Sound and Structural Vibration
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Lecture - 58
Wave Impedance of an Infinite Plate: Fluid Loading

Good morning and welcome to this next lecture on sound and structural vibration, we have come to the end of these lecture series.

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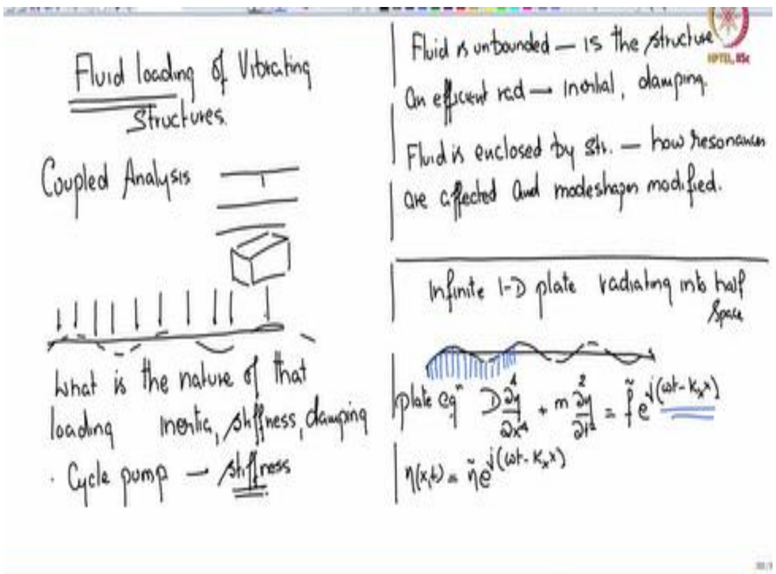

So, the last topic I would like to touch upon is fluid loading of vibrating structures. Now in the course this is something it is not that it has not been touched upon. It has not been spoken in the language of fluid loading, whenever we do coupled analysis which we have done. We have done sound and structure coupled analysis for the classical Crighton’s problem and we have done it for the structural acoustic waveguide and also for the box panel interacting with the cavity.

We have done coupled analysis so fluid loading is there, but this terminology was not used so I will make a few comments on fluid loading. And we talk of fluid loading that means basically a structure vibrates and then it generates pressure, and that pressure is obviously applied back on the structure. So, now what is the nature of that loading that is what is called fluid learning? What is the nature of that loading?

So, that means is it like an inertia, is it like a stiffness or is it like a damping. So, when you use a cycle pump you use a bicycle pump to pump air into your tire you find it very difficult to pump, just a column of air inside so the air is now acting as a stiffness that is fluid loading. So, we have two essential scenarios where the fluid is unbounded and so you are trying to ask the question is the radiator or is the structure an efficient radiator.

There the fluid loading issue is whether it is inertial in which case the fluid locally exchanges energy with the structure and no far field sound is generated, or it could the fluid loading could be in that form of damping which means sound is transported away from the structure. So, essentially that is the question we ask. The other scenario is when fluid is enclosed fluid is enclosed by the structure it is a contained fluid.

There we are mainly looking at how resonances are affected, and mode shapes are modified mode shapes or new mode shapes appear more shapes modified. So, let me make some comments on this fluid loading so we will take the case of an infinite panel first the infinite panel or plate and then it is carrying a flexural wave we have seen this repeatedly. So, infinite 1D plate radiating into half space.

So, the plate equation governing equation is

$$D\frac{\partial^4\eta}{\partial x^4}+m\frac{\partial^2\eta}{\partial t^2}=\tilde{f}\ e^{j(\omega t-k_x x)}.$$

So, I am providing a wave like excitation. It is not localized; the excitation depends on space; it is a continuous excitation that is what this expression means. So, that means I am forcing a wave on the structure.

And therefore, the structure in the steady state will respond accordingly. So, we get the response $\eta(x,t)$is given by $\tilde{\eta}e^{j(\omega t-k_x x)}$.

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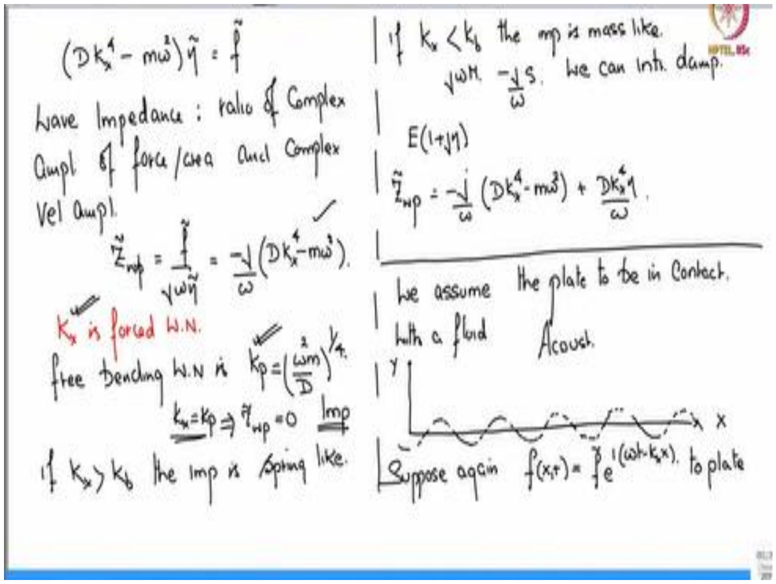

Which results in

$$\left(D{k_x}^4 - m\omega^2\right)\tilde{\eta} = \tilde{f}.$$

Now we define a wave impedance it is the ratio of complex amplitudes, so we are saying complex in amplitude. So, you understand what it means of force per unit area and complex velocity amplitude. So, what we mean is

$$\tilde{Z}_{wp} = \frac{\tilde{f}}{j\omega\tilde{\eta}} = \frac{-j\left(D{k_x}^4 - m\omega^2\right)}{\omega}.$$

So, now k_x is the forced wavenumber.

Now the free bending wavenumber in the plate is let us call it k_p I have been using k_b and k_p interchangeably. So, $k_p = \left(\frac{\omega^2 m}{D}\right)^{1/4}$ and what is that so if now k_x becomes equal to k_p you can see that $\tilde{Z}_{wp} = 0$ so the impedance goes to 0 it is like resonance k_p is naturally the what the panel likes the free wavenumber and $k_x$is what we are forcing into the plate.

So, if those two become equal the impedance will go to 0. Further if k_x is greater than k_p then the impedance is spring like spring like whereas if whereas if k_x is less than k_p then the impedance is mass like you can check that out in one case you will get $j\omega m$ in the other case you will get $\frac{-js}{\omega}$ you get expressions like this. So, now we can introduce some damping also and the damping is introduced as a complex young modulus.

So, the E whatever is there is made complex with a loss factor and therefore along with damping we can write this as

$$\tilde{Z}_{wp} = \frac{-j}{\omega}\left(D{k_x}^4 - m\omega^2\right) + \frac{D{k_x}^4\eta}{\omega},$$

so that is the plate wave impedance. Now the next step is that we now assume the plate to be in contact with a fluid and so this is my geometry again this is y direction.

So, the pressure field is two dimensional my x direction so I have a wave moving on the panel or plate. So, this is now my acoustic half space, now suppose again a force of the form as before is applied again to the plate from below.

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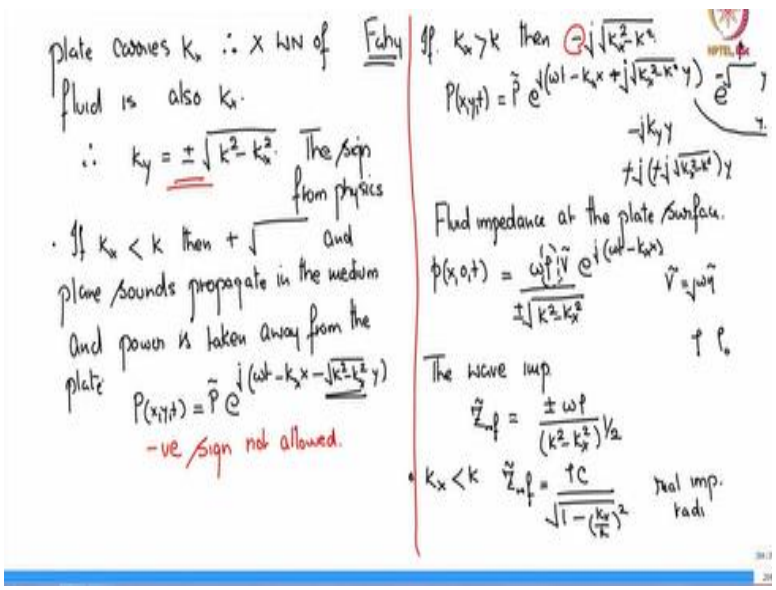

And plate carries k_x because its forced plate carries k_x and therefore the x wavenumber of the fluid is also k_x, so what will k_y be? So, we should know this by now k_y will be the difference between squares of the acoustic wave number and the x direction wave number and then we know by now the sign is decided by whether k is greater or k is less than k_x sign from physics. So, if k_x happens to be less than k then we take the positive real square root.

What does that mean? It means that plane sound waves propagate in the medium. I should mention that all this I am taking from this book by Fahy sound and structural vibration by Fahy propagate in the medium. And power is taken away from the plate why is that because

$$P(x,y,t) = \tilde{P}e^{j\left(\omega t - k_x x - \sqrt{k^2 - {k_x}^2}y\right)}.$$

So, this j and this negative real number mean propagation in the y direction.

And we do not allow the negative sign so negative sign is here and not allowed we had two signs here plus and minus. So, in this case the negative not allowed. On the other hand, if it happens that k_x is greater than k then we have to choose the negative imaginary value $-j\sqrt{{k_x}^2 - k^2}$ why is that because we have $P(x,y,t) = \tilde{P}e^{j\left(\omega t - k_x x + j\sqrt{{k_x}^2 - k^2}y\right)}$.

I have $-k_y y$ with a j that is the starting point. So, with the $-j$ I have to choose a negative imaginary wavenumber $-j\left(-j\sqrt{{k_x}^2 - k^2}\right)y$ so this goes, and I have j square. So, this gives me a negative. So, I have a negative wavenumber, and this is a real number.

So, this is a decaying wave in the y direction otherwise, it will blow up in the y direction at infinity. So, that is why the negative sign here has to be chosen, now what is the impedance fluid impedance at the plate surface which means the

$$P(x,0,t) = \frac{\omega\rho_0\tilde{V}e^{j(\omega t - k_x x)}}{\pm\sqrt{k^2 - {k_x}^2}}.$$

We have seen this before the $\tilde{V}$ is $j\omega\tilde{\eta}$.

Just computing pressure over velocity through the Euler equation. So, the wave impedance of the fluid $\tilde{Z}_{wf}$ is given by $\frac{\pm\omega\rho}{(k^2 - {k_x}^2)^{1/2}}$ I have suddenly used ρ_0, so it is a fluid density, so I have either used ρ for the fluid or ρ_0 for the fluid throughout. So, let me keep it ρ for consistency ρ is the fluid density. Now when k_x is less than k then the fluid impedance is $\frac{\rho c}{\sqrt{1 - \left(\frac{k_x}{k}\right)^2}}$.

It is real and therefore radiation occurs or radiation damping occurs.

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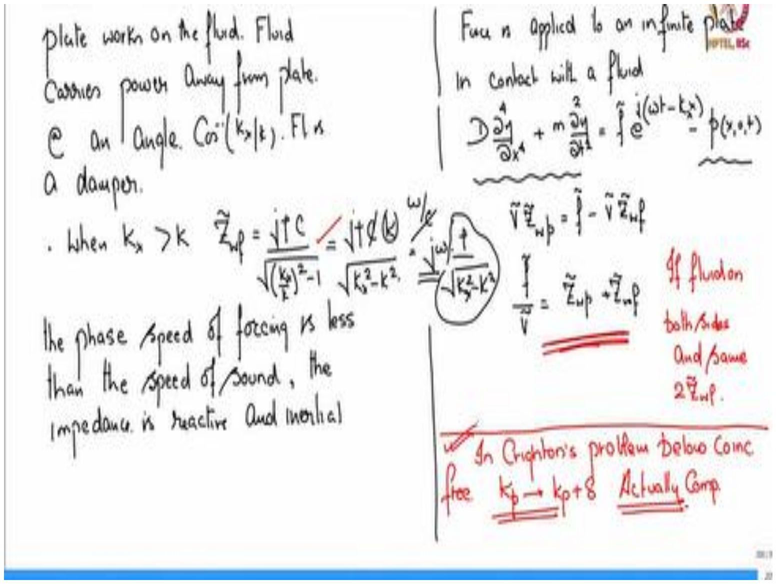

That is the plate works on the fluid and the fluid carries power away on the plate. Now, further at an angle so you get plane waves at an angle $\cos^{-1}(k_x/k)$ so the fluid is a damper causing radiation damping. Now when k_x is greater than k then the

$$\tilde{Z}_{wf} = \frac{j\rho c}{\sqrt{\left(\frac{k_x}{k}\right)^2 - 1}},$$

that means the phase speed k_x is greater so λ_x is smaller.

So, phase speed of the forcing is less than the speed of sound that means the impedance is reactive. So, how an inertial also and y inertial if we open this expression out what do we get we get

$$\tilde{Z}_{wf} = \frac{j\rho c}{\sqrt{\left(\frac{k_x}{k}\right)^2 - 1}} = \frac{j\rho c k}{\sqrt{{k_x}^2 - k^2}} = j\omega \frac{\rho}{\sqrt{{k_x}^2 - k^2}}.$$

So, you have a $j\omega$ times some positive real number so it is inertial. Now when a force is applied let me take this when a force is applied to an infinite plate in contact with a fluid then what do we have. Now we have to do a fully coupled analysis, so we have

$$D\frac{\partial^4 \eta}{\partial x^4} + m\frac{\partial^2 \eta}{\partial t^2} = \tilde{f}\, e^{j(\omega t - k_x x)} - P(x,0,t),$$

so, now what we will get is?

We will get structural impedance from here we will get fluid impedance from here so there is a common velocity at the interface. So,

$$\tilde{V}\tilde{Z}_{wp} = \tilde{f} - \tilde{V}\tilde{Z}_{wf} \,,$$

$$\frac{\tilde{f}}{\tilde{V}} = \tilde{Z}_{wp} + \tilde{Z}_{wf} \,,$$

just to note here that if there is fluid on both sides and same then the $\tilde{Z}_{wf}$ will be $2\tilde{Z}_{wf}$. Now we have seen that in the Crighton's problem.

Let us say below coincidence the free bending wave number k_b or k_p becomes $k_p + \delta$. So, this we actually computed but as we are doing a very descriptive analysis prior to actual computations. So, we will use a different language or thought process that is how to analyse this without if we have not done a detailed calculation. The time is up for this lecture. I will continue in the next class. Thank you.