Crystal, Symmetry and Tensors
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Lecture 1c
Lattice Translation

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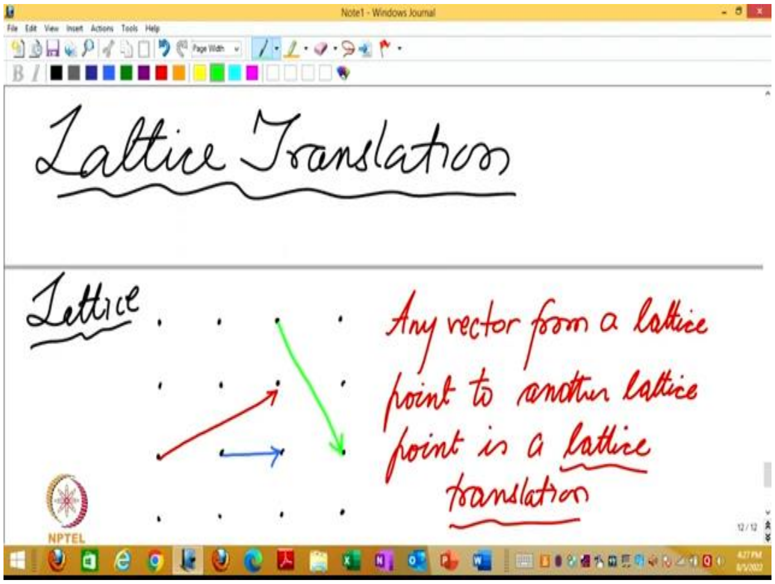

So, now let us look at a concept called lattice translation. So, 1D is too restricted 3D is a little difficult to draw, so many of the drawing we will make, till now I have not drawn any 3D lattice. So, I have drawn only 2D and 1D, so most of the drawing which we will do in the class will be 2D for simplicity of drawing because it is easy to draw it on board or screen. So, let us say that I have a set of lattice points.

So, I do not know what is being repeated, currently I am focused only on how it is being repeated so my focus is on lattice. Then any vector I choose from one lattice point to another is a lattice translation, any vector from a lattice point to another lattice point is a lattice translation. So, a lattice will have many lattice translation, you can have red translation of the blue translation of green translation all these are lattice translations.

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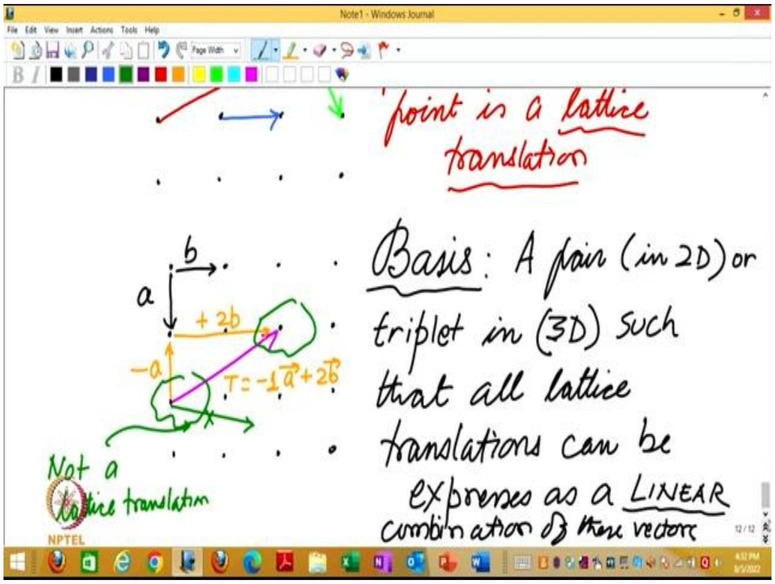

We also saw, we already saw this that if we select, if we appropriately select two translations so, we had done that for with a and b for example, then we can reach all possible lattice translation by integer combination of these translations so that brings us to the concept of bases. So, 2D you required two basis vectors, in 3D you will require three basis vector, expressed as a the technical jargon you probably know linear combination. So, if I have selected a and b in 2D, all translation whatever the translation may be, suppose I have this purple translation so, I can describe it in terms of a and b.

How can I describe it in terms of a and b? Minus a plus 2b, so, if this is T, T minus 1 two times a plus 2 times b. When I gave example of lattice translation I should give some counter example also that vectors which are not lattice translation. So, suppose if I start from a lattice point and end here, this will not be a lattice translation so this will not take me from a point to a translationally equivalent point, it is not guaranteed so, this is not a lattice translation.

So, I should not look for or I should not insist that I am in identical environment, if I translate myself by this vector, but the other vectors by a vector by b vector or minus a plus 2b vector, I am going from a lattice point to lattice point all lattice points are identical. So, I am going by a lattice translation and I can demand or I can expect that I am in an identical surrounding, whatever is the scenery here I should have the same scenery there, if it is not then they are not lattice point and it is not a lattice translation.

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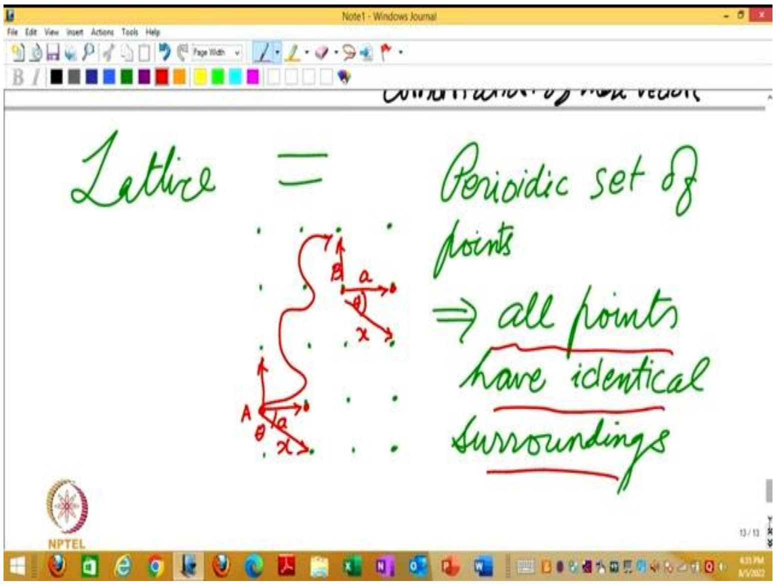

So, that scenery thing which I am telling you is you can describe it in terms of the neighbors so that is another way of defining lattice points, when I say periodic arrangement of points you can also say a set of points such that all points have identical environment. a by 2 be in the previous one, so good question okay I will answer in this diagram because that is already messed up, very good question because I was coming to that. All one not arrangement let me say surrounding.

Now the question so first let me see this surrounding thing, identical surrounding. So, this is a lattice point let us say the lattice point A and this is another lattice point, lattice point B and they have identical surrounding, what do I mean by that? So, if I go by in the horizontal direction by a distance a, I find a neighbor if I go in the same direction that is horizontal direction and in the same sense that is to the right by the same distance I should find an identical neighbor so that is what is happening and not only in one direction so if I go in the vertical direction, so, here from here also I can go in the vertical direction.

If I go at in some angle, if I go by the some angle and the same distance I will again arrive at, so, whatever so if you look around, if you look around if you are a small being walking in the inner lattice and you stationed yourself here and looked around, you will see certain surroundings and then if I blindfold you and take you and leave you to some other lattice point, remove your blindfold and ask, have I left you at the same location or a new location? You will not be able to tell, Charon duniya dekhonge, sara duniya ek hi hai. Okay, so that is very-very important all points have identical surroundings.

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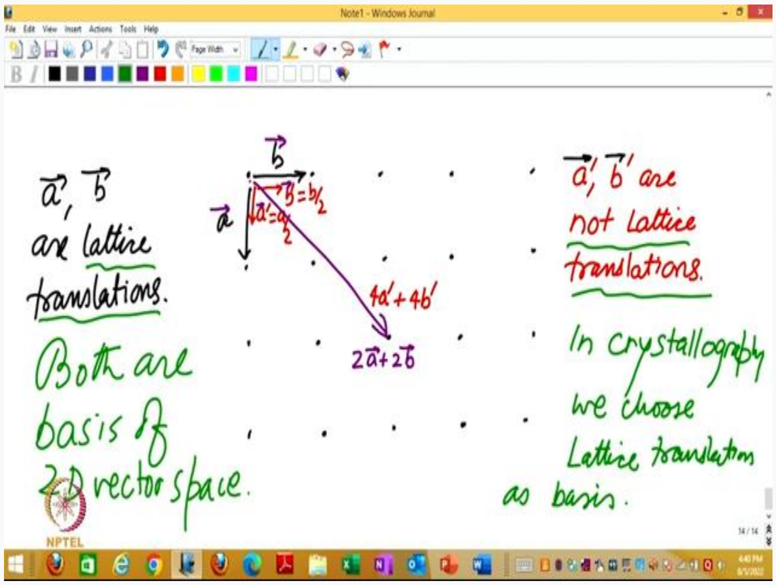

Now, let me try to answer the question which was asked, an interesting question which was asked was, can I not select? In this I selected a and b. The question is, can I not select let us say a by 2 and b by 2? Let me call them a prime, so can I not select a prime and b prime as my basis vector? What will happen if I select a prime and b prime? So, let us select a vector, so let us select this vector.

Now, in the unprimed coordinates a and b this was 2a plus 2b I should keep putting the vector sign which I am missing please add that. So but, what about the same vector in a prime b prime coordinates? 4, it will become 4a prime plus 4b prime because it was half so I have to go twice that amount, fine, that is also a basis. So, a prime b prime are not lattice translations whereas, a and b are lattice translation.

So, in terms of basis for vector space both are basis because both can represent any vector in the space here I have a two dimensional space, any vector I draw in two dimensional space I can express them either as a primed combination or as an unprimed combination.

So, both in that sense, both are basis, but the basis vectors in one case are lattice translations and one case they are not lattice translations, and in crystallography we prefer because calculations become simpler if we choose the basis as lattice translation so, that is our choice to make life simpler, so there is another problem also somebody should point out is reverse of this problem here are shorter vectors were selected what will happen if you select longer vectors? Nm is integers very good. So, that point is very-very important and that brings us to an important classification of these basis also.

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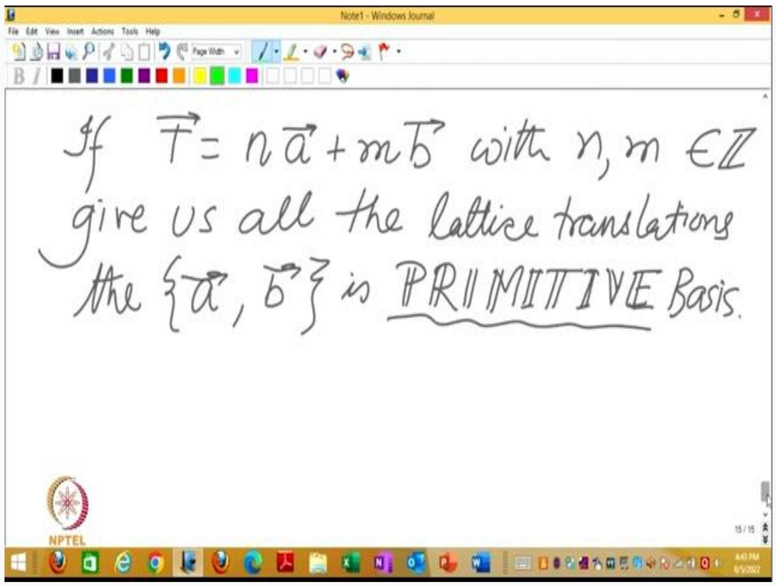

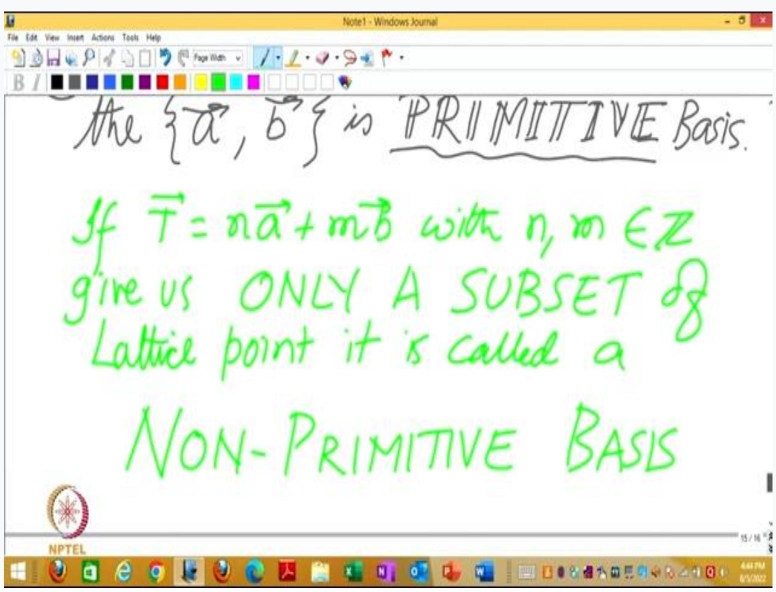

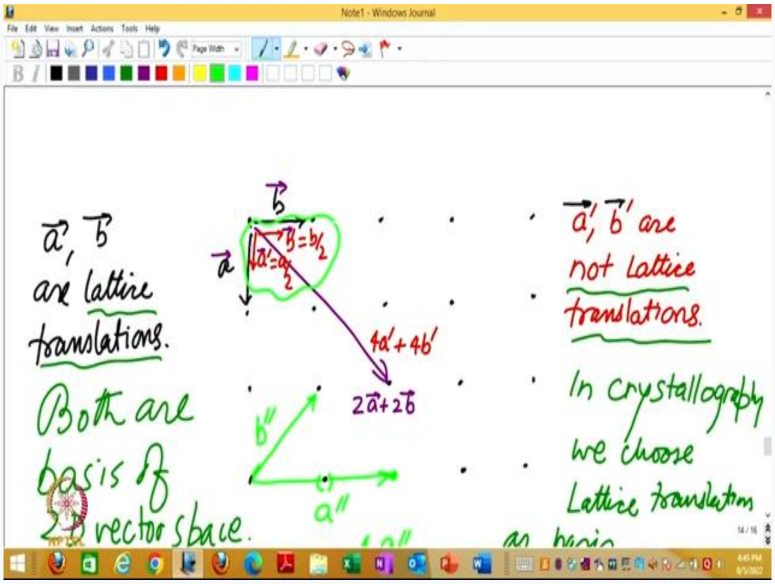

So, if T is equal to na plus mb with n and m as integers give us all the lattice translations, then we gave a specific name to the set a b that the set a b is primitive basis, primitive. This will obviously this a primitive basis has to be made up of lattice translations because we have seen that if the primed vectors we would have selected then some integer combinations like one a prime plus one b prime leaves me in the middle does not take me to a lattice point from and does not give me a lattice translation.

So, problem with a prime b prime was that all integer combinations were not lattice point, the reverse problem which I was telling you and you answered correctly that if I select a larger vector, suppose this a double prime and b double prime so, a double prime takes me to twice the distance of separation of lattice points in that direction so even one a prime is taking me to the next to next lattice point, so, this middle lattice point I cannot come by an integer combination I have to go half a prime so, that is also an allowed basis only its name is given as a non-primitive basis, only a subset is called a non-primitive or non-primitive basis.

And we also discovered one more basis which does not fall in this classification that one is giving me a sub set of points one is giving me all the points and in the third case here a special example which we considered thanks to you gives me points other than lattice point it gives me all the lattice points, but it also gives me additional points which are not lattice points.

So, because it gives me the points which are also not lattice points and our interest is in lattice points that is why this restriction or that is why this choice that why we like to choose lattice translations as our basis that we are assured, if we choose the lattice translations as basis we are assured that linear (combina) integer linear combination of these basis vectors will be lattice points, whether they are all lattice points or a subset of lattice points depends upon their size and shape. But, linear combination has to be a, integer combination has to be a lattice point.