Dynamics and Control of Mechanical Systems
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Lecture - 01
Introduction to Course

Welcome to this NPTEL course on dynamics and control of mechanical systems. My name is Ashitava Ghoshal, I am a professor in the department of mechanical engineering and in the centre for product design and manufacturing and Robert Bosch Centre for cyber physical systems Indian Institute of Science, Bangalore. In this course we will start with the representation of rigid bodies in 3D space.

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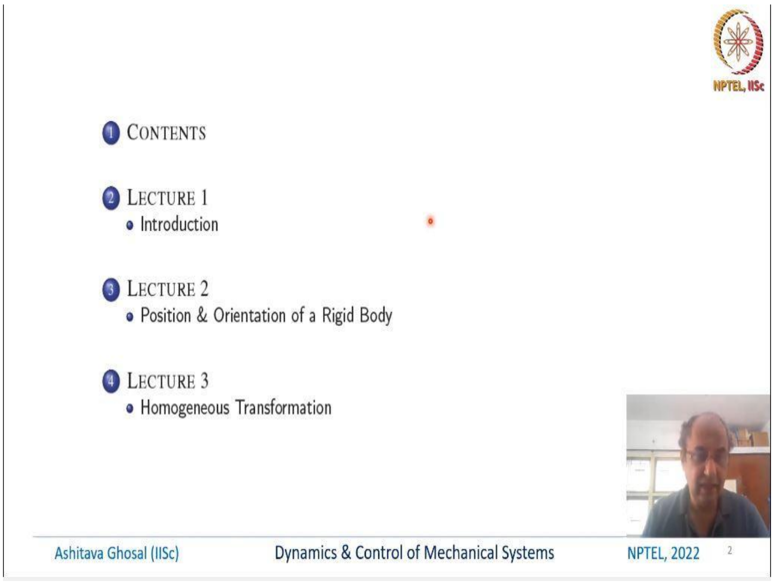

The contents for this week we will first introduce mechanical systems and their various components then, in lecture 2 we will look at position and orientation of a rigid body in 3D space and then in lecture 3 we will look at homogeneous transformation where rigid body can do both translation and orientation.

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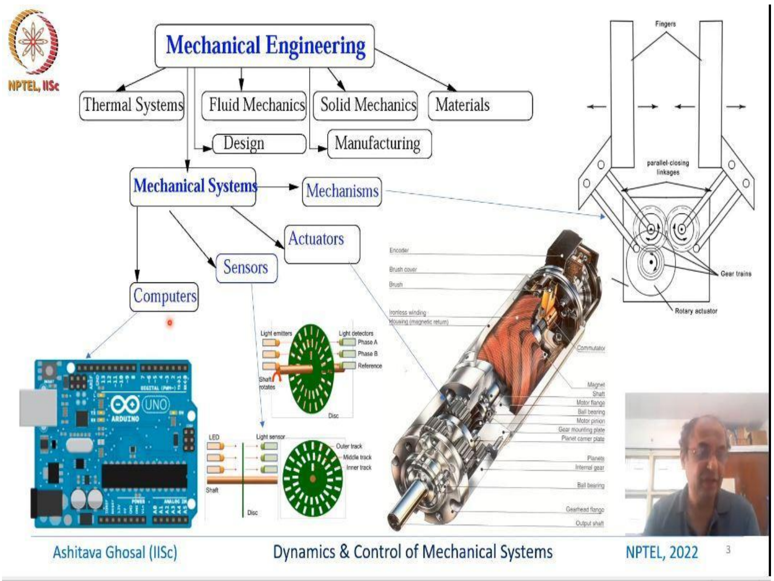

The field of mechanical engineering contains various subfields. So, for example you can have thermal systems fluid mechanics, solid mechanics, materials, design and manufacturing. In this course we will look at mechanical systems. Mechanical systems on their own contains the study of mechanisms, actuators, sensors, and computers. So, for example this picture here shows a four-bar mechanism which is used to actuate or move the fingers of a gripper which is used in robots.

To rotate some of the actuators we have this DC servo motor and the DC server motor on its own contains many elements. So, for example we have the shaft and that to measure the rotation of the shaft we have what are called as encoders. So, this is a picture of an optical encoder where we have some pairs of light emitters and diodes and we can measure the rotation of the shaft by finding out how the light beams are intercepted by this rotating disc.

We also have computers in a DC in mechanical systems the computers are used to measure to do some IO and to do some software and computations.

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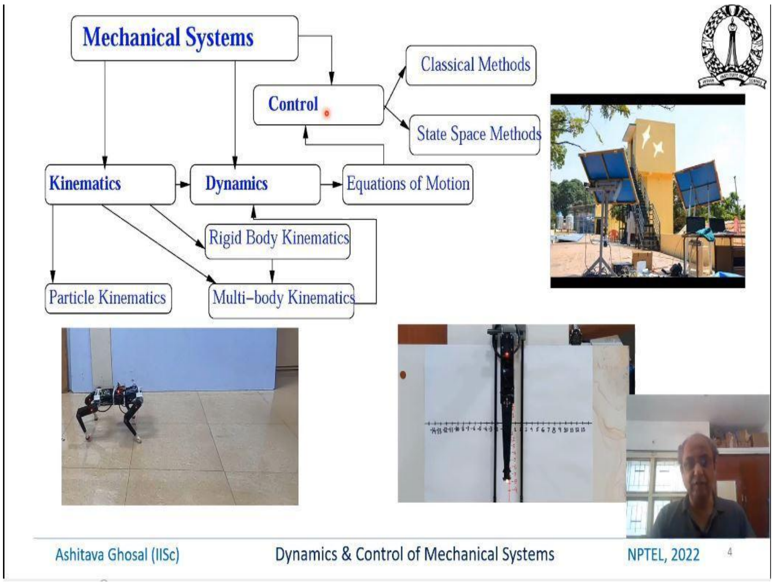

Mechanical system basically consists of three areas of study we have kinematics, dynamics and control. In kinematics we can also have particle kinematics where you consider the element as a point mass then we also have rigid body kinematics where we look at both position and orientation of a rigid body. And several such rigid bodies make up what is called as multibody kinematics and multibody kinematics leads to this idea of dynamics.

So, in kinematics we do not worry about the causes of the motion whereas in dynamics we look at how the motion is caused and the goal in dynamics is to derive the equations of motion. They are ordinary differential equations most of the time and we will deal with ordinary differential equations and natural progression from the equations of motionless control. So, every mechanical system has it is own inherent dynamics.

So, when a force or a moment acts on the mechanical system it will the trajectory or the motion of the mechanical system is defined. However, we would like to have a desired motion of the mechanical system and this is the area of control. There are two main ways of doing control one of them is called the classical methods where we look at what are called as root locus and frequency domain and so on and then there is also state space method.

So, here are three examples of mechanical systems I will show you them and these are short videos.

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So, first is this what is called as a Sun tracking system basically there are two mirrors which track the sun and then they focus the incident beam onto a receiver. This is a picture of a quadruped which is moving around each leg of the quadruped contains two rotary joints and there are two motors. So, here it is shown how the tip of the quadruped the end of the leg is moving up and down.

So, a study of kinematics dynamics and control allows us to analyse and then design systems like this quadruped or the simple two art motion or even the Sun tracker.

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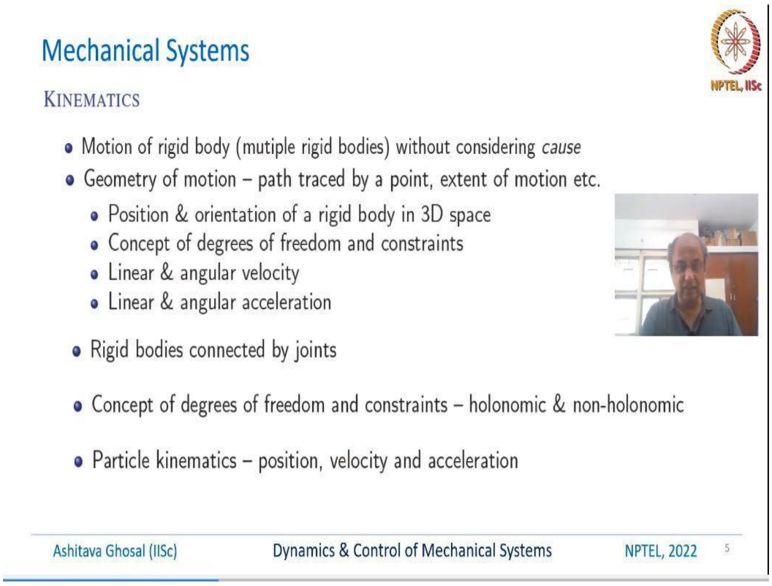

Kinematics on its own consists of or looks at the study of motion of rigid bodies or multiple rigid bodies without considering the cause. So, what do we study? We look at basically the geometry of the motion. So, the path trace by a point or maybe what is the extent of the motion of the rigid of the point or rigid body and so on. So, the main are concepts in kinematics are position and orientation of a rigid body in 3D space.

Concept of degrees of freedom and constraints, linear and angular velocity of the rigid body and linear and angular acceleration. The rigid bodies are often connected by joints. So, when you have rigid bodies connected by joints then they move, or they have different degrees of freedom. So, we will look at this concept of degrees of freedom of a set of rigid bodies connected by joints and the constraints which the joint impose.

So, there are different kinds of constraints one of them is called holonomic and the second one is non-holonomic. We will look at these in more detail in this course. The one classical way of looking at kinematics is what is called as particle kinematics, we assume that it is a point mass and then we study position velocity and acceleration of this point mass.

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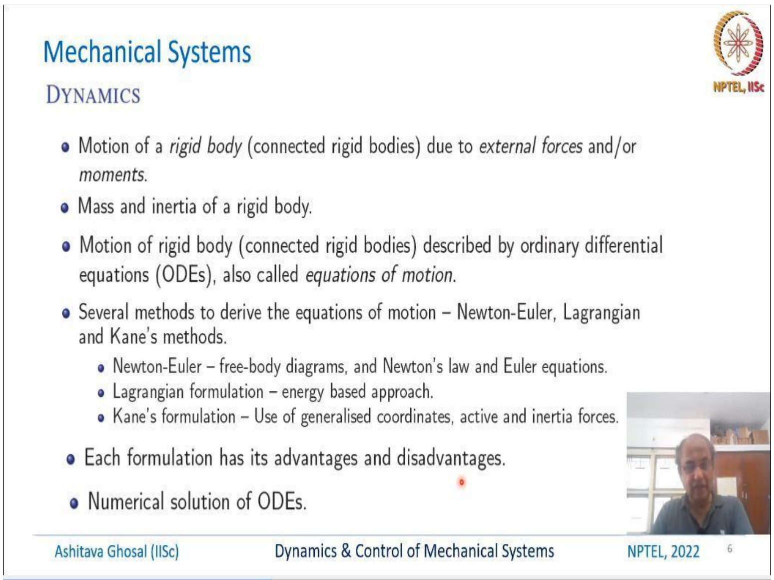

In dynamics we look at the motion of the rigid body or connected set of rigid bodies due to the external force's and or moments. So, the in this case we have to study or consider the mass and inertia of the rigid body. The motion of the rigid body or the connected set of rigid bodies can be described by ordinary differential equations also called the equations of motion. There are several ways and methods to derive the equation of motion.

The common ones are called Newton Euler or Lagrangian and then there is a Kane's method. In Newton Euler we have this concept of free body diagram we also use Newton's law and Euler's equation of motion. In the Lagrangian formulation we find the kinetic and potential energy and this is an energy-based approach. In Kane's formulation there is this use of so-called generalized coordinates and they have this concept of active and inertia forces.

We will not look at Kane's formulation in this course in detail in this course. Each formulation has it is own advantages and disadvantages. And finally in dynamics we also need to look at how to solve the equations of motion. Most of the time we; will be doing numerical solution of the ordinary differential equations of equations.

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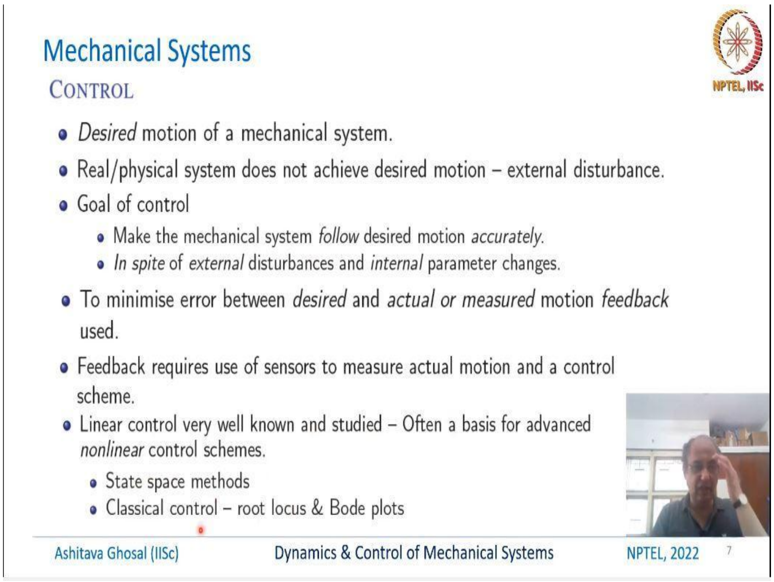

In control, we have basically a desired motion of a mechanical system. So, the real or physical system does not achieve the desired motion due to several reasons one of them is the external disturbance. So, the goal of control is to make the mechanical system follow our desired motion accurately. So, the important words are follow and accurately. So, we will look at how much close we can follow our desired motion to what accuracy.

And more importantly in spite of external disturbances and internal parameter changes. So, the goal of control is also to minimize the error between the desired and actual or measured motion and to achieve this we use this important notion of feedback. Feedback requires the use of sensors to measure actual motion and a control scheme. So, most of the time; we will use linear control schemes because this linear control scheme is very well known and studied.

And it is also a basis for advanced non-linear control schemes. So, we will not look at non-linear control schemes in this course. The two main ways which we look at control is one is called state space methods and the other one is called classical control. So, classical control consists of root locus bode plots and other frequency domain approaches. We will not look at in too much detail the frequency domain approaches.

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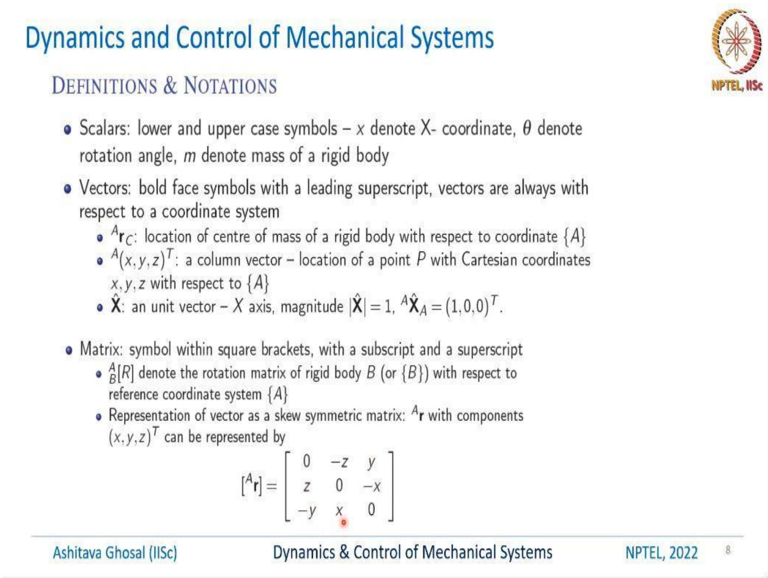

Before, we get in depth into this course let us first look at a few definitions and notations which will be used throughout this course. So, we will denote scalars using lower and uppercase symbols. So, for example x and small x can denote the X coordinate θ will denote the rotation angle m can denote the mass of a rigid body. Vectors will be denoted by bold phase symbols with a leading superscript, vectors are always with respect to a coordinate system.

So, we will define a coordinate system little bit later but vectors are always with respect to a coordinate system. And for example, this quantity Ar_C it locates the centre of mass of a rigid body with respect to the coordinate system $\{A\}$. So, the leading superscript here denotes the reference coordinate system and r is a vector, so it is a position vector and we will see later that position vector can be determined or represented by three scalars.

So, a vector will sometimes be also denoted in this form here A denotes a reference coordinate system and $x,\ y,\ z$ are the components of a vector. So here this T corresponds or it shows that it is a column vector. So, basically this is the location of a point P with Cartesian coordinates $x,\ y,\ z$ with respect to A. We also need to distinguish normal vectors with unit vectors. So, an unit vector has a magnitude 1.

So, in this example capital X bold face X with a hat denotes a unit vector. So, the magnitude of this unit vector is 1 and in the coordinate system A the X_A or the X_A unit vector in its own coordinate system is given by 1 0 0. A matrix is a symbol within square brackets, so with the superscript and a subscript so for example BAR denotes the rotation matrix of a rigid body B or a coordinate system B with refresh with respect to a reference coordinate system A.

So, in the rotation matrix, this superscript denotes the reference coordinate A is the reference and B is the rigid body with rest. And we say that this matrix BAR denotes the rotation of rigid body B with respect to A. Sometimes we need to represent a vector as a skew symmetric matrix. So, for example this vector Ar, r is bold face which means it is a vector with respect to the A coordinate system and with components x, y, z can be written in this form.

So, this is a square bracket Ar to basically denote that it is actually a matrix a vector represented as a matrix and this will be a skew symmetric matrix. So, from linear algebra we know a skew symmetric matrix is one which has the diagonal elements as 0 and this x, y, z will be arranged in this form. So, the second element is - z the third element is y and since it is skew symmetric this will be + z and this is - x and this is again - y and x.

So, we will see later this matrix when you multiply by a vector also denotes the cross product of one vector with the second vector.

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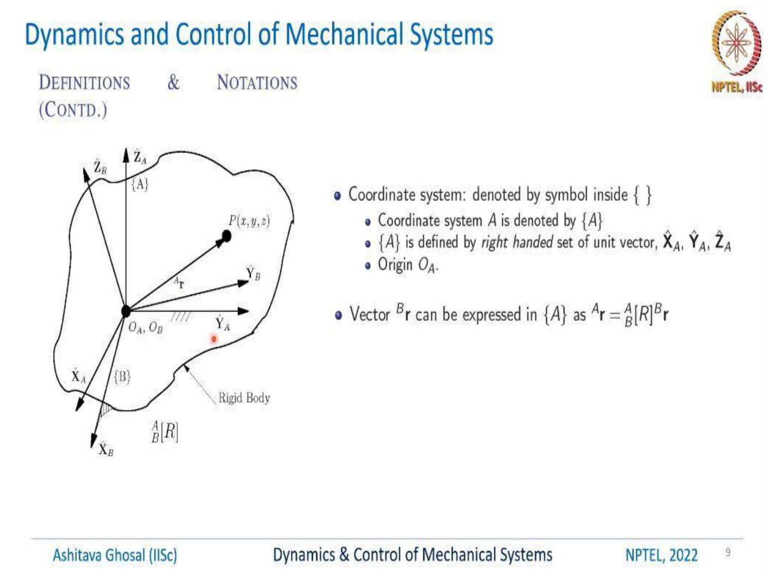

So, let us continue, so in this picture I show a rigid body. So, this is odd looking shape it represents a rigid body in 3D space. So, now in this rigid body there are several important things which are of interest. One is that there is a coordinate system $\hat{X}_A$, $\hat{Y}_A$, and $\hat{Z}_A$, so remember x with a hat denotes a unit vector {A} denotes the fact that it is associated with this coordinate system A we also have an origin O_A.

So, every coordinate system must have three things, one is a set of unit vectors X, Y and Z and origin. And also, you have to tell whether it is a reference and a label. So, we need to make sure that this label tells you that this is the A coordinate system. Likewise in this rigid body I could have another coordinate system which is $\hat{X}_B$, $\hat{Y}_B$, $\hat{Z}_B$ in this instance the origin of the two coordinate system O_B are shown at the same place.

So, sometimes we see these hash lines. So, this means that the A coordinate system is fixed we can also have another set of hash lines which is attached to the rigid body. So, in this figure what it means is this A coordinate system is in some sense like fixed in space whereas the B coordinate system is attached to the rigid body B. So, as I said the coordinate system is denoted by symbol inside these curly brackets the coordinate system a is denoted by curly bracket A.

A is defined by a right-handed set of unit vectors $\hat{X}_A$, $\hat{Y}_A$, $\hat{Z}_A$ and an origin O_A. So, there will be many such coordinate systems for several rigid bodies so we will be labelling each rigid body in this way. So, you can have A B C and so on. A vector in this coordinate system say for example with respect to the B coordinate system as can be expressed in another coordinate system.

So, this is very well known from basic mechanics that if you have a vector in the A coordinate system and you have another vector in the B coordinate system, they are given in terms of by multiplying pre-multiplying Br with respect to a rotation matrix
$BA[R]$. So, just to keep it keep track this notation is very useful, so for example you can think of this B and B cancelling out. So, we are left with a and this vector r which is same as here Ar.

The rotation matrix ${}^{B}_{A}[R]$ gives we will see very soon it gives the orientation of this rigid body B with respect to another coordinate system or with respect to a reference coordinate system A.

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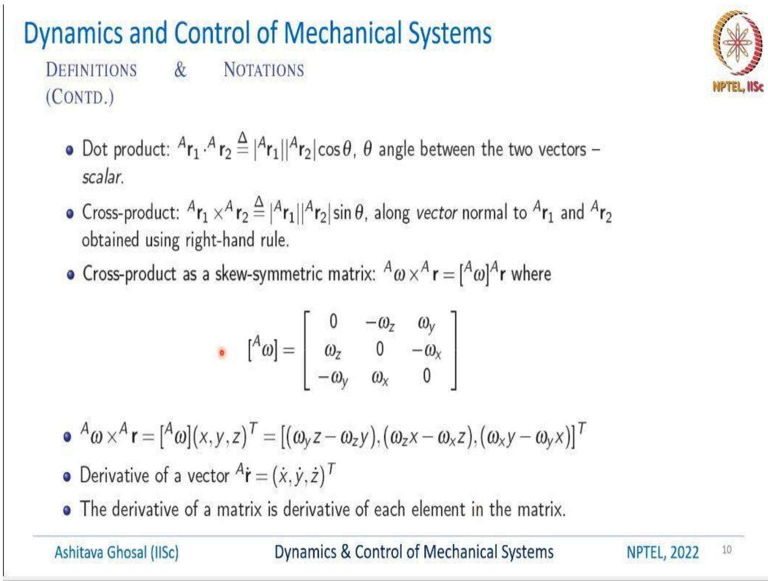

So, let us continue with definitions and notations used in this course. So, the dot product of 2 vectors ${}^A r_1$ and ${}^A r_2$ is defined as the magnitude of ${}^A r_1$ ${}^A r_2$ into cos θ where theta is the angle between the two vectors. The dot product of two vectors is always the scalar. The other important thing in dot product or for that matter in any vector operation between two or more vectors all the vectors must be in the same coordinate system.

So, it should be A here and A here. So, it does not make any sense to do ${}^A r_1 \cdot {}^B r_2$, the cross product of 2 vectors ${}^A r_1$ and ${}^A r_2$ is denoted by the symbol cross is defined as the magnitude of ${}^A r_1$ magnitude of ${}^A r_2$ into $sin(\theta)$ where, θ is the angle between these two vectors. The cross product of two vectors is also a vector and the vector is always normal to ${}^A r_1$ and ${}^A r_2$ and the vector normal to ${}^A r_1$ and ${}^A r_2$ is obtained by the right-hand rule.

So, ${}^A r_1$ is one set of fingers and ${}^A r_2$ is the other one and the normal is along the thumb, the usual way we find cross product between two vectors and using the right-hand rule. The cross product can also be viewed as a skew symmetric matrix, so for example this vector

$[A\omega]\times Ar$ can be also written as this matrix $[A\omega]\,Ar$. Where $[A\omega]$ is this skew symmetric matrix which was introduced last in the last slide.

You know it has 0 in the diagonal terms and the off-diagonal terms are ω_z and - $\omega_{z,}$ ω_y and - ω_y and - ω_x and ω_x. So, the cross product of $[A\omega]\times Ar$ can be written as this matrix multiplying x, y, z and again this T implies it is a column vector can be obtained as ω_y z - ω_z into y. So, this is the x component ω_z into x - ω_x z this is the y component and ω_x y - ω_y x. And this is also a column vector this again remember T denotes it is a transpose or the column vector. The derivative of a vector Ar denoted by $A\dot{r}$ is nothing but the derivative of each of its components. So, it is $\dot{x}\,\dot{y}\,\dot{z}$ if Ar was x y and z, the derivative of a matrix is the derivative of each element in the matrix. So, we will need to use derivative of a matrix later on.

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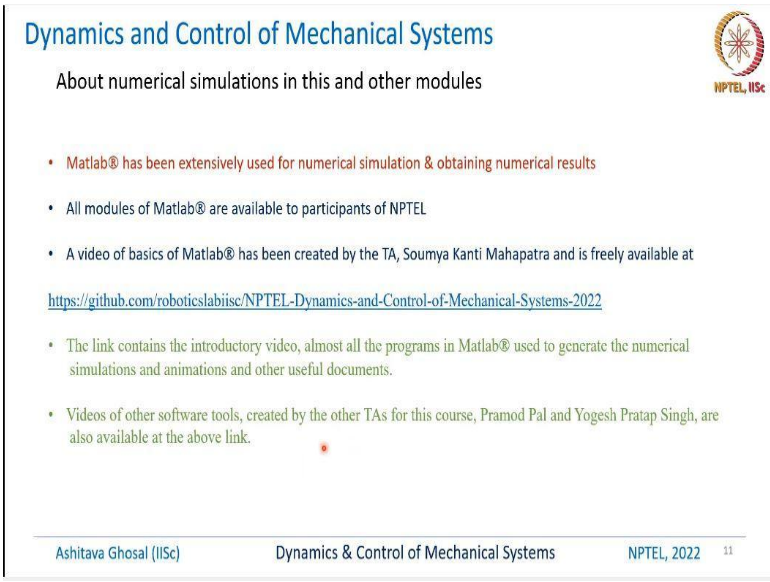

Before, we go into this module a quick word about the numerical simulations in this and other modules. Matlab has been extensively used for numerical simulation and obtaining numerical results. All modules of Matlab are available to participants of NPTEL a very basic video of Matlab and how to use it has been created by the TA, Soumya Kanti Mahapatra and is freely available at this website. Please go to the website and you can explore.

And see what are the material which has been put there. The link contains the introductory video almost all programs in Matlab used to generate the numerical simulations and animations and other useful documents. Videos of other software tools used in this course such as ADAMS and created by other TAs for this course Mr. Pramod pal and Yogesh Pratap Singh are also available at the above link.