Optoelectronic Materials and Devices
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Lecture - 12
Equilibrium Carrier Statistics in Semiconductors: Density of States, Fermi Function and Population Density in Bands

Welcome to lecture number 12. You have so far what we have done is, well in last lecture what you were doing was plan structure of semiconductors. I hope you got a good overview of what the band section looks like, the kind of information we can get from it, we talked about effective mass in terms of devices, we talked about optoelectronic properties, we talked about in that context, we said that if you want a good optical material then it better be a direct band gap semiconductors, so that its absorption is very good.

By same token we will see later that emission also will require exactly the same thing that intuitively you can pretty must see, because if photon has to come out then the electron must relax back to the valence band from conduction band. And in doing so if it involves phonon in in like in indirect band gap semiconductor, then it will be, if the energy release would go to heating the lattice, and it would not come out as light. Whereas, if it is direct band gap semiconductor the, you do not need to involve the lattice, you do not need to involve the phonons and therefore you have hope, you hope that you will get photons out.

And therefore, that is something which information you when you select materials you can select based on the band structures based on how the band structure looks based on that you can select therefore whether what kind, what kind of material will be opto-electronic material which will not be apart from that you saw curvatures, when we looked at, you looked at we looked at effective masses at different places. Accordingly, you can make innovative devices such as something which has a negative differential resistance etcetera, etcetera; there are many, many such things which come out of band diagram

Anyways, that is something they will start using, start using, but obvious question which you may, come to your mind is that well fine, we but then how many if if you go to talk about electrons in conduction band, valence band then how many electrons are there in conduction band. How many holes are there in the valence band at let us say room temperature or the operation, operation temperature of a device which typically would be around room temperature.

And hence at that temperature what is the carrier, what is the number of carriers. When device is operating you will think of carriers present in area of these bands. Over and above what is thermal equilibrium? Point is that whatever is in thermal equilibrium in other words if I take a room, gallium arsenide which is supposed to be optical material. If I take it at room temperature some electrons forms valence band would have gone to conduction band and therefore there will be some holes in the valence band. But then I would not see light coming out, I do not see light coming out of this gallium arsenide material.

Why so? Obviously, this material in thermal equilibrium. Things which are in equilibrium are not giving you, they they cannot be producing extra energy, they cannot be giving out light. In order to get out of light I must create excess over what is the thermal equilibrium value. So, in other words when we start learning about devices then it will be about creating the excess values over and above, above or below, does not matter, either way.

Over and below equilibrium, in other words in order to understand those device at all times you must know at least what is thermal equilibrium, what is the base line above or below which you can then start operating the device and hope to get photons in and out. Therefore, what we going to start now is carrier statistics. Question we will ask is, so let us reduce this problem to different one.

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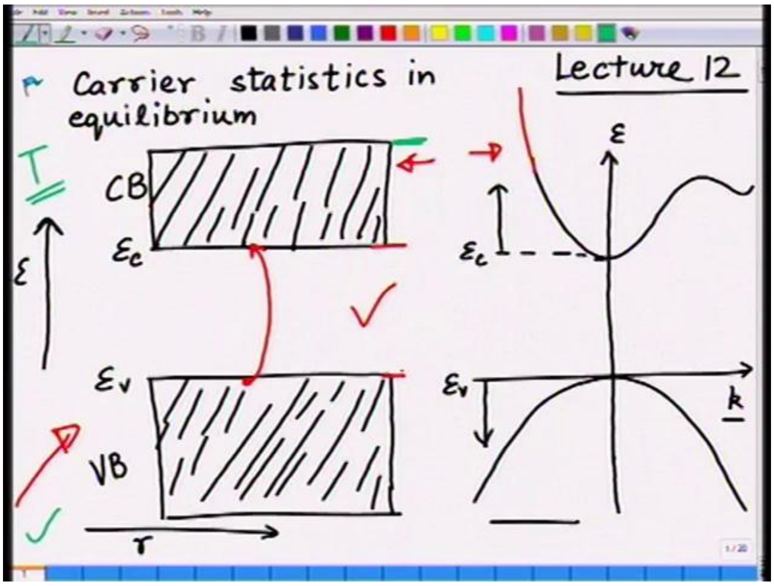

Now, if I think of my semiconductor to be as follows that is here is the valence band energy that corresponds to clearly if I plot it like this, a direct band gap semiconductor let us say I plot, something like this does not matter. So, this make it little better, right here, something like this. This is E versus k. So, if I plot this, so here is my E v, here is my E v energy, this is energy axis and here is my E c energy which is the bottom of conduction band. Here is the bottom of conduction band and here is the top of the valence band.

This is the top of the valence band and this is bottom of the conduction band. I may plot this energy on the energy scale, this is the electron energy scale. This is the electron energy scale, then what do I see? These are all allowed energies. I have a band, allowed energy how far it is going? I have not shown here.How far it is going? I have not shown here. So, let us assume that it is somewhere like this. It gets closed somewhere here, after that some band gap again and then so on.

So,let us say I have this is a band which we are calling as a valence band and I have a band here, a conduction bandand the common features in all semiconductor is at 0 k electrons are filling completely the valence band and conduction band is completely empty and hence, it is insulator. Now, if this is the band, though what kind of picture is this? This kind of picture is like this, that in this diagrams E k diagram, but here I am plotting only energy and therefore I am drawing horizontal lines.

What is this axis then? This axis, you can think of the real axis. Here it is reciprocal line, reciprocal lattice axis or reciprocal axis k was in reciprocal space. Now, you may think of real space point beinglet us say I have, have this as a semiconductor which I am showing you here. Here is the semiconductor which is let us say and I may ask the question at this position where is the conductionband, where is a, and valence band? Then I show that at this energy level this is the energy, where the valence band is and this is where the conduction band starts.

I ask the same question here. What about this point? Where is the valence band and where is the conduction band? Well this is homogeneous material, so therefore, the valence band is at same position and the conduction band is same position. What about this point? Same thing, same position is valence band, same position is conduction band. So, in that sense I can think of this as a real space, space along the axis or on the, in the material somewhere and at all points since is a homogeneous material the energy, energies are same, identical at every point.

Therefore, you can imagine this plot to be in the real space. In the real space to be this plot. So, so why I am going to do is, now abandon this band structure for a minute for doing this thermal equilibrium statistics, I am going to abandon this, this, this E versus k diagram and stick to only energies. We understand that if a electron has to go from here over to here, if electron has to go from here to here we understand that it may have to change its case state also.

Depending on whether it is direct band gap semiconductor or a indirect band gap semiconductor, but may, but knowing that, that there is, that the semiconductor is indirect or direct band gap semiconductor knowing that it may have to change the case state. We will ignore that point right now. We will think of only energy transitions and therefore, use this kind of band gap diagram which everybody is quite familiar with from school days itself where a valence band and conduction band is shown.

So, that is exactly the diagram I have now began to reproduce and while in this particular diagram I am not showing the top of where this, this particular band energy levels terminate. I have in this diagram I have shown that this is where it terminates and this is where the band of valence band terminates in the energy right here. This is the maximum for the energy, right here this is the minimum of conduction band and this is where the valence band terminates.

This is where the conduction band terminates. So, after that there is another band gap and then more bands are there. We are not interested in that, we are and we are interested in only the one which is completely full and the next one which is completely empty. Those are the two we are interested in because those are where that is where the transitions of electrons will take place between empty and filled states. So, this is the picture. In this, in this picture the question we asking is if you have a temperature T?

If you have a temperature T then how many electrons are going to be found in conduction band, how many electrons n per unit volume, number of electrons per unit volume will be found in conduction band. And how many holes, number of holes per unit volume will be find found in valence band.Let me draw this picture, now little bit more clean.

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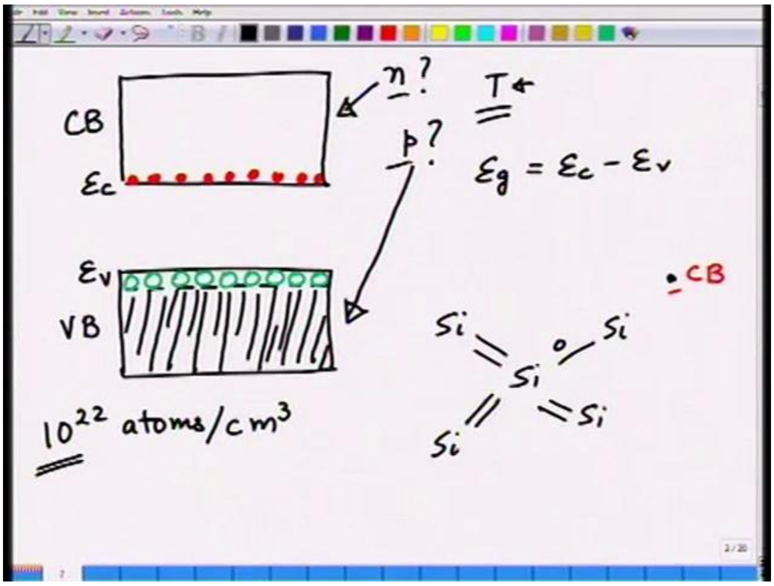

So, here is conduction band and this is energy E c and this is energy E v sorry valence band maxima and here is conduction band minima and this is the valence band sorry, this is not conduction band this is valence band and this is conduction band. All right, what do we have? We are saying that this is full of electrons, but some have gone into the higher conduction band state. So, this is where the electrons are filled up in the dashed region and the top states are where the holes are.

So, that is where the holes are right here. A holes because some electrons are jumped in there, jumped up into the valence band and what has happened is there is some electrons which have, electrons which are jumping have jumped here. So, question we are asking is what is n number of electronsper unit volume of the material, and what is p number of holes in the valence band, number of electrons in the conduction band, number of holes in the valence band.

And the couple of lectures ago I have already explained to you that we, we can speak of holes in the valence band because remember I showed you that in order to calculate current it is sufficient to derive quantity called holes in valence band, where the curvature is concave downwards. And electrons, we can think of in conduction band. All right, this is what basically we want to find out at temperature T, how many will there be. And remember it is in this context Iwas telling you what this band gap is.

Band gap is E g is E c minus E v and that I showed you for silicon is 1. 1for example, for gallium arsenide is about 1.4, E v and I showed at room temperature at at this T how many n n p's question I have asked. And I told you the k T is about 29 milli electron volts or 30milli milli electron volts. Given that thermal energy, yet how many electron and holes are there in conduction band and valence band is the question we are trying to find. In this context you should also think like this.

If you have silicon which is bonded to other silicon atoms like this, tetrahedrally bonded to other silicon atoms, bonding I have shown you, the fact that one electron becomes free, it is all covalent bond. So, all electrons are occupied, all electrons are tied up and they are all in valence band. Now, what happens? A bond breaks, a bond breaks, a bond breaks like this and I get something a hole here and electron become free, corresponding electron becomes free.

This is a free electron which goes in conduction band and this holes is in the, and the free hole in the or the hole which is free I will say is in the valence band. So, I have these green holes in the valence band and a conduction band and the electron in the conduction band which is also free. Alright, if this is happened then remember how many bonds are there? Density of silicon atoms, if you take density and calculate the number of silicon atoms per centimeter cube you would find about 10 to power 22 atoms per centimeter cube in silicon. So, that is the number you need to remember for most materials, this is the 10 to power 22 atoms per centimeter cube.

So, if you look at the bonds also number of bonds with it, approximately similar order of magnitude same number of bonds 10 to power 22 bonds per centimeter cube. So, if you going to break these bonds how much maximum you can get into any of these is 10 to the power 22. Obviously, is not going to happen otherwise material is all gone. So, this number of n n p is going to be significantly less than 10 to power 22 electrons, or holes n n p in with respective bands per centimeter cube.

So, there is some some some boundaries of numbers I am giving you. And now let us begin to see how we will derive this quantities n n p. So, that is what we are going to start in this lecture and probably will take three four lectures, we build up this idea for three four, next three four lectures. So, let us start with it.

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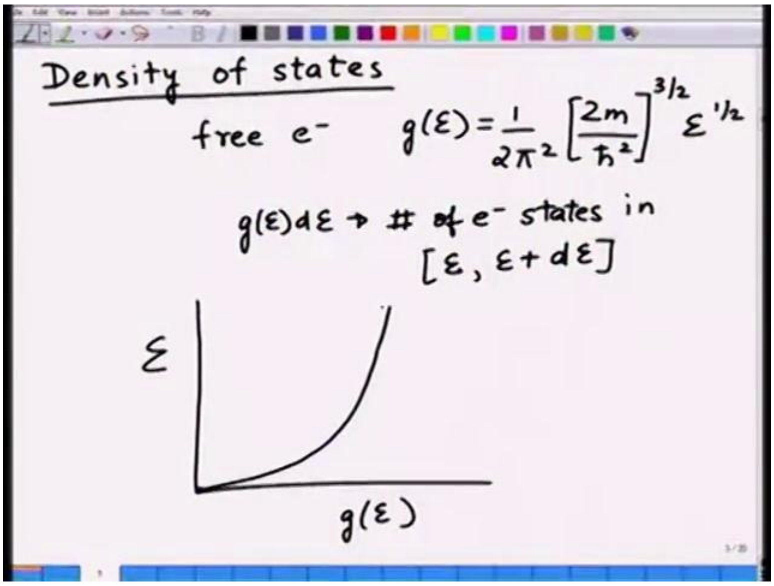

Density of states

free e^- $\quad g(\varepsilon) = \frac{1}{2\pi^2}\left[\frac{2m}{\hbar^2}\right]^{3/2} \varepsilon^{1/2}$

$g(\varepsilon)d\varepsilon \rightarrow$ # of e^- states in $[\varepsilon, \varepsilon + d\varepsilon]$

And in doing so we will begin to use many of the concepts which we have already used. So, let us start with this as follows. Let us start with idea of density of states which you have already been introduced. First let us start with density of states.You shall recall that I have derived for free electrons, if a free electrons I had derived density of states as a quantityequal to 1 over 2 pi square 2 mby h bar square3 by 2 energy to power half. That is the quantity I have derived.

Earlier as density of state and you will recall g of E d E gives you, gives you number of electron statesin E and E plus d E, that was the definition. This is the number of electron states available in this region. If I plot this then this looks like as follows, as energy increases if I plot g of E, then what happens? As energy at 0 energy this is 0 or the band edge at 0 energy and then it gradually increases as a square root of E, as a square root of E it starts increasing like this. That is the nature of g versus E curve.

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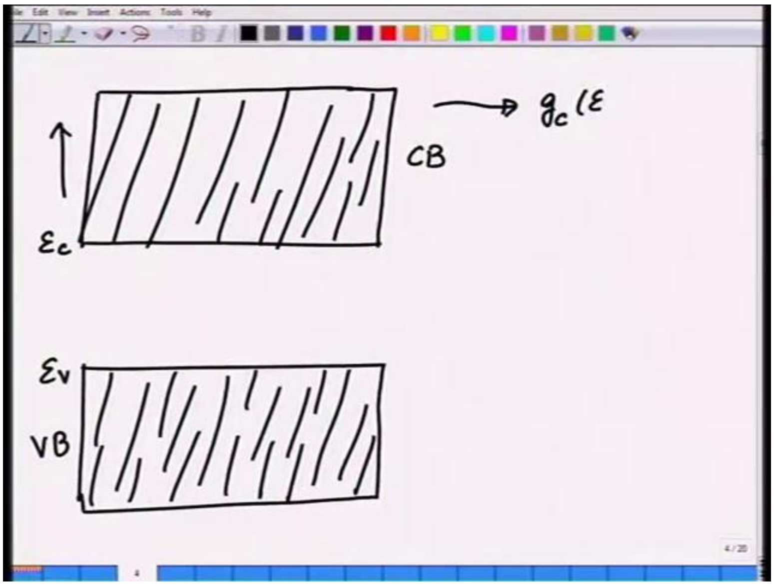

Having done this now let us start applying this. What do we have? We have a conduction band. We have a conduction band looking like this and here is E c, here is E c and it is going up here. That is a conduction band. This I am showing only the conduction band. Of course, there is a valence band also. There is a valence band also, now I am going to use this density of states, but with following modification. I am going to write density of states in conduction band as symbol c, I will give and this is the density of states in conduction band.

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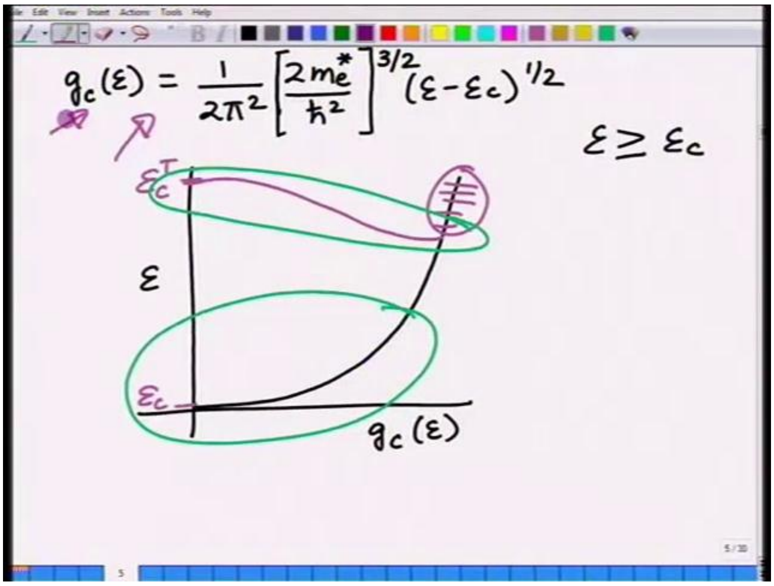

I am going to write this as 1 over 2 pi square 2 m of electron. So, I am going to put symbol here. I am going to put a star for a effective mass, I am going to put the same thing h bar square and I am going to write this as energy minus E c to power half. You can begin to see why I have done this. First of all you notice that this is valid for all energies greater than or equal to E c. All energies greater than equal to E c, why? Because you see at E c, below E c there is no conduction band, there is no this allowed state. Therefore, I must think of all energies.

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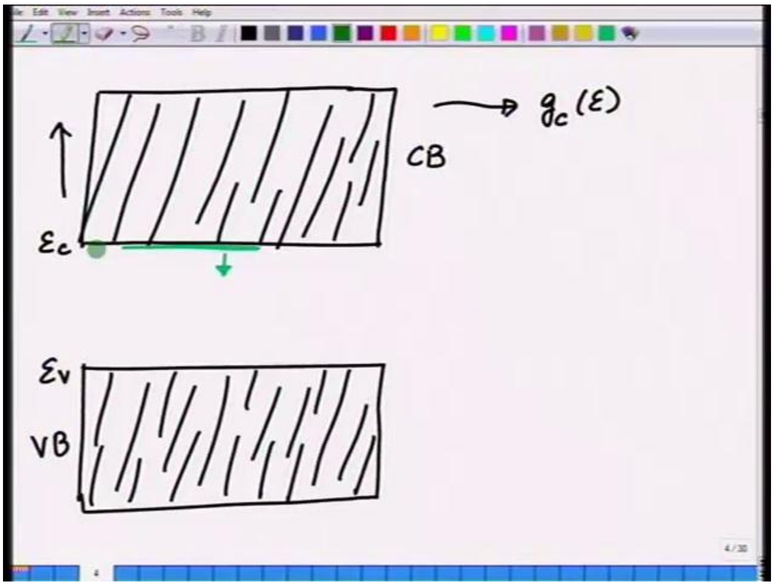

Above that, since the band just starts here, band just starts here.

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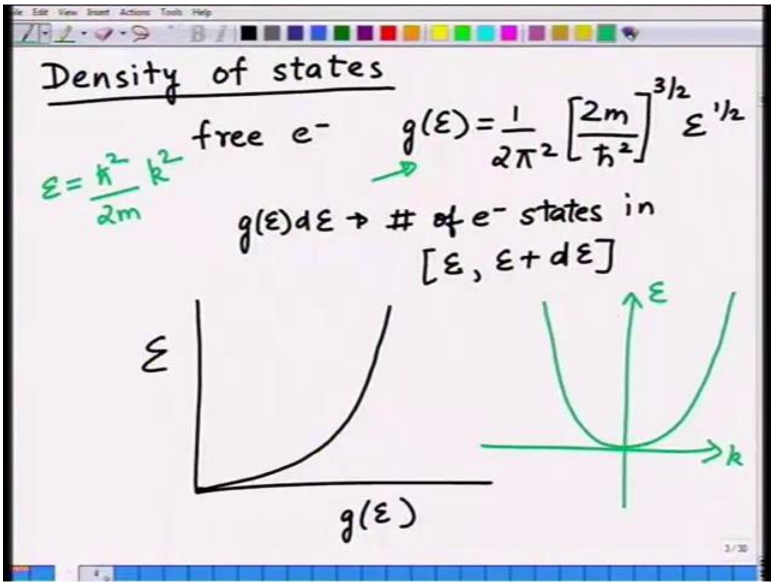

Density of states

free e^- $\quad g(\varepsilon) = \frac{1}{2\pi^2}\left[\frac{2m}{\hbar^2}\right]^{3/2}\varepsilon^{1/2}$

$\varepsilon = \frac{\hbar^2}{2m}k^2$

$g(\varepsilon)d\varepsilon \rightarrow$ # of e^- states in $[\varepsilon, \varepsilon + d\varepsilon]$

So, there are 0 density of state just at E c just like I had my 0 in this expression was at E equal to 0. At energy equal to 0 I had a 0, n energy equal to 0 I had a 0 here, 0 of g c, density of state was 0, 0 at equal to 0, but now since this 0 is at E c level for conduction band for conduction density of states of electron in conduction band its 0 is at energy equal to E c, therefore I have written as this as E minus E c. So, that at E equal to E c this density of state becomes 0 and I start (( )) same curve. That is the first change I have made. Second change is that recall that now I taken into account in free electron theory I had E versus curve which was exactly parabola. Energy was and this is k versus energy, where the exact parabola, that is what this this quantity was.

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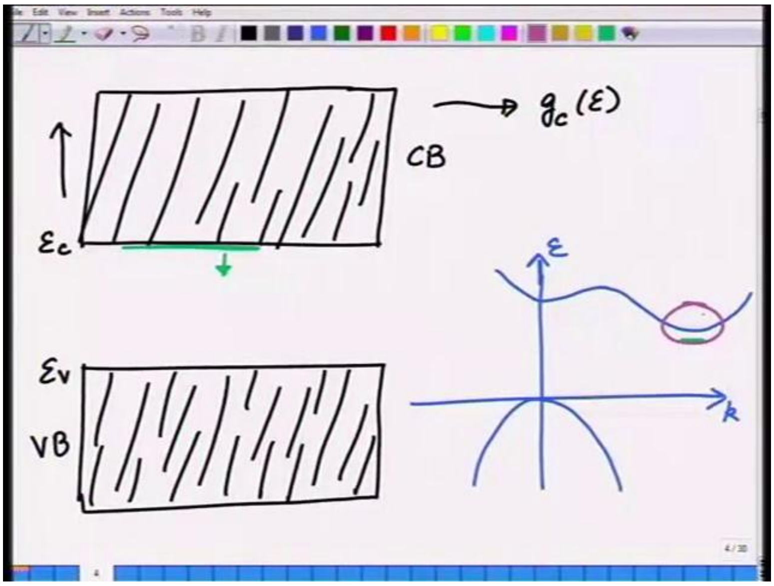

Now, my my my situation right now is if I plot, if I see, if we look at silicon or gallium arsenide band diagram then some, in this band diagram I see some behavior like this.This portion E versus k, this portion which I am plotting here, this is the portion which I am interested in, this is where the minimum of conduction band is where E c is. I am plotting this region. When I am plotting this region then effective mass at this point of time at the, at the, it may not be a parabola for one thing, that it may not be a parabola at all.

Second thing is the curvature may be different than what the electrons mass is. 9.11 into 10 to power minus 31 kgs. It may be different from that. Therefore, you remember this is precisely the region reason we derived expression called effective mass. So, what I am going to do is I am going to take E versus a curve, actual E versus a, E versus k curve sorry not a curve E versus k curve. And I am going to calculate the curvature and I am going to take inverse of that mass and I will try to somehow reduce it to a single quantity and that quantity I will call as effective mass.

What precisely we do in this case, I am ignoring that question, but therefore, but however what I am saying is that in this expression, therefore, if I replace it, I am going to think of equivalent density by, I am going to fit it the density of state to actual density of state. I will fit this formula by replacing by mass by an effective mass depending on the curvature of E versus k diagram. So, this therefore becomes my effective density of states in conduction band. That is one thing. Second thing notice that if I plot this density of state, if I plot this density of states again then g of c E if I plot versus energy then it will continue to increase likethis.

But notice that again I will encounter a band edge somewhere here. So, the according to this expression which I am just plotting it continuously increases. Somewhere here is my E c and here is that E top, let us call it E, E c top, this is the top of E c, this is E c top, let us call it E c top. So, if this is E c top then clearly my density of state at top again becomes 0. So, it must somewhere be something like this. Actual density of state should be really something we should not have this portion, but should have, should behave like this.

Fine, but this expression does not carry, this expression does not carry that it is only increasing. We will stick nonetheless to this expression and I will show you that this portion or the curve is irrelevant at the end and therefore, we are, it is sufficient that we are going to be operating, we are going to bedoing all other calculations only in this region right here and hence, our density of state which I have just derived for conduction band should be sufficient. Let us move next page.

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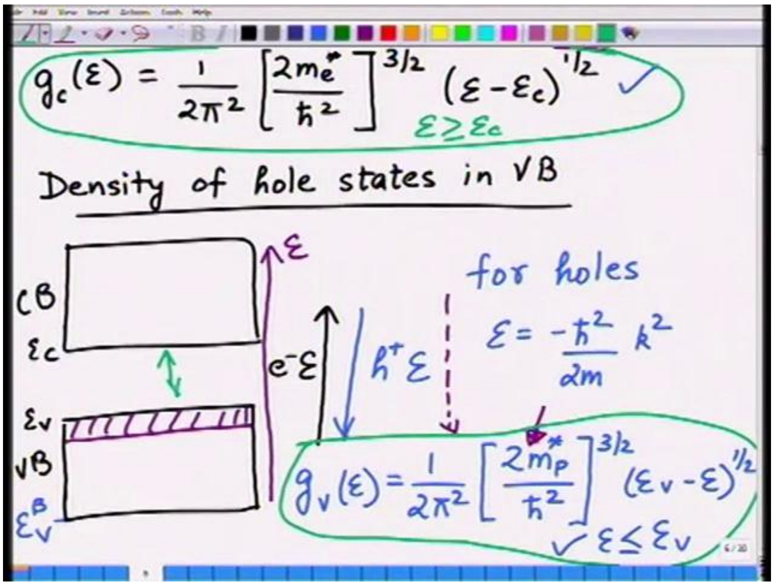

$$g_c(\varepsilon) = \frac{1}{2\pi^2}\left[\frac{2m_e^*}{\hbar^2}\right]^{3/2}(\varepsilon-\varepsilon_c)^{1/2} \quad ✓$$

$$\varepsilon \geq \varepsilon_c$$

Density of hole states in VB

for holes

$$\varepsilon = -\frac{\hbar^2}{2m}k^2$$

$$g_v(\varepsilon) = \frac{1}{2\pi^2}\left[\frac{2m_p^*}{\hbar^2}\right]^{3/2}(\varepsilon_v-\varepsilon)^{1/2}$$

$$✓\ \varepsilon \leq \varepsilon_v$$

So, I am going to produce this density of states of conduction band in energy E, therefore again I am going to write it as 1 over 2 pi square 2 m electrons effective mass. I have replace this electrons mass by effective mass, so that I can fit it according to E versus k curve, actual real E versus k curve. E minus E c to power half. Now, I am also interested in density of, this is the density of electrons states. So, likewise I am interested of density of holes, whatever density of holes, hole states not electronic states hole states in valence band.

Clearly, if electron in in energy in this picture, this is the conduction band and this is the valence band. This is the valence band and this is the conduction band. If electron energy increases like this here E c, here is E v. So, electron energies increasing like this then hole energy must be increasing like this. Hole energy, electron energy, electron energy and this is holes energy. This holes energy must be increasing this way. So, by same token if I start saying for this holes, for holes, therefore if I first think of energies as going minus h square by 2 m k square, first of all start thinking like this.

They are going down then second thing I can do is, I can use a same formula now for density of states. I can write g vof E as equal to 1 over 2 pi square times 2 m. Now, I am going to write h star, let us use, for holes we will stick to p, m p square, m p by h bar square to power 3 by 2 and now I am going to write it as E v minus E to power half for energy is less than or equal to E v. For energy is less than equal to E v in the same way, in same range just I have written like this, same logic applies here when I am looking at hole states.

Except, now I am writing E v minus E because remember, what will happen? Density of states here will be 0 at E equal to E 0, E at E equal to E v density states will be 0 which is what I have done. And as we go deeper and deeper, lower and lower energy towards let us write it E bysame way E t bottom, this is energy E v bottom, lower energy. So, as you move from E v to E v bottom, what will happen? Density of states will continue to increase, density of states would continue to increaseand therefore, I have written first of all E v minus E c in a way where if E is decreasing at this electron energy, E on this scale.

If this is, this is a scale for E. So, if as E as we going down in this, what will happen in energy? That means as E is becoming lower and lower then therefore, then what should happen our density of state should increase, should be increasing. Therefore, I have written as E v minus E here and same, before the same reason for the curvature of valence band.Though curvature, remember which is hole like surface.

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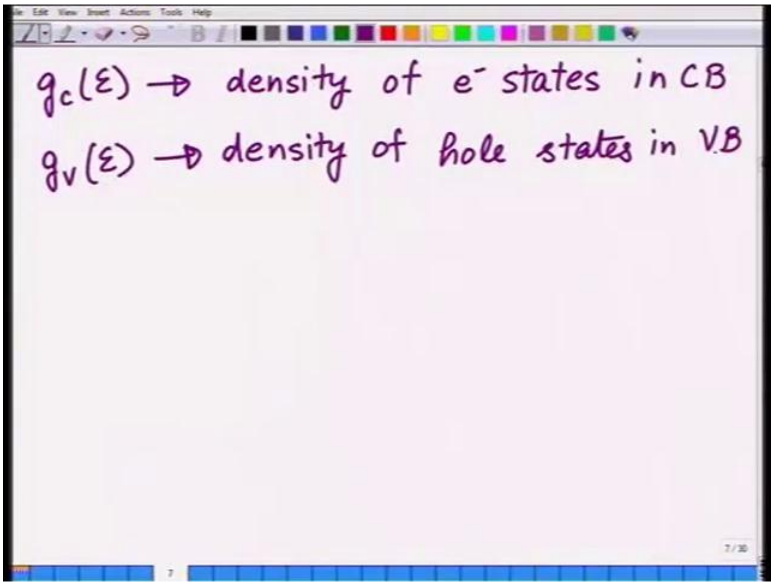

$g_c(\varepsilon) \rightarrow$ density of e^- states in CB

$g_v(\varepsilon) \rightarrow$ density of hole states in V.B

I have, therefore calculated effective mass for that also as substituted in there with the same logic which we did for conduction band, density of electron states, density of electron states in conduction band. Now, I have calculated density of hole states in valence band. So, same logic applies as I go down and then as I reach towards the bottom of the band, again my density of states should go down to 0 which is not predicted by this equation. This equation says it keeps increasing continuously.

So, but I will show you to it later that only portion that we are going to apply this equation is really just near this region, this E v region, near the E v region you going to apply only in this small region. And therefore, we are okay to use this expression that I will prove to you later, little bit later, later that we can continue to use that. So, the two things which we have done is let me now encircle this. So, this is one expression which you are going to use, which is the density of states of the electronsin conduction band.

Second, we want to use this expression which is for density of states in and this of course is for E greater than or equal to E c and no expression is needed. There is no density of states between E v and E c because I can see there is no density that is why it is called band gap. The no density of states between these two states between these two energies. So, that defines our density of states. Now, I am going to define another quantity first.

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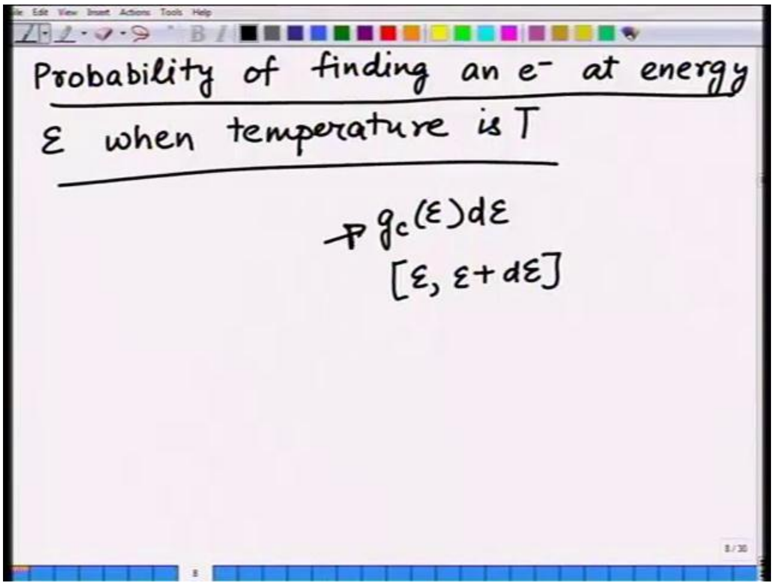

Probability of finding an e^- at energy ε when temperature is T

$\rightarrow g_c(\varepsilon)d\varepsilon$

$[\varepsilon, \varepsilon + d\varepsilon]$

Now, I am going to define probability of finding an electron. So, this is what we want to do now. Why so? On the side you can see that if g c E is d E is the number of, g c E d E is the number of electron states in conduction band in energy range E and d E. Then if I multiply these are the states, if I multiply this by probability of occupation of the states then I will get actual number of electrons in energy range E and E plus d E. Between E and E plus d E if I want to know how many electrons are there, then I know that this is the number of states, electrons states.

These many electrons could be there. Maximum, that is the number of electrons that could be there in this energy range. And if I, therefore multiply this by probability of finding electron at this energy E then I would get actual number of electrons in the same energy states. So, that is what we are heading towards. I have already defined this quantity. Now, I am going to define the probability. Once, I have the probability I will multiply the two and I will get my number of electrons.

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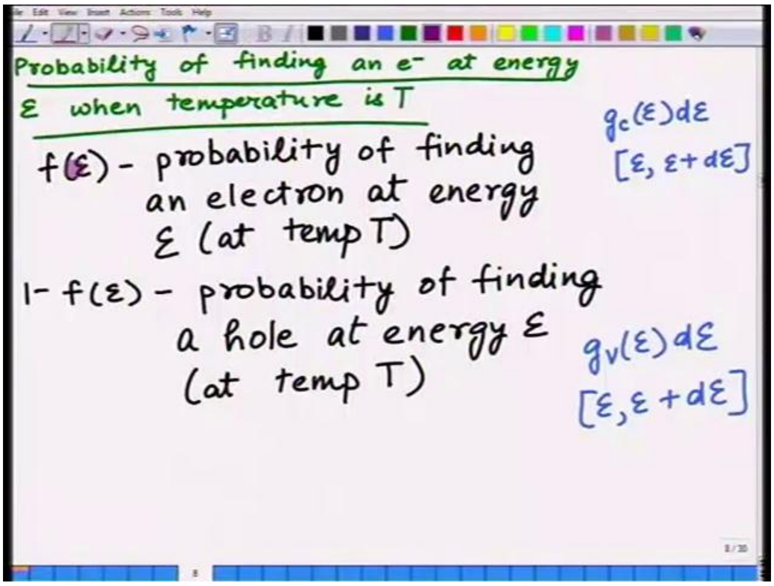

Probability of finding an e^- at energy ε when temperature is T

$f(\varepsilon)$ - probability of finding an electron at energy ε (at temp T)

$1-f(\varepsilon)$ - probability of finding a hole at energy ε (at temp T)

$g_c(\varepsilon)d\varepsilon$

$[\varepsilon, \varepsilon+d\varepsilon]$

$g_v(\varepsilon)d\varepsilon$

$[\varepsilon, \varepsilon+d\varepsilon]$

That is the strategy we are adopting. So, f E is the, let us define f E is that probability. F E is that probability of finding an electron in energy at energy state E. So, this is the probability. If so then please also note that 1 minus f E is probability, well electron is not, so 1 minus f E must be that probability. So, in valence band if I want to find how many holes are there then I obviously know g v E d E is the number ofhole states.

That is the number of hole that can be in energy range E and E plus d E. In energy range this much that can be the hole, holes, if I multiply this by 1 minus F e therefore. Then I will get how many holes are actually there in valence band. So, that is why we if I know what f E is then I will know 1, 1 minus f E, what 1 minus f E is. Now, let us do this. Now, electrons are fermions.

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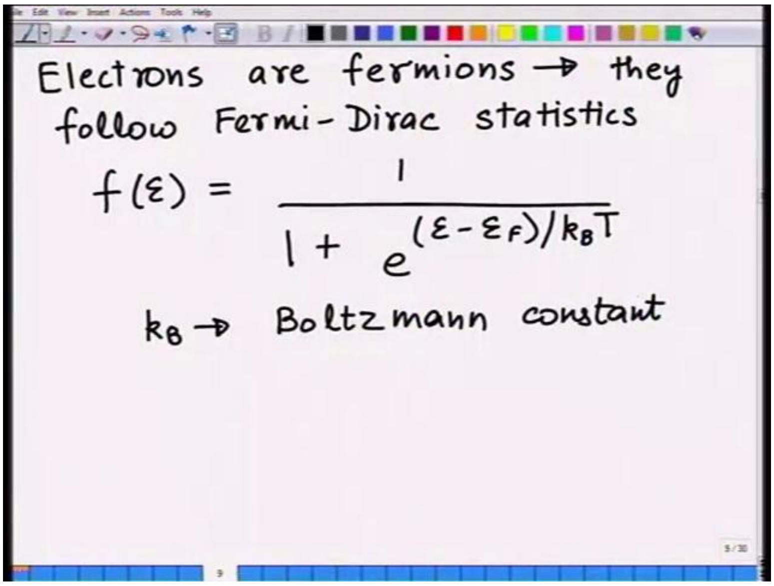

Electrons are fermions → they follow Fermi-Dirac statistics

$$f(\varepsilon) = \frac{1}{1 + e^{(\varepsilon - \varepsilon_F)/k_B T}}$$

k_B → Boltzmann constant

Electrons are, electrons are fermionsthat means the (( )), the half spin in there. Just think of it like that only at this point of time. Electrons are fermions. They follow Fermi statistics, Fermi-Dirac statistics. So, what is the Fermi-Dirac distribution? F of E is equal to 1 over 1 plus e to power E minus E F by k T. Maybe I should be little bit more careful here now, for time being I should write it as k B T. I want to try to remember to keep it k B, but in context please remember also, you also please remember if I miss sometimes then this k B starts for, stands for Boltzmann constant, k B is Boltzmann constant.

So, and since we have been using a small k earlier also, therefore I am putting a b sign, b subscript also. The times I may forget to put this b subscript. So, in context remember this k is not the reciprocal, vector in the reciprocal space rather it is a, it is Boltzmann constant. So, let us plot this.

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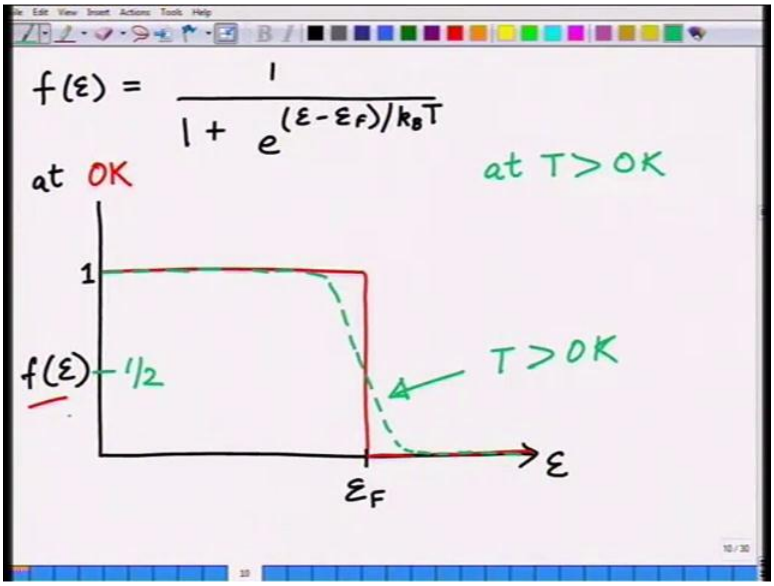

Let us plot this first at 0 K. So, this is the Fermi, this is the probability of finding a electron at energy E and there is a parameter called EF. What this parameter is, we will see in a minute. So, let us plot this at 0 K first, at 0 K. At 0 K let us do this. So, if I plot as follows, hence the energy axis and hence f of E. If I plot this and here is quantity called EF this parameter. This energy EF is in here. So, for all energies E less than E f, all energies E less than EF, E minus EF is of course negative and some negative (( )) divided by 0 leads to, therefore minus infinity.

So, e to power minus infinity is 0. So, I am left with 1 divided by 1. So, I have up to here this probability is equal, is 1. This probability is 1. For all energies greater than E f, for all energies greater than E f, E minus E f is greater than 0 and then some number greater than 0 divided by a temperature which is 0 gives me plus infinity. The e to power plus infinity is infinity, so 1 divided by infinity is therefore 0. So, this curve looks like this at 0 K. This is 1 and then 0, interesting.

So, that defines my E F. Now, recall context metal, metals free electron theory what did we say? We did all the, our calculations were at 0 K we said electron start putting on K states, start putting in electrons and whatever number of electrons you have up to whatever point they fill up, whatever the energy, that is what we call as the Fermi energy. And then we talked about Fermi sphere. That is the highest energy they are filling up, up to. Why? Because at 0 K probability of finding electron was 1.

So, there was no difference between whether there is a state or whether the state is occupied. Because we know that probability of finding electron was 1, therefore if there is a state then it will be occupied up to EF, up to there as you can see f E, value of f E is 1 up to EF. That means up to EF where, if there is a state electron would have occupied it at 0K and hence, we did not distinguish between state and the presence of electron. And above EF, therefore all the energies were all the states were empty.

As you can see probability of finding electron above the EF was 0. And therefore, even if there are states, since there is no probability of them being occupied, therefore there were no electrons above EF. And in that sense, now we can see that this EF quantity right here is the Fermi energy. This is what we call as Fermi energy. Now, let us start looking at for temperatures, at temperatures greater than 0 K. What happens to this probability? Now, you see that some electrons which are filling up to E F.

Now, as you raise the temperature some of these electrons will now jump to a higher state leaving behind holes. So, what will happen? This curve should begin to change and should acquire a character something like this. A character which is like this at T greater than 0, 0 Kelvin. That should be the temperature and that you can clearly see from here itself for energy, naught T is of a finite number. T is finite number, so this is going to fall of exponentially like this. So, you can go ahead and plot this.

So, that is the nature of our Fermi energy curve and then this, this should and this value at EF this act E equal to E f then this E minus EF will be 0, 0. So, E (( )) and since T is finite, so there will be E to power 0, E to power 0 is 1 and therefore, I have 1 plus 1 right here and this has 2. So, 1 divided by 2, so the probability is half. So, at E equal to EF this number should be half. So, the another definition now of, another definition of Fermi energy.

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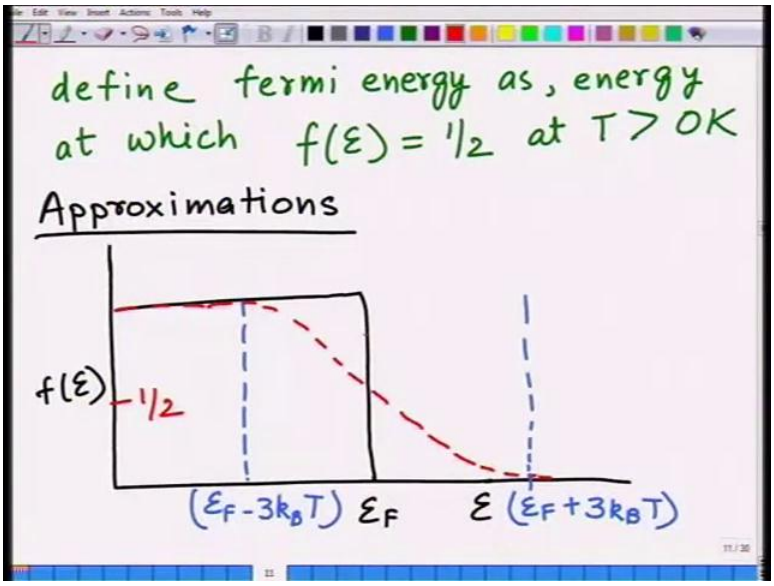

define fermi energy as, energy at which $f(\mathcal{E}) = 1/2$ at $T > 0K$

Approximations

So, we can define now, define Fermi energy which probability of finding an electron is equal to half. So, Fermi energy is that energy where probability of finding an electron is half at T greater than 0 K. So, that is another way of defining Fermi energy which will be useful in context of semiconductors as you can almost see if you, if you have not seen it yourself I will point it out to you eventually. Now, let us make some approximations.

So, make some approximations. Let us make some approximations as follows. So, I have continued to plot this E versus f of E as this curve right here. That is, this is E F and on top of that now I am introducing this curve like this. And I am going to pick few points. I am going to pick a point right here. We will use different pen here. Let us pick this point here which is EF minus 3kT, k BT and this point let us pick from the another point right here. This is EF plus 3 k BT.

So, this 3 is a number pulled out of hat as a arbitrary number, but what the significance is I will show you, it may not be 3. If you are happy with 1, you work with 1. If you are not happy with 3 you may work with 5 also. Either way you will see what it meansand once you understand that then you can choose, free to choose whatever you like and this of course, is half. If that is the case then let us make a approximation.

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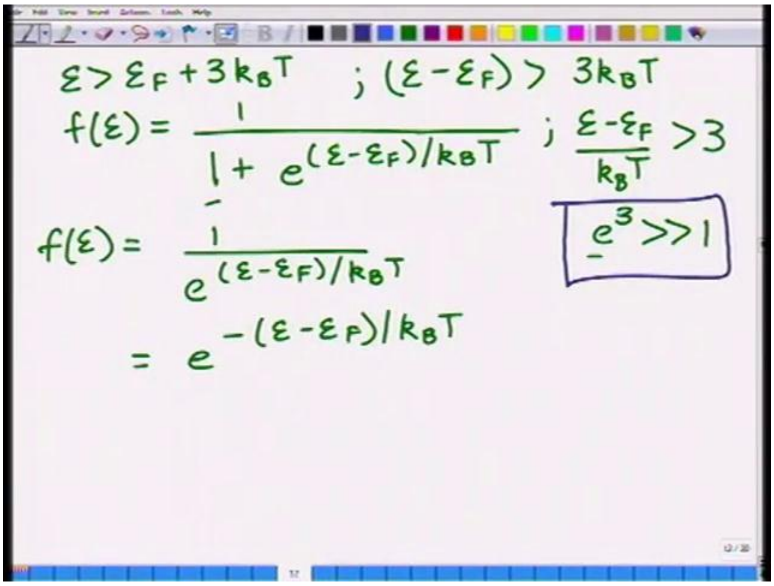

$$\varepsilon > \varepsilon_F + 3k_BT \quad ; \quad (\varepsilon - \varepsilon_F) > 3k_BT$$

$$f(\varepsilon) = \frac{1}{1 + e^{(\varepsilon - \varepsilon_F)/k_BT}} \quad ; \quad \frac{\varepsilon - \varepsilon_F}{k_BT} > 3$$

$$\boxed{e^3 >> 1}$$

$$f(\varepsilon) = \frac{1}{e^{(\varepsilon - \varepsilon_F)/k_BT}}$$

$$= e^{-(\varepsilon - \varepsilon_F)/k_BT}$$

For E greater than EF plus 3 k T that means for energy is beyond here. For energies greater than E plus 3 k BT, what happens? What happens to my f of E? f of E is 1 over 1 plus E minus E f exponential by k BT. So, if E is greater than, if E is greater than EF that means E minus EF is greater than 3 k BT. So, E minus EF is greater than, greater greater than3 k BT then EFE minus EF by k BT is greater than 3. In other words this, what is exponential is greater than 3 and e cube, if I think if I can neglect if E cube, you think E cube is much, much greater than 1 then I can neglect this quantity.

I can just neglect this quantity 1 in favor of E cube and that case I can simply make the approximation and write this as 1 over e to power E minus EF by k BT which is equal to e to power minus E minus E f by k BT which notice is like a Boltzmann distribution. So, for and what is significance of 3? 3 k B T? That you can make this approximation. If you think that e cube is much, much greater than 1 then take 3 k BT. If you not satisfied, if you satisfied by e to power 1 which is 2.7, if you are satisfied by e to power 1 as being much much greater than 1 then you can take this number 3…

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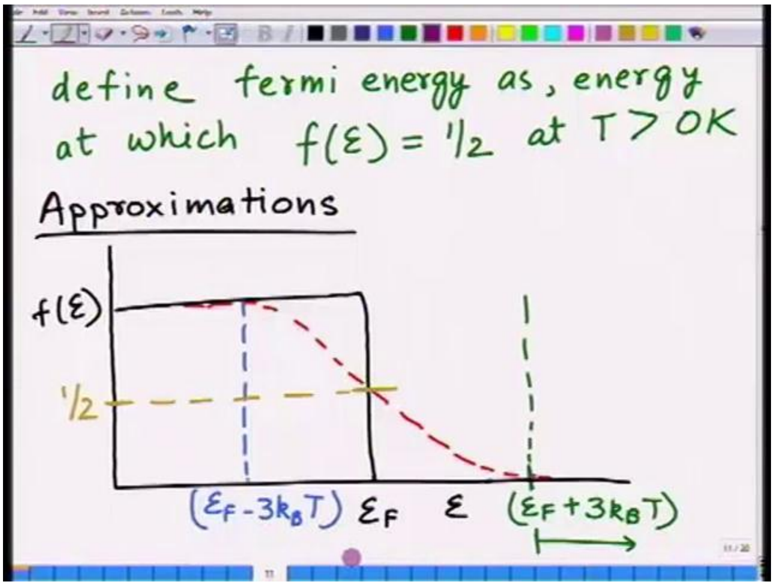

define fermi energy as, energy at which $f(\mathcal{E}) = 1/2$ at $T > 0K$

Approximations

Instead of 3 you can take 1 or if you do not like 3 to be toocoarse in an approximation and if you wanted to be 5 then if you think e to power 5 is actually 11 much much greater than 1 then you can think of this number as 5. The choice is yours. For most cases E cube is or 3 k BT is sufficient enough, but that once you understand it to what accuracy you want to calculate things you can make your choices. But the point what important point is a Fermi direct distribution for these energies E greater than E plus k BT, k BT has reduced to a Maxwell Boltzmann, Boltzmann types of, Maxwell Boltzmann distribution.

It reduces to Maxwell Boltzmann distribution. Similarly, if I solve for 1 minus f E, I am interested in1 minus f E. Remember, why I am doing that? I am interested in number of holes, number of electrons at higher temperature, what all electrons have jumped to high energies. Correspondingly they have left holes here. They have left holes here. So,I am interested in 1 minus that blue shaded region 1 minus f E and red region I am interested in, red shaded region I am interested in f E, the probability of finding electron and blue region I am interested in 1 minus f E which is the probability of finding holes.

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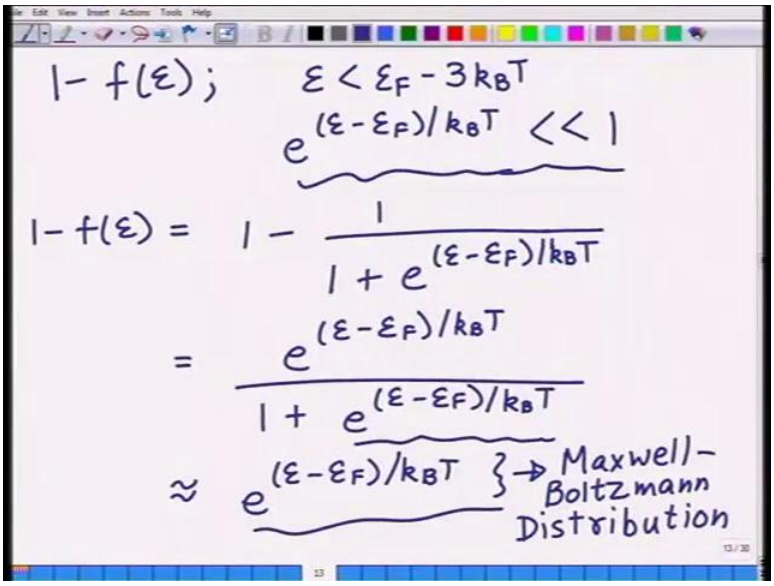

$1 - f(\varepsilon)$; $\quad \varepsilon < \varepsilon_F - 3k_BT$

$$\underbrace{e^{(\varepsilon-\varepsilon_F)/k_BT} << 1}$$

$$1 - f(\varepsilon) = 1 - \frac{1}{1 + e^{(\varepsilon-\varepsilon_F)/k_BT}}$$

$$= \frac{e^{(\varepsilon-\varepsilon_F)/k_BT}}{1 + \underbrace{e^{(\varepsilon-\varepsilon_F)/k_BT}}}$$

$$\approx \underbrace{e^{(\varepsilon-\varepsilon_F)/k_BT}} \Big\} \rightarrow \text{Maxwell-Boltzmann Distribution}$$

So, if I am looking at this then then what, then I am looking at for conditions where E is, E is less than EF minus 3 k BT or I am interested in region where e, e to power E minus EF by k BT is much, much, much, much less than 1, is much much less than 1. Therefore, by same token what we have been doing there, since we can neglect e cube in favor, e cube we can neglect, is much much greater than 1. Same reason where we are interested in this approximation where I can make this approximation.

So, let us write 1 minus f E. If I write my 1 minus f E, then I am going to write this as e to power, e to power e. Since, this is much much smaller than 1. Since, this quantity much smaller than 1, then therefore I can drop this term, I can drop this term in favor of 1 and this approximately, approximately; therefore, is equal to e to power E minus EF by k BT which is again Maxwell Boltzmann distribution. And remember this quantity is a negative quantities.

All e's are less than EF. Therefore, this quantity is a negative quantity in the bracket. So, this is Maxwell. If that is the case, now I can figure out what the population density is. Now, you see that now we have understand that if you are from Fermi energy, whatevertheFermi energy is, if you, as long as you do not ask the question very close to it, then the distribution is Maxwell Boltzmann type of distribution. So, now let us look at this from following point of view. So, let us look at like this. So, what is…? Let us go back to our original question now. How many electrons in conduction band?

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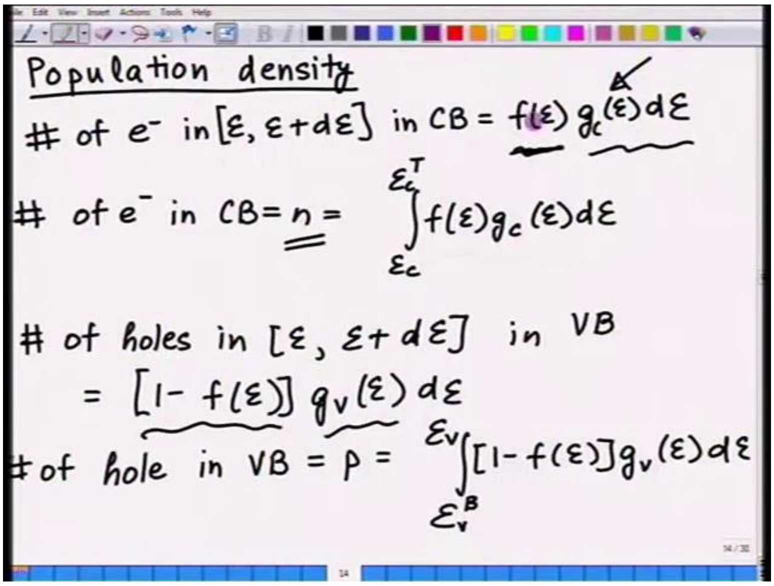

Population density

of e^- in $[\varepsilon, \varepsilon + d\varepsilon]$ in CB $= f(\varepsilon)\, g_c(\varepsilon)\, d\varepsilon$

of e^- in CB $= n = \int_{\varepsilon_c}^{\varepsilon_c^T} f(\varepsilon)\, g_c(\varepsilon)\, d\varepsilon$

# of holes in $[\varepsilon, \varepsilon + d\varepsilon]$ in VB

$= [1 - f(\varepsilon)]\, g_v(\varepsilon)\, d\varepsilon$

# of hole in VB $= p = \int_{\varepsilon_v^B}^{\varepsilon_v} [1 - f(\varepsilon)]\, g_v(\varepsilon)\, d\varepsilon$

So, I am going to write this as population density. I am interested now in population density, population density of electrons in conduction band, of holes in valence band. So, that density, so now remember, now this n if you wish I can write it as number ofelectrons in e to E plus d E rangein conduction band is going to be equal to f of E g of…It is only going to be this quantity, because this is the number of states multiplied by number ofelectrons and number of probability of finding electron in this state. So, with that multiplication I should have number of electrons actually there.

So and therefore, number of electrons in conduction band which we define as n is simply equal to integration of this, from where? From E c which is the bottom of conduction band to E c top, top of the conduction band, if we integrate through and through, f of E g c E d E. Of course, you can raise the objection that you kept saying that g c, I am not, g c is not valid near E c top because at that point density of state should become 0. But yet our expression, in our expression which we are using, going to use is going to increase. Anyway, that is true.

That objection may be true, but at this stage since I am not writing what the expression of g c is, if we have written correct expression of g c at least this expression is correct. Then we will see that if we use that definition of that expression of g c, which we have derived then of course, we have to answer the question that I have just raised. But to, up to this stage we are okay. Similarly, we can write number of holes in E and E plus d E in valence band as equal to 1 minus f of Eg v of valence band d E.

This is the density of holes in valence bond, band and multiplied by the probability of finding a hole on that side. Then I have number of holes in a valence band in this energy land. So, number of of holes, I remember all these aresince g c is (( )) material volume per centimeter cube, per meter cube whatever it is. Therefore, then quantity which we will derive is per unit volume. Number of electrons per unit volume, therefore number of holes in valence band now will be equal to p which should be equal to 1 minus f of Eg v E d E and now we were integrated from from E v bottom to valence band edge up to valence band edge.

So, that is what the integration, this integration would be equal to. So, that is what we need to do to evaluate these quantities and we can now get what we are interested in n n p. In order to do so let us first do it graphically. So, we are going to do this integration first graphically and show youwhat this density n n p will be and what its behavior will be. That will take little time. We do, we are running out of time today. So, I will start that in next lecture, but just give you idea that what we are going to do is first we are going to plot this Fermi function, we may plot this Fermi function, we have to plot this g c function and then we will plot multiplication of the two.

Similarly, we will plot this 1 minus FE function, we are going to plot this g v function and then we will plot multiplication of the two. And the area under the multiply, multiplied curve then would give you the population which we are looking for. So, that is the graphical integration we going to perform and I am going to make this picture in the next lecture and show it to you. Once, (( )) have seen that and after that we will start deriving our expression in a analytical form.

Thank you.