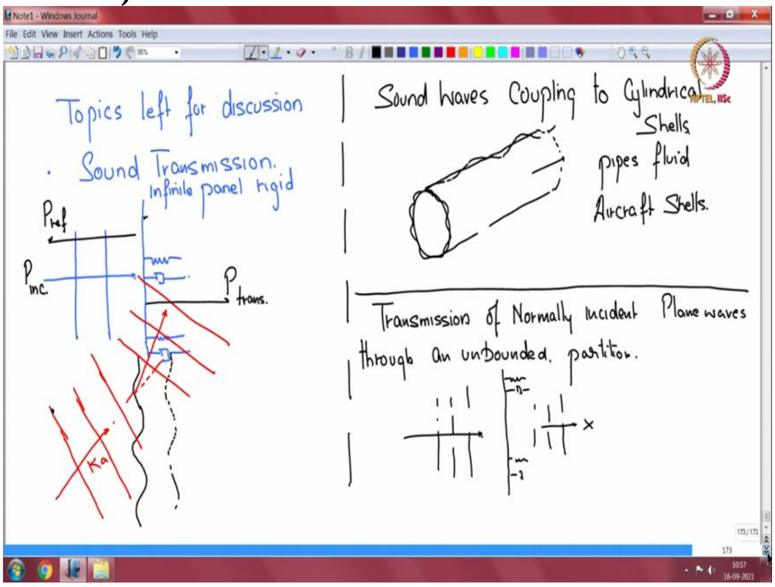
Sound and Structural Vibration
Prof. Venkata Sonti
Department of Mechanical Engineering
Indian Institute of Technology - Bengaluru

Lecture - 50
Transmission of Sound through a Rigid Panel with Flexible Mounts

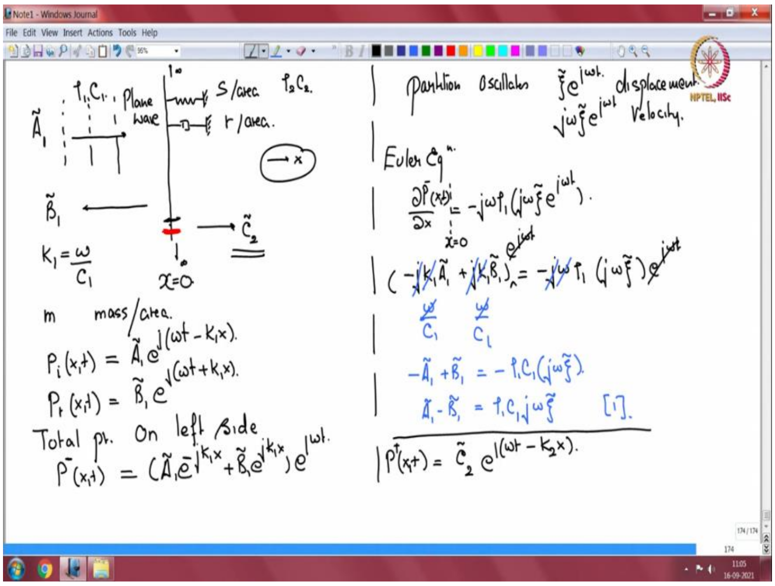
Good morning and welcome to this next lecture on sound and structural vibration we are going to start looking at a new topic today.

(Refer Slide Time: 00:32)



Which is transmission, transmission of normally incident plane waves through an unbounded partition. So, there is an unbounded partition which goes off to infinity up and down and there is a plane sound wave incident on it, there is a plane wave incident on it, and this is mounted on some springs and dampers and so forth and we want to find out what is transmitted to their site? So, let me draw the geometry properly.

(Refer Slide Time: 02:00)



So, this here is my unbounded partition goes off to infinity in two directions up and down this is my x axis over here and the partition has a mass m per unit area it is mounted on springs, the spring stiffness is also s per unit area and mounted on viscous dampers which are also r units per unit area and the left side has medium properties $\rho_1 c_1$ density ρ_1, speed of sound c_1 and right side has $\rho_2 c_2$.

So, now a plane wave is incident, or it is a plane wave is just a plane wave fronts of plane no change happens on the wave front. So, plane wave is incident it has an amplitude $\tilde{A}_1$, tilde for complex, 1 for the medium as it hits this partition so, this partition will obviously start vibrating. Then a sound wave gets reflected which is with an amplitude $\tilde{B}_1$, 1 for the medium and tilde for the complex.

And because this partition now oscillates or vibrates, it is a harmonic plane wave with single frequency. So, therefore, there is going to be a transmitted sound that amplitude is $\tilde{C}_2$ and two for the second medium. So, we have to find out what is C in this situation? So, now, let us say the $P_i(x,t) = \tilde{A}_1 e^{j(\omega t - k_1 x)}$, k_1 is the wave number in the left side k_1 is $\frac{\omega}{c_1}$ there is a reflected wave.

Reflected wave given by $P_r(x,t) = \tilde{B}_1 e^{j(\omega t + k_1 x)}$. So, the total pressure on left side is given by $P^-(x,t) = (\tilde{A}_1 e^{-jk_1 x} + \tilde{B}_1 e^{jk_1 x}) e^{j\omega t}$. Now let us say the partition oscillates as $\tilde{\xi} e^{j\omega t}$ that is the displacement and therefore, velocity is $j\omega\tilde{\xi} e^{j\omega t}$ that is the velocity.

So, at $x = 0$ this is my x as I said at $x = 0$ where the partition is if I use my Euler equation what do I get?

$$\frac{\partial P^-(x,t)}{\partial x} = j\omega\rho_1\left(j\omega\tilde{\xi}e^{j\omega t}\right).$$

Now, what is $\frac{\partial P^-(x,t)}{\partial x}$ again at $x = 0$ what is that value so, if we do that we get

$$\left(-jk_1\tilde{A}_1 + jk_1\tilde{B}_1\right)e^{j\omega t} = -j\omega\rho_1\left(j\omega\tilde{\xi}\right)e^{j\omega t}.$$

Now, I will cancel that j.

Then this j can go this j can go this j can go this j can go then k_1 is ω/c_1. Therefore, this can go, and this can go so, I have

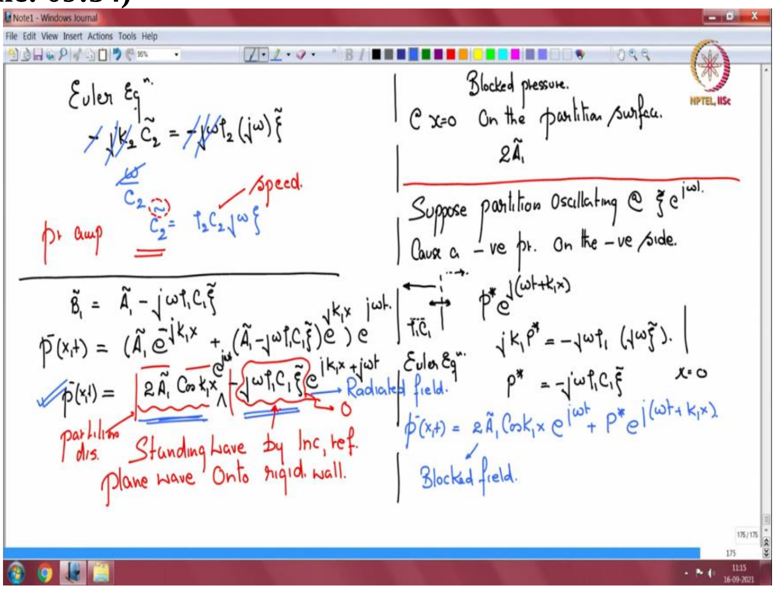
$$-\tilde{A}_1 + \tilde{B}_1 = -\rho_1 c_1\left(j\omega\tilde{\xi}\right)$$

or

$$\tilde{A}_1 - \tilde{B}_1 = \rho_1 c_1 j\omega\tilde{\xi}.$$

So, this is an important equation let us say it is 1 now, the pressure on the transmitted side $P^+(x,t)$ plus let us say is equal to $\tilde{C}_2 e^{j(\omega t - k_2 x)}$. Because the medium is different the wave number is different.

(Refer Slide Time: 09:54)



Now again if we use the Euler equation, I get

$$-jk_2\tilde{C}_2 = -j\omega\rho_2(j\omega)\tilde{\xi}.$$

So, if we again do the cancellations minus goes with minus j goes with j, k_2 is ω/c_2. So, then ω goes with ω and k_2 goes. So, I have $\tilde{C}_2 = \rho_2 c_2 j\omega\tilde{\xi}$ now, we should not confuse between the c_2, $\tilde{C}_2$ tilde is the pressure amplitude and here c_2 is speed of sound.

So, this tilde will be the distinguishing factor. Now, we can write $\tilde{B}_1$ instead of in terms of $\tilde{A}_1$. So, my $\tilde{B}_1$ is $\tilde{A}_1 - j\omega\rho_1 c_1 \tilde{\xi}$. So, what is the pressure on the negative side? $P^-(x,t)$ what is the pressure? That is given by $\left(\tilde{A}_1 e^{-jk_1 x} + \left(\tilde{A}_1 - j\omega\rho_1 c_1 \tilde{\xi}\right) e^{jk_1 x}\right) e^{j\omega t}$.

So, if I now add I get $2\tilde{A}_1 \cos k_1 x \, e^{j\omega t} - j\omega\rho_1 c_1 \tilde{\xi} \, e^{jk_1 x + j\omega t}$, that is my pressure on the negative side. Now, let us see what are these two terms? These two terms here we have what are these two terms? So, this term does not have any partition displacement related term.

So, if the partition displacement is set to 0 if this is 0 then the pressure you would have is this. So, this is just the standing wave that is caused by an incidence and reflection of a plane wave on to a rigid wall or rigid partition so, such a pressure this pressure is called blocked pressure and so, at $x = 0$ on that partition surface you get an amplitude of twice $\tilde{A}_1$ tilde plus of course the time term.

Now, what is this next term here? What is this this term here? So, if we separately try to find out suppose, suppose the partition is oscillating at $\tilde{\xi} e^{j\omega t}$. So, it will cause a negative going pressure on the negative side. So, the partition is oscillating and therefore, it will cause a pressure to go negative it will cause a pressure to go positive also, but we are not talking about that yet.

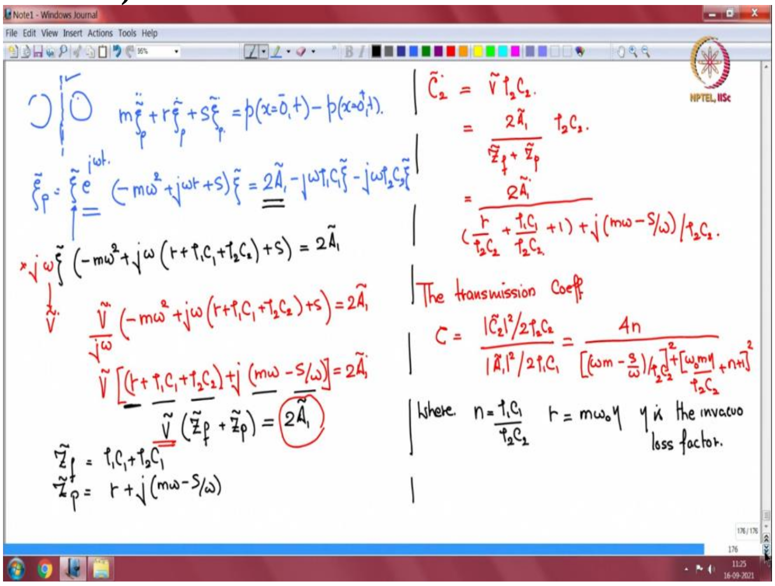
So, this oscillating partition because there is a medium $\rho_1 c_1$ here will cause a pressure to go leftward suppose that is $p^* e^{j(\omega t + k_1 x)}$ suppose that is what it is then again if we use our Euler equation we get

$$jk_1 p^* = -j\omega\rho_1 \left(j\omega\tilde{\xi}\right)$$

this is evaluated at $x = 0$. So, this gives me what again cancelling all that I get p^* given by $-j\omega\rho_1 c_1 \tilde{\xi}$.

So, that is what we have here along with the minus so, this expression now it is worth looking at this expression P^- involves a pressure term which is the blocked pressure which can be set due to the rigidness of the partition on top of it there is a pressure moving leftward due to the oscillation of that partition. So, these are the two terms over there. So, my negative side pressure $P^-(x,t)$ can be written as $2\tilde{A}_1 \cos k_1 x \, e^{j\omega t} + p^* e^{j(\omega t + k_1 x)}$. So, this is called the radiated field, and this is called the blocked pressure blocked field.

(Refer Slide Time: 20:00)



Now so there is a partition now, there is a pressure field here, there is a pressure field here oscillated pressure fields, this there is a partition with mass stiffness damping, so, we should be able to write a per unit area equation. So, that equation of motion per unit area is

$$m\ddot{\tilde{\xi}}_p + r\dot{\tilde{\xi}}_p + s\tilde{\xi}_p = p(x = 0^-, t) - p(x = 0^+, t).$$

Why is did I do that because I can now say $\tilde{\xi}_p$ is $\tilde{\xi}e^{j\omega t}$ that is why all earlier equations were in the terms of this the amplitude. And therefore, if we use the harmonicity I get

$$(-m\omega^2 + j\omega r + s)\tilde{\xi} = 2\tilde{A}_1 - j\omega\rho_1 c_1\tilde{\xi} - j\omega\rho_2 c_2\tilde{\xi},$$

this is being evaluated at $x = 0$.

So, if I keep only $\tilde{A}_1$ on one side I get

$$\tilde{\xi}(-m\omega^2 + j\omega(r + \rho_1 c_1 + \rho_2 c_2) + s) = 2\tilde{A}_1$$

we will convert it to velocity so, we will multiply by $j\omega$ we will multiply by this by $j\omega$ and make it $\tilde{V}$. So, what we now have is

$$\frac{\tilde{V}}{j\omega}(-m\omega^2 + j\omega(r + \rho_1 c_1 + \rho_2 c_2) + s) = 2\tilde{A}_1.$$

If we now bring this $j\omega$ inside I get

$$\tilde{V}[(r + \rho_1 c_1 + \rho_2 c_2) + j(m\omega - s/\omega)] = 2\tilde{A}_1.$$

Now, this so, now, the $m\omega$, s/ω and r are the partition impedances $\rho_1 c_1$, $\rho_2 c_2$ are fluid impedances so, what this means is that $\tilde{V}$ the velocity amplitude is given by the fluid impedance plus the partition impedance operated upon by twice the amplitude.

So, $\tilde{Z}_f$ is a fluid impedance which is $\rho_1 c_1 + \rho_2 c_2$ and $\tilde{Z}_p$ is $r + j(m\omega - s/\omega)$. So, response of the fluid loaded structure is directly related to the blocked pressure. So, it is interesting. So, the response velocity is directly related to the amplitude of the blocked pressure. So, we started off by saying we want to know $\tilde{C}_2$. So, that is equal to $\tilde{V}\rho_2 c_2$, which is what equal to $\frac{2\tilde{A}_1}{\tilde{Z}_f+\tilde{Z}_p}\rho_2 c_2$.

So, if we expand it, if we write it in an expanded manner, we get

$$\frac{2\tilde{A}_1}{\left(\frac{r}{\rho_2 c_2} + \frac{\rho_1 c_1}{\rho_2 c_2} + 1\right) + j(m\omega - s/\omega)/\rho_2 c_2}.$$

So, now the transmission coefficient is given by the τ given by the ratio of transmission transmitted and incident intensities. So, that is given by

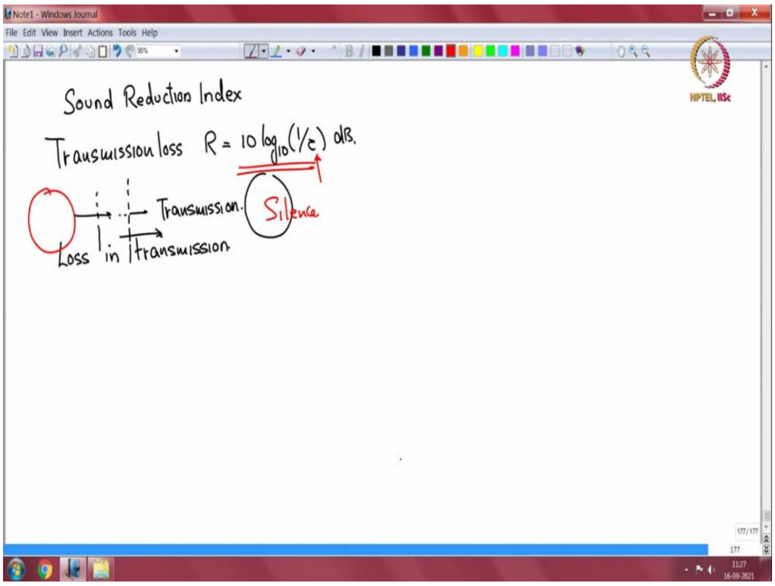
$$\tau = \frac{\left|\tilde{C}_2\right|^2/2\rho_2 c_2}{\left|\tilde{A}_1\right|^2/2\rho_1 c_1}.$$

This can be written after putting all the expressions this can be written as

$$\frac{4n}{\left[\left(\omega m - \frac{s}{\omega}\right)/\rho_2 c_2\right]^2 + \left[\frac{\omega_0 m\eta}{\rho_2 c_2} + n + 1\right]^2},$$

where n is given by this fluid impedance ratio and the viscous term r has been replaced by $\omega_0 m\eta$, ω_0 is the natural frequency of the panel and η is the invacuo loss factor of the panel.

(Refer Slide Time: 29:57)



One more thing that is we will talk about a sound reduction index or more commonly known as transmission loss symbolised by R as $10\log_{10}(1/\tau)$ it comes in decibels so, the way to understand transmission loss is that if I have a partition and I am sending a sound wave if there is most of it is transmitted that is of course transmission. So, transmission is good transmission, good amount is transmitted.

However, transmission loss is loss in transmission, if transmission loss is high then very little got transmitted across the partition. So, sometimes you want silence on this side you have some machinery running on this side and you want silence on this side which means what you want high transmission loss, and it is defined in this way. The inverse of transmission itself is an indicator. So, the time is up for this lecture I will close we will start from here. Thank you.