

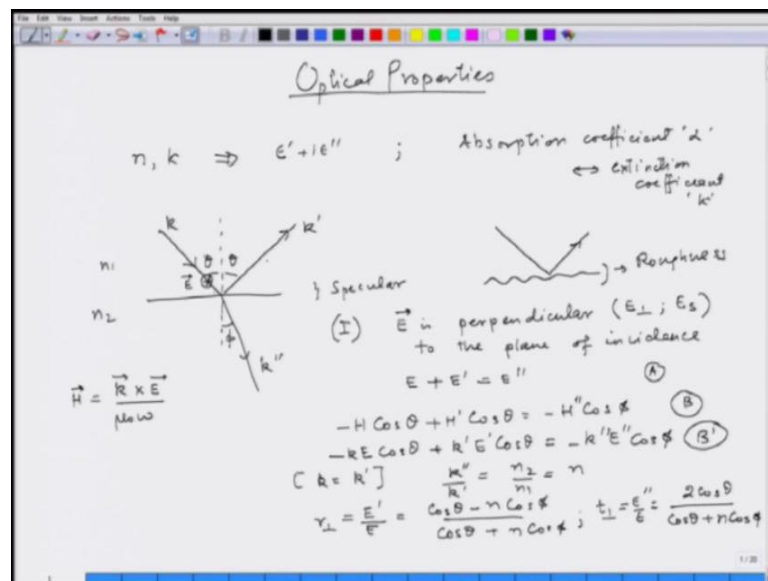
**Optoelectronic Materials and Devices**  
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**Module - 3**  
**Optoelectronic Device Physics**  
**Lecture - 26**

**Optical properties of single interfaces: fresnel reflection coefficients**

Today, we are going to continue our discussion on optical properties. What we have done in optical properties so far is that we told you where what is the genesis of optical properties it is basically interaction of light with the matter that what leads to a optical properties of material. The basic optical parameters property parameters are n and kappa which when we look into the details of how light interacts with the material they are derived from the dielectric constant, compact dielectric constant of the material epsilon prime and epsilon double prime.

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So, we have seen how this basic property optical properties of materials are arrived at, then I said that this is this is good, but generally we do not measure n it is reflective index, kappa which is the extinction coefficient for light. We end up measuring the intensity of light and hence the parameters, the derivative optical properties which come out of this n and kappa are absorption coefficient and we derived an expression for absorption coefficient and for the relationship between absorption coefficient and

extinction coefficient  $\kappa$  and  $\alpha$ .

So, we saw how what we measure is related to the basic parameters of the property of the material, then we were interested in trying to figure out reflectors. And so looking at the  $n$  behavior, we look at the problem where we have material and we look at a specular material. Because as I mentioned that the light coming to a material we are interested in its properties are how it reflects it is not necessarily just a property of the material it can depend on how the surface is. So, for example, a surface which is rough the light will be the how much is reflected or how it is reflected is the function of what is the roughness, the roughness of the surface.

So, that is a different problem the problem that we are looking at is assuming that you have already sharp interface between two variance  $n_1$  and  $n_2$  and so it is not a respective it is called as a specular interface. And the light is coming so the interface wave vector  $k$  and then I looked at a problem where it is being reflected back to wave vector  $k'$  and may be reflected the wave vector  $k''$ . We looked at a problem as we started with the incident angle  $\theta$  and then we have shown that actually through using the boundary condition that the dimensional component of the electric field vector and the magnetic field vector of the light must be continuous across the boundary for a material which is insulating and does not have any charges. We had shown that for refraction  $\theta$ , it is reflected light is at the same angle and that also is what you have learnt earlier.

We also derived the Snell's law which gives the relationship between  $\theta$  and  $\phi$  in this case. So, up to that point now we know the relationship on how the incident light is coming and being reflected back or refracted. Now, what we are interested in is actually also knowing what is the reflectance, how much light is reflected back? So, let us take that case now and see what happens, so we divide this problem into two cases, because the light is coming in this direction in the wave vector direction, but the direction of the  $e$  vector or the magnetic vector. So, let us define for the  $e$  vector could be any direction if it is a unpolarized light. So, we would like to define the problem a little bit and say in the first stage we will see when the electric field vector of the light is perpendicular and we are going to denote this as  $e_{\perp}$  some books also write this as  $e_s$ .

So, this is perpendicular to the plane of incidence, what is the plane of incidence? The

plane of incidence is the plane, which is this plane for the screen that I am lighting on which is consisting of the incident reflected and refracted wave that particular plane is known as the plane of the incidence. So, if I am saying that the  $e$  vector is perpendicular to its either its going into the plane or coming out of the plane. So, let me give updates where it is coming out of the plane and it is perpendicular so the direction of the  $e$  vector here it is coming out of the plane.

And for argument sake let me say that all these  $e$  vectors are coming out so if this is not true then my solution will be a negative answer, but for now I will say at the  $e$  vector for a incident light plus the  $e$  vector for the reflected light they are all coming out of the incident plane is equal to the incident vector of the reflected light across the interface and the condition here is that the electric field vector remains continuous across this interface. So, this could be the condition that one satisfies, that is one condition for electric field vector.

Now, if I look at the magnetic field vector we know the magnetic field vector the condition given by, from earlier lecture we know that the magnetic field vector will be given by the rate vector pass product, electric field vector,  $\mu$  naught  $\omega$ . So, if that is the case then I can assume that all the electric field vectors for the incident reflected and refracted are coming out of the surface. Then I can write the same boundary condition at the interface for the magnetic field vector and that would be for incident like  $\sin \theta - \sin \theta' + \sin \theta''$  will be given by negative of  $\sin \phi$ , so that would be the condition b.

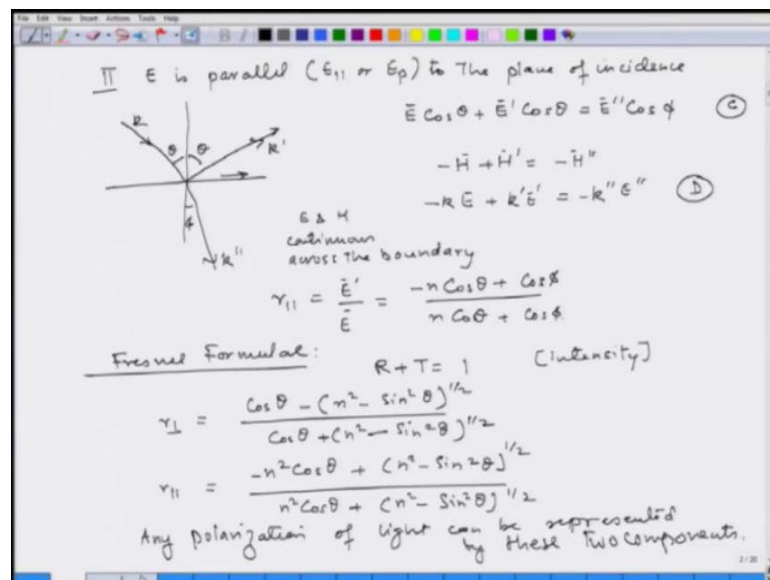
Now, I need to solve a and b, so say this is in form of  $h$ , but I can substitute  $h$  for  $k$  using this equation, so I can write instead of  $h$  I can take write that in terms of  $e$ . If I do that I will get the expression  $k \cos \theta + k' \cos \theta' = k'' \cos \theta''$ . We have shown earlier in the earlier lecture that  $k$  is same as  $k'$  in magnitude, but I am retaining this here just to keep track that was for the incident and this is the reflected ray.

So, now I can solve a and that is the b prime, so solving a and b prime are we get an expression which by using this condition and also by knowing the condition which we derived in the last lecture between  $k''$  and  $k$  is nothing but  $n_2^2 / n_1^2$  or which is also written as  $n_2$  at times, which is basically if  $n_1$  was the a tangent 1 and

this could be the refractive index from what we get. So, if I write it in this terms and what I am interested in is what is the magnitude? What fraction of the magnitude of electric field vector is reflected in this case of the perpendicular case?

So r perpendicular is nothing but the magnitude of the reflected ray divided by the magnitude of the incident ray and this will be given by solving these two equation one finds it should be given by cos of theta minus n, this n reflective index, cos of phi divided by cos of theta plus n cos of phi and the amount transmitted, t perpendicular is given by e double prime over e and this is 2 of cos theta divided by cos of theta plus n cos phi. So, this is first expression when the electric field vector is perpendicular to the plane of incidence.

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Let us look at the second case, the second case what happens to the how much amount is reflected? Will be given by when the electric field vector when the e is parallel is also written as e parallel or sometimes it is also written as e of p, this parallel to the incident, plane of incidence. So, this equation there now it is a same problem, but now we are saying is this light which was incident and reflected at a distance theta, this is pi, we are saying that now for the incident reflected and refracted ray the e vector is in the plane. So, let me assume that all the e vectors for all three are in one direction again this answer it is not, my assumption is not right I will get a negative sign for the solution.

So, if I assume that then I can write the boundary condition, which boundary condition is

as we discussed is basically electric and the magnetic field vectors are continuous across the boundary. So, with this condition I can write  $E \cos \theta + E' \cos \theta = E'' \cos \phi$ , this will be my then first condition and then when I write it for the magnetic field vector this will become  $-H + H' = -H''$ . Once again I will use the in order to solve it I want to change this back into the electric field vector, so I would go for, I am not writing the field signal, but these are all vector properties you know these are all the vector quantities.

So, this plane can be changed back into using the same expressions, this will become  $-k E + k' E' = -k'' E''$ , second condition that we need to have. So, if we then try to find what magnitude of  $E$  is reflected back, so  $E' / E$  you get an expression for this  $\frac{-n \cos \theta + \cos \phi}{n \cos \theta + \cos \phi}$ . Now, these equation that we have just derived and similarly we can write for  $d$  parallel are known as Fresnel formulas.

What are they telling us? They are telling us that when the light is if you are homogenous, uniform, medium, very sharp interface than the amount of light which is reflected or refracted from an interface, these are the coefficients that give you what magnitude of the light is reflected. There is no absorption here so basically the amount transmitted plus amount transmission reflected should be equal to 1, that would be in terms of the intensity.

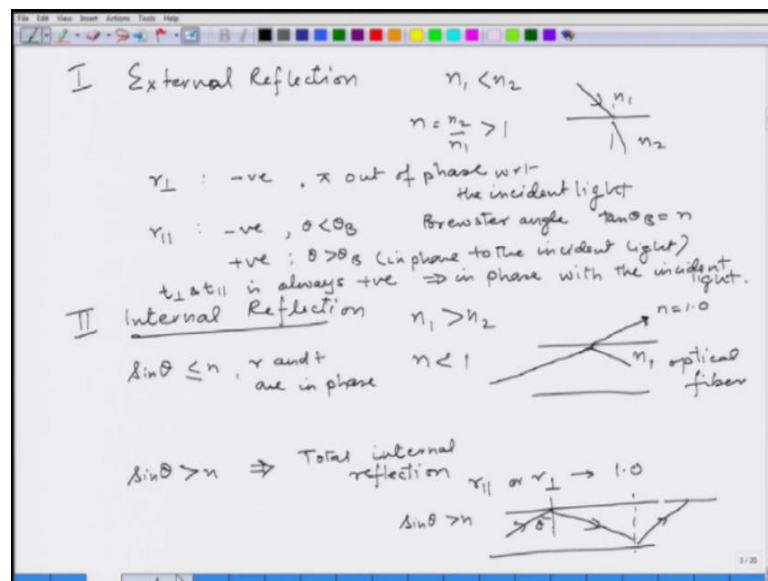
So, in this case these are the Fresnel coefficients which give that number, so the Fresnel coefficient if we use the concept and we want to take away the  $\cos \phi$  using the Snell's condition. Then Fresnel formula or the perpendicular or perpendicular polarization will be given by  $\frac{\cos \theta - n^2 \sin \theta}{\cos \theta + n^2 \sin \theta}$ , the incidence angle is what we know, we want to write it in terms of a incidence angle eliminating the refracted angel of  $\phi$ .

This is for perpendicular polarization and for the parallel polarization expression is given by  $\frac{-n^2 \cos \theta + \sin \theta}{n^2 \cos \theta + \sin \theta}$ . Now, the nice thing about writing this expression in terms of perpendicular and the parallel polarization is that if I have any random polarization, then I can always make it into two components of parallel and perpendicular and then use a Fresnel coefficient to figure out what section or what is the

reflected light like electric field magnitude is and then I can calculate for the general polarization component also.

So, this is the advantage of writing within that terms, so if you have any other polarization. So, in terms of Fresnel coefficients any polarization of light can be represented by these two components. Now, using the basic parameter of the optical parameter  $n$ , we have found an expression for what is the magnitude of the electric field vector which gets reflected or refracted. Before we go into relating it with what we actually see, because when we see a material, we see reflectance, we do not see the electric field vector, we see the square of the magnitude of the electric field vector which is the intensity of light. So, before we analyze it in terms of the intensity of light, let us take some special cases of what is happening to this reflection from a surface a specular surface, so we can talk about particularly Fresnel coefficient in terms of two conditions.

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So, again case one which we can call as external reflection, external reflection basically means when  $n_1$  is less than  $n_2$ , which is I am coming from a rare medium to a dense medium, optically rare medium which is  $n_1$  to a dense medium, so ((Refer Time 19:43)) then towards it to a optically dense medium. So, when  $n_1$  left and  $n_2$  which is like coming into a glass from air whatever, then  $n$  which is defined as  $n_2$  over  $n_1$  it is less than it is greater than 1. So, that condition you will say as external reflection and a second case which we want to see is internal reflection, so what happens to Fresnel

coefficients when we do this. The internal reflection case is when  $n_1$  is greater than  $n_2$  or in other words  $n$  is less than 1.

Now, this is a case for example, if you have an optical fiber, so this is an optical fiber and the light is traveling in there in this fiber. And then if it has to come out the optical fiber is a dense medium,  $n_1$  is a dense medium here and the light is if it is the interface and it wants to come to the air which is 1.0. So, this could be an internal reflection case. So, that would be the case for  $n$  less than 1 because in the two cases we get different values. So, if I look at the first case the external reflection first then and I look at Fresnel coefficient, what I find is for  $r$  perpendicular case, for the ratio of the magnitude of reflected to the incident electric field vector are perpendicular.

If I evaluate that it will come out could be negative, it will come out to be negative for  $r$  values, so what are it means? When it says negative which means the assumption that I made, the assumption that I had made in that case that  $e$  vectors all going to be in the same state in the same way it is not going to be right. And then negative basically means that this is going to be  $\pi$  out of phase, the reflected light is going to be  $\pi$  out of phase from the incident light, out of phase with respect to the incident light.

If I look at the  $r$  parallel that is interesting case we call, it is negative for the values of  $\theta$  less than a angle which we call as Brewster angle, so Brewster angle is such that  $\tan \theta$  Brewster angle is equal to  $n$ . So, that means Brewster angles  $\tan$  inverse of the  $n$  value, so it is this thing is negative for  $\theta$  less than  $\theta_B$  which means it is  $\pi$  out of phase with respect to incidence light. But it is positive for  $\theta$  greater than the Brewster angle which means that the reflected light is now in phase to the incident light. And if we look at the transmitted one for both cases, for the perpendicular as well as the parallel case the transmitted one is always positive, which means it is always in phase with the incident light.

Now, look at the case for the internal reflection, for the case of internal reflection we will define two conditions, one condition when  $\sin \theta$  is less than equal to  $n$  and the other condition when  $\sin \theta$  the angle at which it depends on a angle at which the light is coming is greater than  $n$ . When  $\sin \theta$  is less than equal to  $n$  then  $r$  and  $e$  for all of them  $r$  perpendicular is parallel  $r$  and  $t$  are in phase, the solution that we get from final equation is positive.

For sin theta greater than n this is the special case and this is a condition for total internal reflection, basically means that in this condition you have r is going to be 100 percent, r percent parallel or r perpendicular this is going to be 1.0, reflection is going to be 1.0, basically total light is getting internally reflected in show that case the light is completely reflected at this angle. Angle theta where for angle sin theta greater than n you have complete total internal reflection, so this is with the help of the Fresnel equation we know for a optical material if we know the refractive index optical properties, how the light behaves how much is reflected and how much it reflected back.

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Intensities of reflected light

$I \propto n|E|^2$

Reflectance  $R = \frac{I'}{I} = r^2$

Transmittance  $T = \frac{I''}{I} = n^2 t^2$

$R+T=1$  (No loss of light)

Normal incidence

$R_N = \left(\frac{n-1}{n+1}\right)^2$

$T_N = n\left(\frac{2}{n+1}\right)^2$

Grazing incidence  $\theta \rightarrow \frac{\pi}{2}$

$R_G \rightarrow 1.0$

$T_G \rightarrow 0$

} A technique to increase the reflectance of a surface

Now, taking this further to the property that we are aware of so basically we want to see what is the intensity, so intensity in such emissions last time also we generally do not measure the electric field vector or the magnetic field vector, we measure intensities of reflected light or transmitted reflected or transmitted light. So, if I am looking at intensities as discussed earlier also the intensity of light comes out to be proportional to the medium in which the refractive index of the medium and the magnitude of the electric field vector is square of that.

So, if I am looking at that, that means the reflectance for a surface, a specular surface r is given by i prime the light the intensity of light, which was reflected divided by incident light and that is nothing but r square, for both perpendicular and parallel cases one will write r square. And the transmittance for a single surface how much is transmitted will be



given by  $i$  double prime over  $i$  and that will be since now the transmittance is being measured in the second medium that would be  $n$  times  $t$  square. So, here if I have taken is that  $n$  is equal to 1 for the incident medium  $n$  for the second medium into  $n$  as we were looking at  $n_2$  and  $n_1$  earlier.

So, this could be the transmittance and you can show from the Fresnel equation that  $r$  plus  $t$  is always 1 since there is no absorption no loss, no loss of light. Now, this is a special case because if you have a scattering along with reflection and transmission there will be loss of light, so when we will include the scattering event of the of the light also then this condition will not apply, but the condition that I am considering so far which is homogenous, nice specular interface and materials without any without any in homogeneities in the optical constant  $r$  plus  $t$  will come out to be 1 and so that one can also then appreciate some special cases of this reflectance and transmittance.

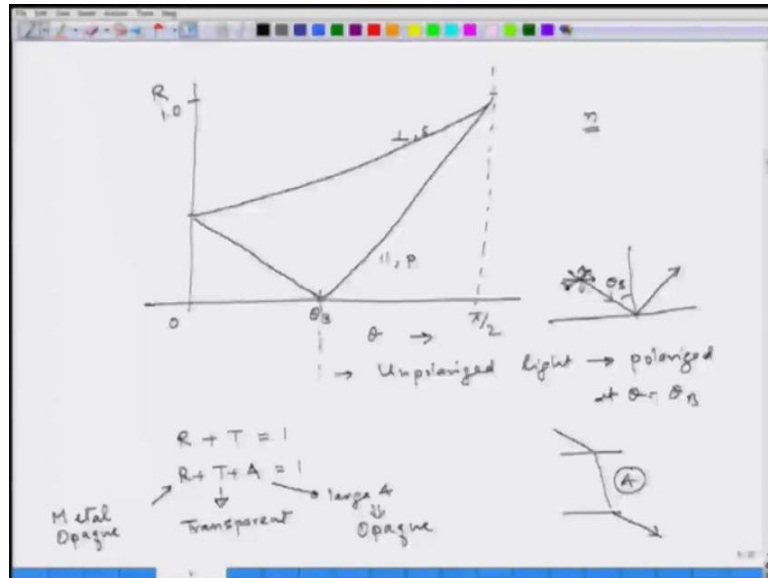
So, for normal incidence you would find that the reflectance it does not matter what is the polarization etcetera it should always be this, all whether internal or external reflection and transmission would be  $n^2$  over  $n + 1$  square and you can check that both should adapt to 1. For grazing incidence and for normal incidence is a light coming right at 90 degree the amount reflected is  $r_n$  and transmitted is  $t_n$ . For the grazing incidence if the coming is very close to the surface, whatever is reflected back is  $r_g$ . And you can show for the grazing incidence it basically plane theta is going towards  $\pi/2$ , but  $r_g$  is going towards 1 and amount transmitted is going to 0.

So, basically this is the way if you want to increase the reflectance of a surface instead of coming normal to the surface you should come at the grazing entrance, which is basically then this becomes a technique to increase the reflectance of the surface. So, very important part point here that we are showing that using the basic optical parameters  $n$  and  $k$  you can come up with the absorption coefficient of a material, then for a particular one interface we are showing what would be the reflectance of the material,

Very important is reflectance is not something which is going to be a constant value it is going to be dependent on how at what angle you bring in the light, which is a function of how you bring in the light it is not going to be a fixed value. And the second thing which is also very important it is also going to be a function of what is the polarization of light. So, if we look at if we look at both we have to look at the two cases and one case where

it was perpendicular to the plane of incidence and in the other case when it was parallel to the plane of incidence.

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So, if we take those Fresnel equations and plot  $r$  reflectance from the specular surface as a function of  $\theta$  and this remains same, the reflective index of the material is same, then what you find is reflectance where you are at the angle which is grazing incidence. The incidence angle is close to  $\pi/2$ , when you are at normal incidence then both perpendicular and the parallel modes are same and when you are at the grazing incidence it comes to the 0 to almost 1.

And this will be this manner this could be the perpendicular one and for the parallel we know that by the interesting behavior that at Brewster angle close to 0 and then it will go back to 1 at a mole  $\pi/2$ . And this is for the parallel case that sometimes written as  $p$  which is also known as  $s$ . So, reflectance of a surface, material  $n$  and specular surface is a function of  $\theta$  and the polarization of the light. This Brewster angle is very interesting because if you see in terms of application if you bring any light if it is a unpolarized light coming to a surface at a Brewster angle, so this light is unpolarized all possible  $e$  vectors are there.

This coming to the surface the light which is reflected is not going to have any parallel component it is only going to have the perpendicular component. So, unpolarized light will become polarized at  $\theta$  is equal to  $\theta_b$ , because the reflectance of the parallel

component goes to 0, so only the perpendicular component is reflected. So, this is one way of polarizing light by bringing it at the incidence angle of the Brewster angle to a surface.

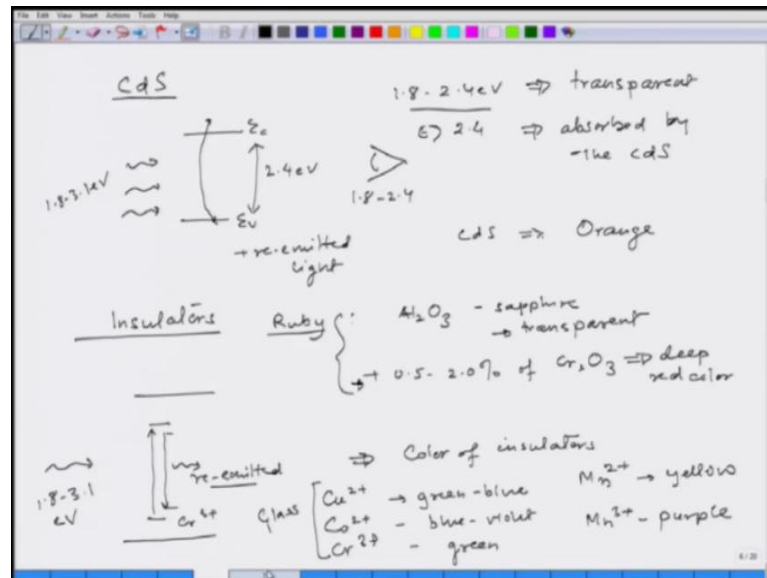
Now, based on this behavior of reflectance transmittance let us look at some of the materials that we know, so if I basically what we have look at is for single surface the intense the reflectance plus transmission is going to be 1, but if there is absorption in the material which means the surface is thick enough, so it gets reflected, but then as it comes out there is some absorption in the material also. You have looked at this case earlier without getting into the concept of reflectance and transmittance here, then total  $r$  plus  $t$  plus  $a$  should be 1.

What different type of materials are based on this behavior, if a material has very high reflectance and you can think of this material you know very well metals have very high reflectance, if  $r$  is very large and which is a case of the metal. Then transmissions is going to be very small and in case of as metals help may be some absorption which basically means the material will be opaque the light cannot be pass through. On the other hand if the material does not reflect too much and has good transmission, if it has good transmission it is going to be a transparent material, as it is generally a case when you have insulating or a semiconducting material.

And it may have little bit of absorption than it will not be completely transparent it would be partially transparent. On the other hand if the material has very large absorption large  $a$ , then also it is going to be a opaque material. So, depending on the optical property of the material you can have different kind of optical materials as we see in the regular light we call a material transparent, translucent or opaque and it all depends on the  $n$  and  $k$  values with defined the reflectance and absorption of the material.

Now, having described that let me give some examples of some materials which behave as transparent or translucent. So, if I take some examples here so metals generally have very high reflectance, some absorption and they are opaque because they do not transmit anything.

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On the other hand insulator for semiconductors for example, cadmium sulfide, they would not be completely transparent, but they will exhibit some sort of colors, so in case of cadmium sulfide which is a semiconductor with a bandgap of 2.4 electron volts and if we are seeing it in the, because our eyes are sensitive to only the visible range which is around 1.8 to 3.1 electron volt. If we are seeing the light through this material cadmium sulfide, what we have learnt is that all the wave lengths which are smaller than the 2.4 will not be absorbed because there are no states in between.

So, from 1.8 to 2.4 electron volt whatever light is coming to this material to some interface and passing through, the material is going to be transparent through that. So, as an observer here I will see 1.8 to 2.4 while incoming light might be white, I will see 1.8 to 2.4 the light which is for energy greater than 2.4 it is being absorbed by the cadmium sulfide, so it is absorbing some of this light and taking the electrons larger than 2.4 and taking putting in conduction band. Some of it is actually reemitted we know that if it is a direct band gap semiconductor, some of it will reemit so you may have some reemitted light because of this absorption also.

So, overall in effect what the color that you would see would be due to what the light which is for which it is transparent and the light which re-emits, so if we see the full of act of it that cadmium sulfide sought of look a little bit of orange in color, cadmium sulfide has a orange color and that is because some of the light it does not absorbs,

whatever it absorbed and reemits that the fact cadmium sulfide looks orange. So, this is why materials have colors and as we call them absorption and reflectance behavior in different. Let us take another example were material cadmium change colors and which is important for many applications and that is insulators.

And this is a interesting example, but we know in a common daily life the example of ruby, ruby is basically  $Al_2O_3$  sapphire, ruby gem is basically  $Al_2O_3$  sapphire and if it is completely pure sapphire it is completely transparent, it is a very wide bandgap insulator, it is a transparent material. But if I add to that insulator something like 0.5 to 2 percent of chromium oxide and this gives a deep red color sapphire and this is what we call ruby. What is happening here? This is not a optical absorption  $Al_2O_3$  wide bandgap semiconductor when I dope it with chromium oxide, it is creating some deep levels here, inside the band gap and now when I excite, when I bring the visible light which is 1.8 to 3.1 electron volt.

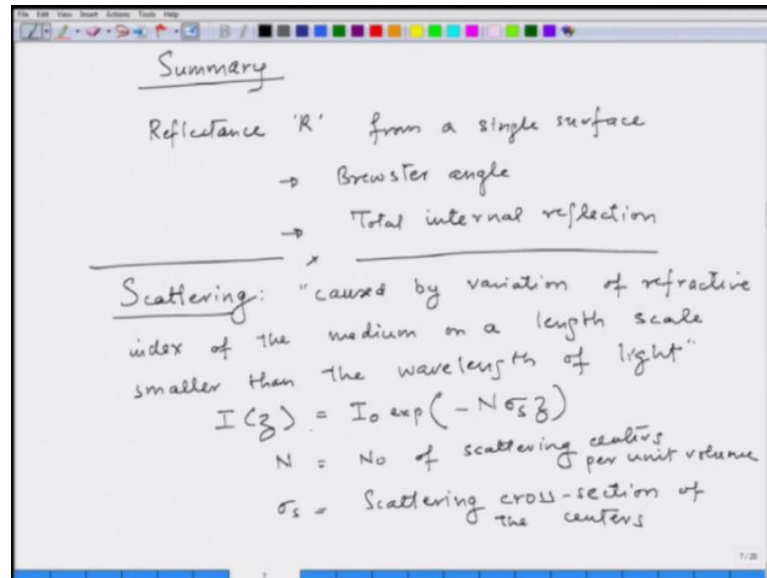
Some of this light is being absorbed by these center, see the chromium ion center, some of this light is being absorbed and then it also comes down and emits may be at a different energy level, so it is reemitted. And because of this process when chromium ion as a impurity level create states within the insulating layer and they then absorb light and reemit and this leads to the color of the insulators. A very interesting process because this is what then leads to different color glasses, for example, if you want green and blue, in the pseudo aligned glass you should dope with carbon carbon 2 plus to get green and blue color.

And this emission by the way which is coming from chromium c plus is because of the transitions within the atomic levels of the chromium ion, it has nothing to do with the  $e_c$ ,  $e_v$  of the alumina. It is the transitions between in the atomic electronic states of the chromium ion which are giving the colors, so they are like color centers in an insulating material. Copper 2 plus will give you green blue color, cobalt 2 plus is going to give you blue and violet.

Chromium 3 plus it is c is that depends on where you put if you are putting it in the glass or soda lime glass, then chromium 3 plus gives actually green, in alumina it is giving you red and the manganese 2 plus will give yellow and manganese 3 plus will give you a purple color. So, this is very interesting phenomena of the color of different materials

how we get it then it partly depends on what is there in the material and how different light is reflected transmitted or absorbed or reemitted and that gives the overall colors for the material.

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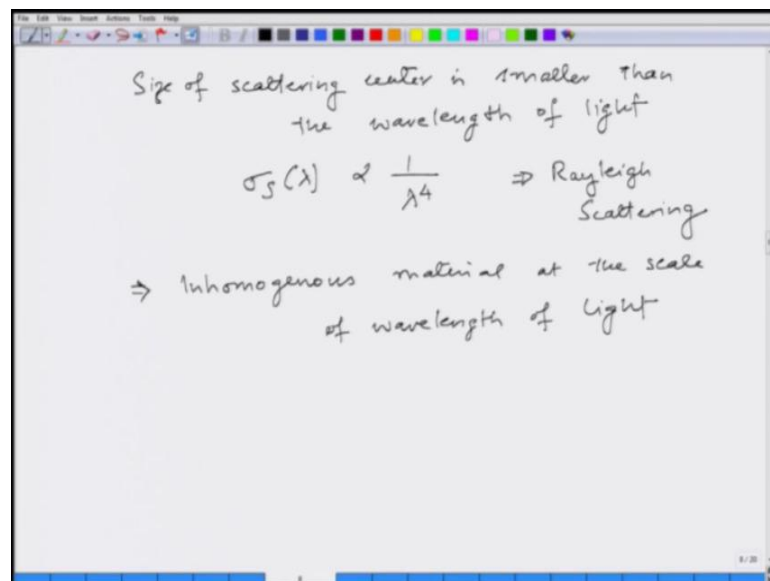
So far what we have done is to give a summary of our optical properties of materials, we had looked at absorption earlier, in this lecture we looked at how reflectance that is r from a single surface changes with polarization and the angle at which you are coming in this process we learnt about Brewster angle and we learnt about the total internal reflection. We looked at the behavior of internals of then reflectance and transmittance and absorbance of the materials, which are basically derived from n and kappa you can see a material being opaque, transparent or translucent, depending on what you have in the material and how it behaves through the light, how it interacts with the light.

Now, all this was assuming a very homogenous material and as I mentioned earlier if the material is not homogenous, then there will be scattering. So, if your interface was not specular then there would be a phenomena which is different from reflectance as we discussed so far and we all that is scattering, scattering of light its extremely important because this happens more in real life then not. So, what is scattering of light? Scattering of light is a occurring when we have variations in the case we have variations of which is being caused by variation of reflective index of the medium on a length scale which is a smaller than the wave length of light.

So, whenever scattering is taking place all whatever has been discussed by us in terms of reflectance, transmission and absorbance that is going to be affected because then you cannot use those equation and you have to correct for how much light is scattered and hence the equations will not apply when the case of scattering is there, one has to find new formalism to account for scattering as well. And the scattering phenomena by itself can be accounted by the change in intensity due to scattering is given by incident light generally with the expression exponential minus capital N perfection for scattering times z.

So, this is what is generally given its a formula very similar to absorption, but has different genesis where n here is number of scattering centers, which are different in optical properties within a medium. So, n is total number of scattering centers per unit volume and sigma s is the scattering cross section of the centers. So, if one has a scattering then one has to also look at the change in intensity due to the scattering events.

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If the size of the scattering center is smaller than the wavelength of light, the size of scattering center and its probably you have seen earlier is smaller than the wave length of light. In that case the perfection for given wavelength is normally proportional to 1 over 4 power of the wave length and this has been used by proposed by Rayleigh, this is proposed by Rayleigh and it is known as the Rayleigh scattering effect. So, this has to be a accounted for whenever we are dealing with, so scattering has to be accounted for

whenever we are dealing with inhomogeneous material, material at the scale of the wave length of the light.

And the changes all the other equation that we developed earlier which work for a homogenous uniform material, when scattering is not taking place. So, we need to account for scattering also when we are talking about the optical properties of the light. With this in terms of optical properties we understand what are the properties of the material? And then we cover it up with how it is important to the optoelectronic materials and in optoelectronic materials it is not just a one interface which is important. Because in a optoelectronic material you are going to use number of layers so what is the behavior of materials when you have number of layers which are at the thickness of the range of the wave length of the light, so that is what we will take up next and here.