Sound and Structural Vibration
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Lecture – 54
Cylindrical Shell Vibration

Good morning and welcome to this next lecture on sound and structural vibration. We are discussing sound structure interaction in cylindrical geometries and we started just looking at the cylindrical vibration. So, we have come so far where we let me see showed the three equations of motion in the 3 directions using stress resultants.

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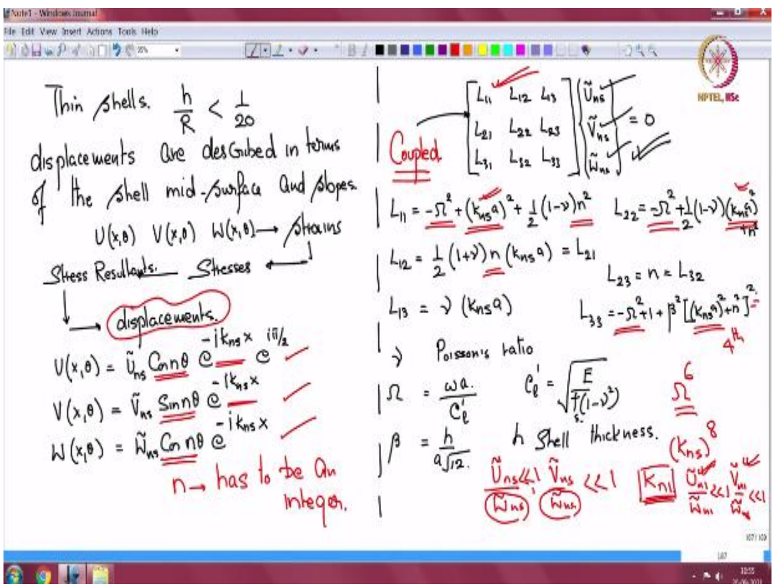

And the stress resultants can be further written in terms of displacements or other midplane displacements because it is a thin shell and once you do that then you assume this form because this form is suitable to the equations. So, this is the wave number in the x direction or propagator in the x direction and this is the circumferential description. Once you substitute that then you get that is matrix sorry, I should do this and you get

$$\begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \begin{Bmatrix} \tilde{U}_{ns} \\ \tilde{V}_{ns} \\ \tilde{W}_{ns} \end{Bmatrix} = 0.$$

Where

$$L_{11} = -\Omega^2 + (k_{ns}a)^2 + \frac{1}{2}(1-\nu)n^2,$$

$$L_{12} = \frac{1}{2}(1+\nu)n(k_{ns}a) = L_{21,}$$

$$L_{13} = \nu(k_{ns}a)\ ,$$

and then

$$L_{22} = -\Omega^2 + \frac{1}{2}(1-\nu)(k_{ns}a)^2 + n^2,$$

$$L_{23} = n = L_{32},$$

and

$$L_{33} = -\Omega^2 + 1 + \beta^2[(k_{ns}a)^2 + n^2]^2$$

the whole set of equations is now non dimensionalized.

So, ν is the Poisson's ratio then we have Ω the non dimensional frequency given by $\frac{\omega a}{C_l'}$, C_l' is the speed of the longitudinal wave in the material as a plate $\sqrt{\frac{E}{\rho_s(1-\nu^2)}}$, then β is another non dimensional parameter which is $\frac{h}{a\sqrt{12}}$ and h is shell thickness. Now you can see that you look at the diagonal terms which is has Ω^2 , L_{22} has Ω^2, L_{33} has Ω^2, so it is 6th order in the frequency.

And if you look at k_{ns} right here, it is square and square, so 4th order then you will get a square from L_{11}, you get a square from L_{22} so it is 8th order in k_{ns}, it is 8th order to the power 8 in k_{ns}.

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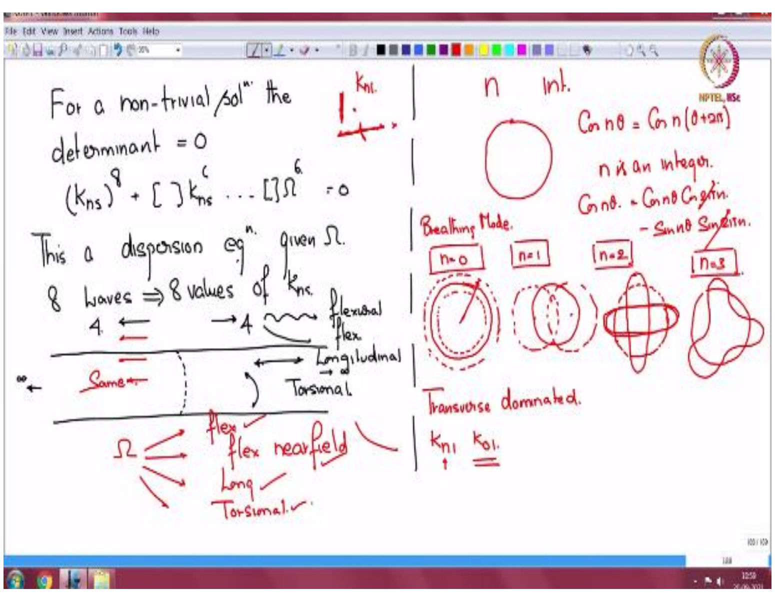

So, if you take for a non-trivial solution of this equation, this particular equation if you do not want $\tilde{U}_{ns}$, $\tilde{V}_{ns}$, $\tilde{W}_{ns}$ to be simultaneously 0 then the determinant of this must be 0. Nontrivial solution the determinant must be 0 and determinant has a polynomial in k_{ns}, $(k_{ns})^8 + [\ ]k_{ns}{}^6 \ldots.[\ ]\Omega^6 = 0$. So, this is the dispersion equation.

So, for a given an Ω you will have 8 waves and 8 values of that means 8 values of k_{ns} and what will they be if you have a shell and let us say I am locally exciting here this shell is going to infinity in this direction. Then I will have 4 waves, 4 waves going this way 4 waves going that way. So, that symmetry holds those are the 8 values and typically you will have a propagating flexural wave, you will have a decaying flexural wave, you will have a propagating longitudinal wave and a propagating torsional wave same on this side 4 waves and those are the 4 waves typical.

Now for $n = 0$ I am not going to present all the details here. So for $n = 0$ that means if I put $n = 0$ here why is that important? One more thing this shell this n here has to be an integer it is no surprise has to be an integer. So the reason will be n is an integer. The reason is that if you go around the cylindrical shell once as you come back to the same point the displacement should be continuous. So that means what $\cos n\theta = \cos n(\theta + 2\pi)$. So, this is possible only when n is an integer.

So, this will be equal to $\cos n\theta\ \cos 2\pi n - \sin n\theta\ \sin 2\pi n$. Now, if n is an integer then you will get 0 here you will get a 1 here then $\cos n\theta$ on both sides if n is not an integer that will not happen. So, n has to be an integer so, if n is an integer first let me describe that also for $n = 0$ what sort about displacement can we expect we can have let us say that transverse dominated displacements let us describe transverse dominated, again what do I mean by transverse dominated.

So, let us say I pick a particular Ω and I get 4 waves I get a flexural wave, I get a flexural near field decaying like this, then I get a longitudinal wave, then I get a torsional wave. So, if I plug any particular wave number back into this matrix why because k_{ns} is the number I have then that determinant is 0. So, rank drops. So, I can compute $\frac{\tilde{U}_{ns}}{\tilde{W}_{ns}}$,I can compute $\frac{\tilde{V}_{ns}}{\tilde{W}_{ns}}$.

And based on the wavenumber chosen I could have this to be a small value which means that wavenumber is speaking of a dominant transverse displacement. So, a particular wavenumber let us say k_{n1} says that $\frac{\tilde{U}_{n1}}{\tilde{W}_{n1}}$ will be very small. Similarly, $\frac{\tilde{V}_{n1}}{\tilde{W}_{n1}}$ will be very small which means this is a transverse displacement dominated wavenumber it will not be only transverse displacement there will be a small component of axial movement displacement and torsional displacement because this set of equations are coupled. So, I am trying to say why I said as a dominantly transverse.

So, if I have a particle then it will dominantly move up and down, but it will have a small part axial and will have small part torsional. So, k_{n1} could be that sort of a wave whereas k_{n2} could be dominantly axial so, that particle will be moving axially x direction, but it will have a small component of transverse small component of torsion. So, it is complicated but dominantly axial. So, let us now look at transverse dominated displacements for various ends how they could be. So, $n = 0$ what happens is that you have the displacement oscillates.

So, you will have a radius going up and down, like a breathing mode is called the breathing mode. So what I am saying is we have chosen let us say k_{n1} here n is 0, chosen k_{01} amongst these waves. So we have substitute $n = 0$ already into that matrix. And we are looking at that particular wavenumber where transverse displacement is dominant. So for $n = 0$ what happens is the radius breathes in and out previous breath in and out. So, the breathing mode we did that same thing for $n = 1$.

Then I will draw it smaller. So, we will have the nominal radius and it will oscillate back and forth. This will oscillate back and forth the centre will oscillate back and forth, that is the kind of transverse displacement you will get. If you have $n = 2$, you substitute $n = 2$ and find that wavenumber which is dominantly flexural, then it will have this nominal radius nominal cross section, then it will oval, it will oval like this. This is exaggerated, but that will how it is breath. And if it is $n = 3$, I do not have space here.

So, if then $n = 3$ the nominal cross section is like this and it will acquire the shape at some point in its vibration, it likewise is also exaggerated but and so forth. So, now, for $n = 0$ we will have

4 waves, $n = 1$ will have 4 waves, $n = 2$ we will have 4 waves $n = 3$ you will have 4 waves and so forth $n = 3$ and so, the n kind of makes it orthogonal $\cos\theta$ orthogonal to $\cos 2\theta$ orthogonal to every $\sin\theta$. So, it would be a logical to look at modes with n as a standard number.

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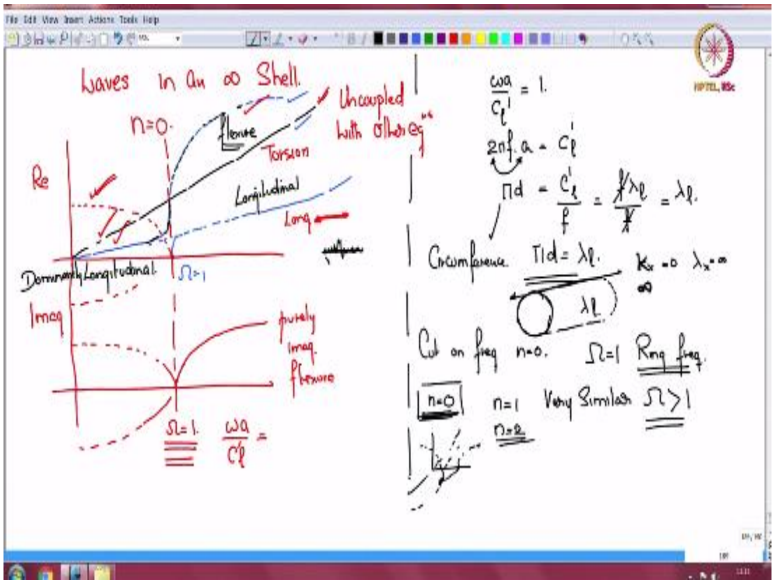

So, if we look at the waves in an infinite cylindrical shell for $n = 0$ you have to have 4 waves. So, how will it look like there could be real wavenumbers this could be complex wave numbers as could be purely imaginary wavenumbers. So, this is the real, this is the imaginary, and this is a line where I say $\Omega = 1$, Ω is what $\frac{\omega a}{c_l'}$ the non dimensional frequency and that is equal to 1 is some line here.

Now below this I will have two complex waves with various alternating real and imaginary signs. You will have negative also and negative is also negative imaginary so you have two of those below this $\Omega = 1$ frequency. And you will have two real waves so one is going like this let us say one is going at some angle and continues forever. The other let us say blue comes like this very sharply turns and then starts to behave like a flexural wave omega to the power half behavior and the other one beyond $\Omega = 1$.

This behaves as though it is continuing it behaves as though there is this continuity as though now as far as the imaginary so, now beyond $\Omega = 1$ we have one real wave, two real wave, three real waves and we will have one purely imaginary wave so, be beyond $\Omega = 1$ we can talk of a flexural wave this one is at torsion, this is a torsional wave this is again a longitudinal wave particles moving back and forth in the axial direction and this is the purely imaginary flexure decaying wave below $\Omega = 1$ it is a little complicated.

We have two composite complex waves and two real waves. Now, what is interesting here is that this is torsion it remains torsion and for $n = 0$ it is uncoupled with the rest uncoupled it uncouples with other equations and other displacements you can see that here, so, L_{12} for $n = 0$ you can have let us see, L_{11} survives L_{13} survives. So, L_{31} survives and L_{33} survives. Whereas, you have L_{21} and L_{12} go to 0 and L_{22} alone survives. So, for $n = 0$ this is 0 this is 0 this is nonzero and it is tied to this.

And therefore, this is 0 this is 0 and these 4 values are tied. So, the actual displacement and transverse displacement are coupled together and torsion completely decouples. So, below this $\Omega = 1$ frequency what did I say? we have two complex wave numbers and one torsion which is completely decoupled from the rest and this blue line it begins as a longitudinal wave and $\Omega = 1$ it very rapidly transitions to flexure it begins is longitudinal means what it is dominantly longitudinal. As I said all displacements are coupled.

So, you cannot say purely longitudinal or purely flexure because the equations are coupled every type of wave having a dominant motion has other components other motions in it. So, the particle is dominantly longitudinal or it will have as its torsion is decoupled but it will have small transverse displacement. So, this dominantly longitudinal wave suddenly turns and becomes dominantly flexure and it again cuts on it or at $\Omega = 1$ and it continues as longitudinal.

And what is this $\Omega = 1$, so, this is an observe behavior is a computed behavior experimentally observed behavior and numerically analytically computed behavior. So, what is this $\frac{\omega a}{C_l'} = 1$. So, which means $2\pi f\, a = C_l'$ speed of the longitudinal wave in the material. So, twice a I will put it as the diameter, so, I have $\pi d = \frac{C_l'}{f} = \frac{f\lambda_l}{f}$. So, this is equal to λ_l. So this is the circumference.

The circumference is equal to the wavelength of the longitudinal wave so, just at this frequency what has happened is that you have the circumference at just become equal to the longitudinal wave in the material and in the other direction in the axial direction the wavelength is actually infinity. So, what happens is that this is a cut on frequency of a wave which has infinite wavelength in the x direction or k_x give the notation is 0 or λ_x is infinity and the circumferential length or radius is equal to λ_l the cut on frequency of the $n = 0$ mode.

So, this is very important it just happens that below this frequency the shell is unable to acquire displacements and just at $\Omega = 1$ the displacement start to open out the wavelength of wave is being accommodated in the circumference. So, this shell starts to vibrate kind of or explodes in vibration and therefore, this longitudinal wave suddenly transits and becomes a dominantly flexural wave. So, this is called the ring frequency this $\Omega = 1$ is called the ring frequency. And it is very important parameter or physical frequency.

This will feature very heavily in our future discussions. So, this is a description of just $n = 0$ waves $n = 1$ waves are very similar beyond $\Omega = 1$ but below $\Omega = 1$, they get complicated. They have some peculiarities of their own. So there will be complex wavenumbers. I am just going to give you a picture not a detail. So, there will be complex wavenumbers which will arrive to some value and then they will purely become imaginary for some time in the frequency.

And then they will start cutting on whereas you will have a flexure cut on at some frequency and torsion etcetera. So it is complicated. So $n = 2$ again will have its own peculiarity below $\Omega = 1$ beyond $\Omega = 1$ 1 it is again very similar. So I have just given you how $n = 0$ looks like. Now so we know about something about the type of waves that we are going to see.

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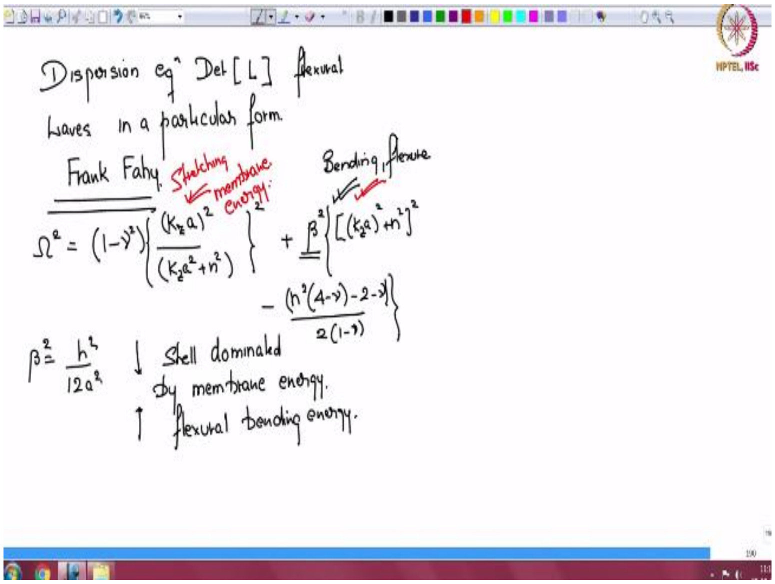

Now we even know that there is a dispersion equation which comes from the determinant of that matrix let me call it be L matrix. Now, we can arrange that dispersion equation that describes flexural waves in a particular form to understand their behavior now, let me just add one thing here that this set of equations are called Donnell Mushtari equations there are several shell theories maybe 30, 40 Shell theories with various terms added for various accuracies.

So, each shell theory has its own specific aspects and the more one of the most basic or the most basic is Donnell Mushtari set of equations for a thin shell theory. Now, so, we can arrange the dispersion equation in a particular manner I should also mention that this entire portion I have taken from the book by Frank Fahy you can find it in that book it is known in structural vibration. So, this particular form of the dispersion equation for flexural waves I write here.

$$\Omega^2 = (1-\nu^2)\left\{\frac{(k_z a)^2}{((k_z a)^2+n^2)}\right\}^2 + \beta^2\left\{[(k_z a)^2+n^2]^2 - \frac{n^2(4-\nu)-2-\nu}{2(1-\nu)}\right\}.$$

Let me put it here so, this is the form a convenient form. Now, this portion talks of bending energy and therefore flexure. And this parameter I said β is $\frac{h}{a\sqrt{12}}$ so, β^2 is $\frac{h^2}{12a^2}$. It is another very important parameter if β^2 is small. Then the shell is dominated by membrane energy.

Whereas, if it is any acquires any importance or dominance then the shell is dominated by flexural or bending energy. So, it is a very important parameter. So, this term describes bending energy and this term describes membrane energy stretching, stretching energy stretching or membrane energy, we are out of time so I will discuss in next class thank you.