, now, what we do is? We use that we use this new relation is are very lengthy so, I need more and more horizontal space.

So, let us see I will denote my one constant E as $\frac{\rho_a c k_a{}^2 V_0{}^2}{4\,\pi}$ that is my E just to shorten the presentation. So, now what happens my power integral we will see if I can fit in one line

$$p(\omega) = E\int_0^1\int_0^a\int_0^b \left(\sin\frac{m\pi x}{a}\,\sin\frac{n\pi y}{b}\right)\left(\sin\frac{m\pi x_1}{a}\,\sin\frac{n\pi y_1}{b}\right)\cos\left(\beta k_a(x-x_1)\right)J_0(\alpha k_a(y - y_1))\frac{\alpha}{\sqrt{1-\alpha^2}}\,dx\,dy\,dx_1\,dy_1\,d\alpha.$$

So, they are all in one line, this term will come up here, this term will come up here they are only one line one integral. Now we see that x comes here you know x_1 comes here. So, I have $dx\ dx_1$, then I have $x-x_1$ coming here. So, we will just look at the x and x_1 integrals.

$$\int_0^a \sin\frac{m\pi x}{a}\int_0^a \sin\frac{m\pi x_1}{a}\,\cos\left(\beta k_a(x-x_1)\right)\,dx\,dx_1,$$

$$= \int_0^a \sin\frac{m\pi x}{a} \int_0^a \sin\frac{m\pi x_1}{a} \frac{e^{j\beta k_a x} e^{-j\beta k_a x_1} + e^{-j\beta k_a x} e^{j\beta k_a x_1}}{2}\, dx\, dx_1.$$

So, this is now simple enough so, you will get two terms you will get

$$\frac{1}{2}\int_0^a \sin\frac{m\pi x}{a} e^{j\beta k_a x}\, dx \int_0^a \sin\frac{m\pi x_1}{a}\, e^{-j\beta k_a x_1}\, dx_1\,,$$

so, that is this part that is this part and similarly this multiplied by this part, and they are the same so, you will get a doubling.

$$\frac{2}{2}\int_0^a \sin\frac{m\pi x}{a} e^{j\beta k_a x}\, dx \int_0^a \sin\frac{m\pi x_1}{a}\, e^{-j\beta k_a x_1}\, dx_1.$$

If you carry out this integral. So, carry out what do you get you will get

$$\frac{4m^2\pi^2}{a^2}\left(\frac{m^2\pi^2}{a^2} - \beta^2 {k_a}^2\right)^2 \cos^2(\, a\beta k_a/2\,)\ \ or\ \ \sin^2(\, a\beta k_a/2\,)\,.$$

So, cos you use when m is odd you use cos you use sin when m is even simple. So, within this 0 to 1 integral we have done the x part of the whole integral it is done, so this will now come only under 0 to 1 integral. There is no y in here.

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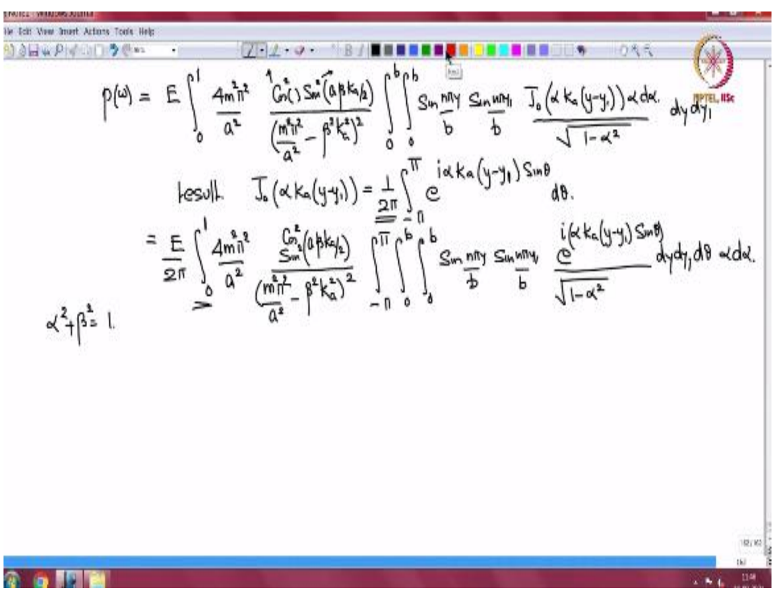

So, let us see. So, how does the whole integral look for example So, let me see

$$p(\omega)$$

$$= E\int_0^1 \frac{4m^2\pi^2}{a^2}\frac{\cos^2(\,a\beta k_a/2\,)\ \ or\ \ \sin^2(\,a\beta k_a/2\,)}{\left(\frac{m^2\pi^2}{a^2}-\beta^2 {k_a}^2\right)^2}\int_0^b \sin\frac{n\pi y}{b}\int_0^b \sin\frac{n\pi y_1}{b}\,\frac{J_0(\alpha k_a(y-y_1))\alpha}{\sqrt{1-\alpha^2}}\,dy\,dy_1 d\alpha\,.$$

So, one more result we use what is that the result to be used is this

$$J_0\left(\alpha k_a\left(y-y_1\right)\right)=\frac{1}{2\pi}\int_{-\pi}^{\pi} e^{i\alpha k_a(y-y_1)\sin\theta}d\theta.$$

So, what happens to our result we have this

$$\frac{E}{2\pi}\int_0^1 \frac{4m^2n^2}{a^2}\frac{\begin{matrix}\cos^2\\ \sin^2\end{matrix}(a\beta k_a/2)}{\left(\frac{m^2\pi^2}{a^2}-\beta^2 {k_a}^2\right)^2}\int_{-\pi}^{\pi}\int_0^b\int_0^b \sin\frac{n\pi y}{b}\sin\frac{n\pi y_1}{b}\,\frac{e^{i\alpha k_a(y-y_1)\sin\theta}}{\sqrt{1-\alpha^2}}\,dy\,dy_1\,d\theta\,\alpha\,d\alpha.$$

α relates to this integral so please do not forget and remember $\alpha^2+\beta^2=1$. Now so we look at this integral the time is running out let me stop here we will continue thanks.