Heat Treatment and Surface Hardening (Part–1)
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Lecture Number 33
Avrami Kinetics - 2

In this lecture we continue our discussion with Avrami Kinetics.

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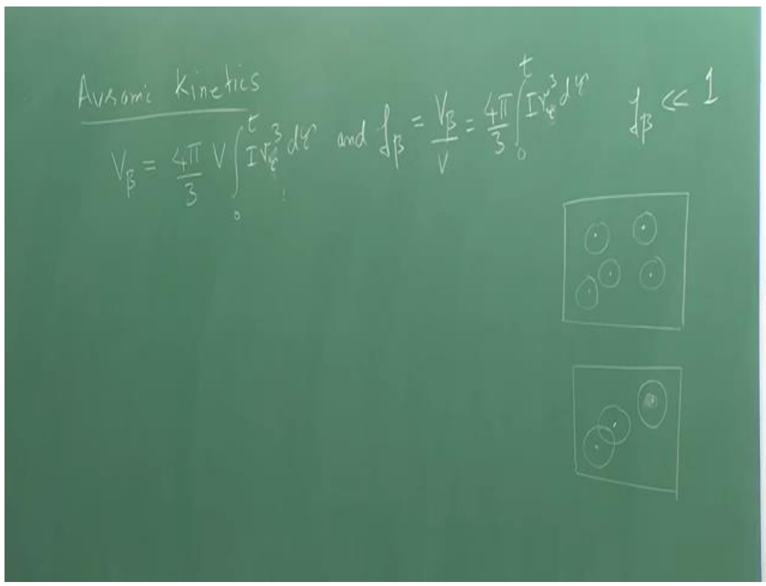

In the last lecture we had arrived at these two expressions where the amount of beta that is formed in s system of volume V is given by (thi) this expression. Where I is the nucleation rate, r tau is the radius of the nucleus that was formed at time tau and integrating the whole thing. If we divide this particular expression by the system volume V, we get the fraction beta transformed as this with basically the volume term cancelling out.

In the last lecture I had also mentioned that this is true for small values of transformation or small fraction of beta transformed. Now why is that, well this model that has been presented so far allows nucleation to take place randomly anywhere in the material. Hence, and it allows the nucleus that was formed to grow indefinitely. What that means is let us say we have some nuclei that are formed and then they are allowed to grow.

As long as they do not impinge on each other this these expression this expression or the fraction transformed given by this relationship will remain valid. However one can visualize a situation where two nuclei could well impinge on to each other. But the mathematical model does not allow for this impingement. You can have a situation where a nuclei a beta phase is already existing and a new nucleus is nuclei can form randomly anywhere a new nuclei could form and start to grow within the region which is already transformed.

In fact it can be best if we illustrated it by the simple simulation of beta particles nucleating and growing.

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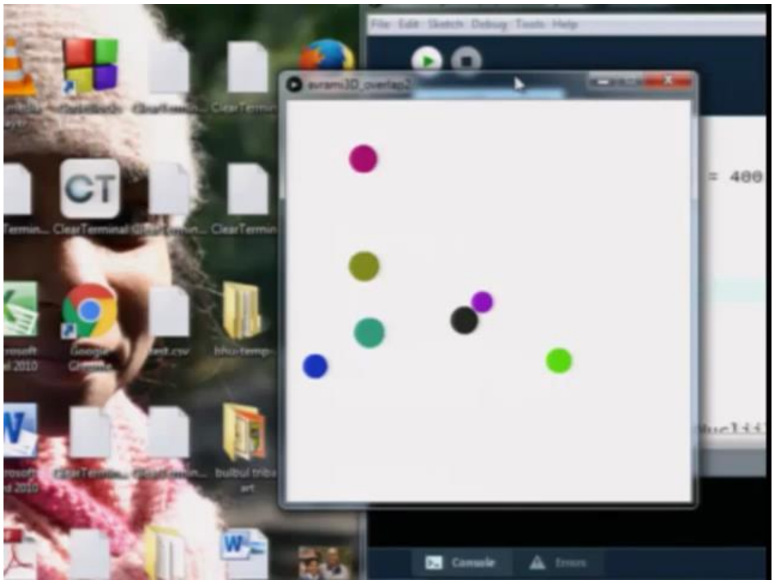

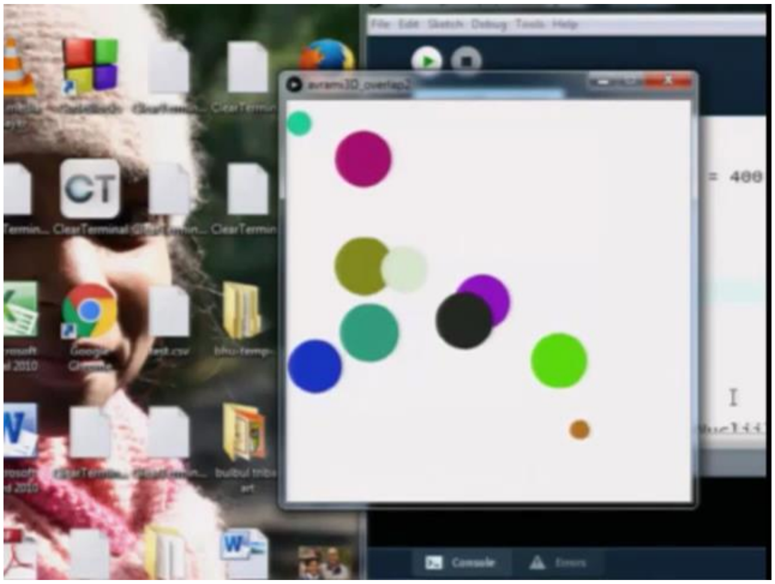
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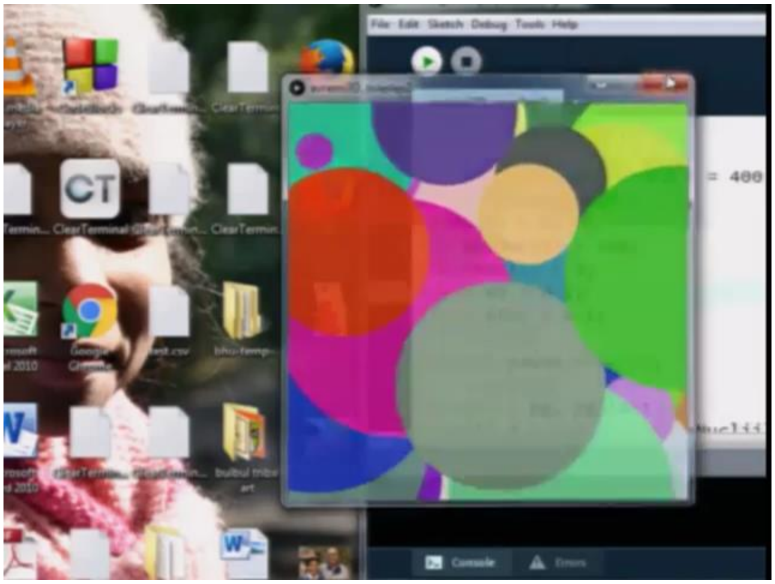

So far as the amount of (nuc) fraction transformed is small they do not impinge on each other, whatever we have arrived is a correct but we allow further growth to take place then you can clearly see you can (clear) as further impingement goes as further growth grows.

So as a growth continues to take place you can see that the particles are growing into each other. And as transformation is allowed to continue further we will also see nuclei forming inside the already transformed material which of course is not physically possible but this model has this particular flow which needs to be sorted out and I can also see that the volume transform can keep growing indefinitely.

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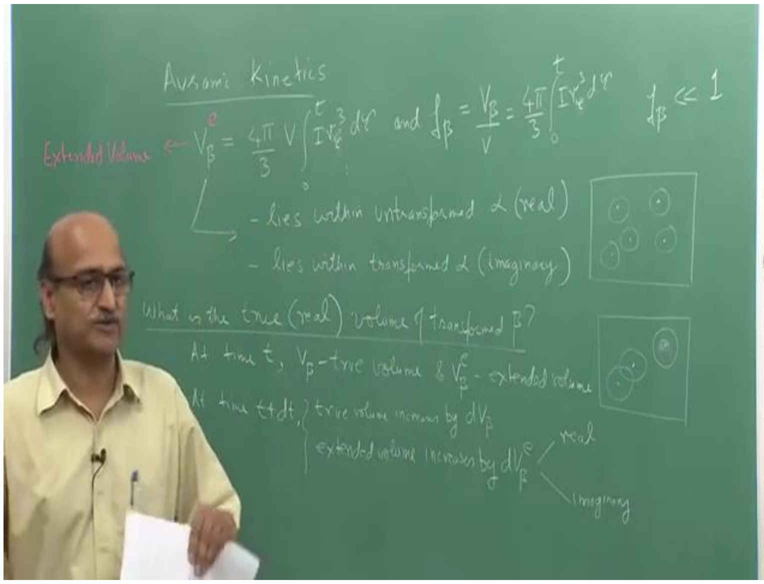

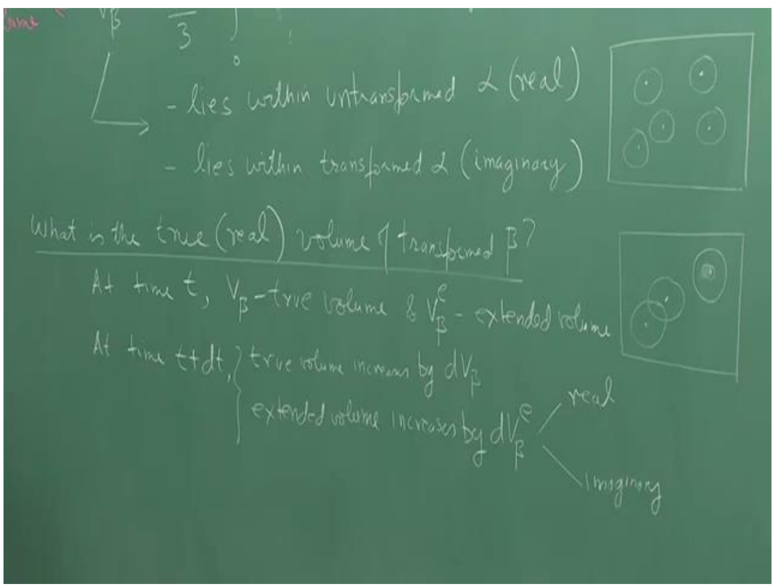

So how to take care of this and let us look at that how do we take care of that. So as we have seen in that simple simulation that the volume of beta transformed increases without bounds. So this volume transformed V beta is represented as V beta super script e, where e stands for extended volume. And this extended volume as time progresses increase increases without any bounds.

And we would also have already seen that part of this extended volume lies within the previously untransformed alpha. And part of it lies within the already transformed alpha. So the volume which lie which lies within the previously untransformed alpha is the real volume. While the volume which lies within the transformed alpha is imaginary. And we have to take into account the both the real and the imaginary volumes.

So we need to determine what is the true or real volume of transformed beta? How do we do this, let us say at time t V beta is the true volume and V beta e is the extended volume. At time t+dt, true volume increases by dV beta and extended volume increases dVV beta e. And again this increment in the extended volume part of it is real and another part of it is imaginary which has extended into the previously transformed alpha, while the real one is the increment in the previously untransformed alpha.

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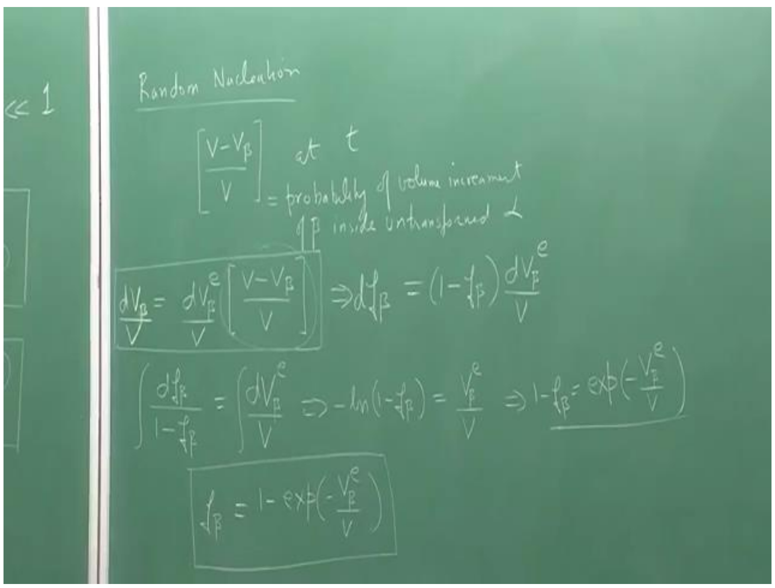

Now we had make an made an assumption that we had random nucleation inside the entire volume of the system. What that means is that at any given time V-V beta upon V, where V is the volume of the system, V beta is the true volume. So therefore true volume of the transformed beta so V-V beta upon V would be the fraction of untransformed alpha at any instant time t at t any instant this is this represents the fraction untransformed.

Hence if the nucleation is random then probability of a nucleus forming inside the untransformed alpha will be V-V beta upon V, that is the fraction of untransformed alpha. So this represents probability of volume increment of beta inside untransformed alpha. Alternatively it can also be looked at that the part of dV beta e, as we in increment of time dt the increment in the extended volume the fraction of it which lies inside the untransformed alpha is given by this probability.

And hence the incremented true volume of beta as we go from time t to t+dt, is then given by dV beta e times this probability [V-V beta upon V]. So this then is the key relationship between the increment in true volume to the increment in the extended volume. If we divide both sides by the volume of the system V, then I can write this as f beta on the left hand side dV beta upon V sorry is equal to df beta the change in the beta fraction.

This is equal to if I look at this square brackets first well this can be written as (1-f beta) times the increment in extended volume upon the volume of the system. Now this is a straight forward equation from here we can now get a relationship between V beta and V beta extended. So if I look at this expression I can write this as df beta upon 1-f beta equals dV beta e upon V.

If I integrate both sides I will get –ln (1-f beta) equals V beta extended divided by the total volume of the system. This takes me to 1-f beta equals exponential (-V beta e upon the total volume). And the final expression we would get for the (fract) true fraction of beta transformed. From here just rearranging the terms equals 1-exponential (-V beta e upon V). So this gives me a general form of the true fraction transformed in the transformation from alpha to beta being given by 1-exponential minus the extended volume upon V.

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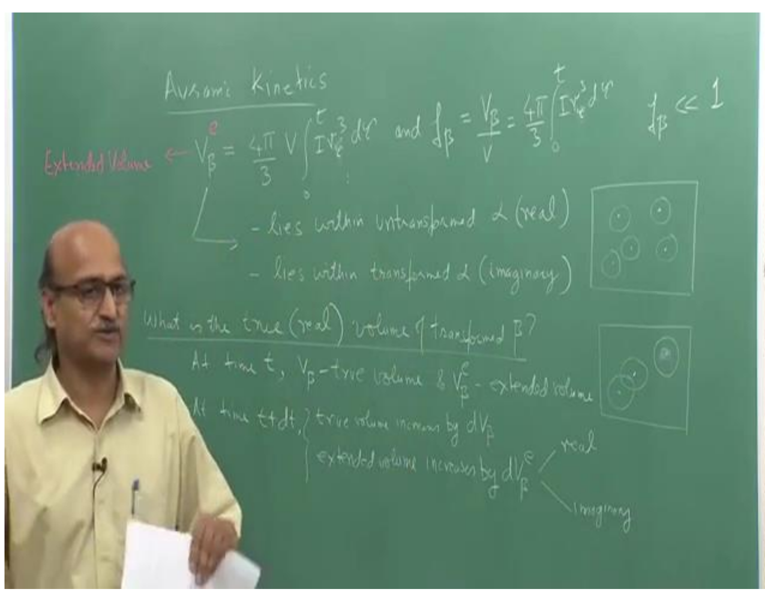

And how do we know what the extended volume is, the extended volume is given by this relationship. Which means now we to plug in the nucleation rate here, how the nucleation rate is changing if it is changing or it is constant how the size of the beta particles are changing what kind of growth kinetics they are following if we plug this in substitute in this we would get the kinetics of phase transformation.

So let us start with the few simple cases in fact just start with the simplest case, where we assume that the nucleation rate and the growth rate are constant and the nucleation takes place throughout the transformation at a constant rate.

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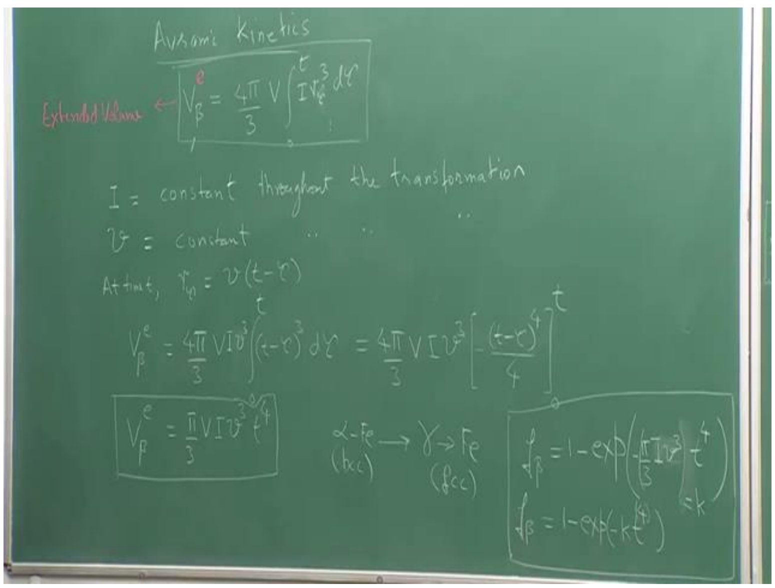

So the nucleation rate I is constant throughout the transformation. And the growth rate v is also a constant throughout the phase transformation. What this means is that at any time t, so at time t. The nucleus which formed at time tau its size is simply as we had already noted in the last lecture is simply v time (t-tau). And we rewrite the expression for extended volume as 4 pi by 3 V, since the nucleation rate is constant I comes out of the integral and within the integral replace r tau by v times (t-tau) and in fact since v is also constant will come out of the integral and become v cube and inside the integral you would have (t-tau) cube d tau.

And if I integrate this would be 4 pi by 3 times the volume of the system times a nucleation rate times the growth rate v cube and the integral after integration would be (t-tau) to power 4 divided by 4 with the limits 0 to t. So my extended volume then become I put in the limits at t this will be 0 at 0 if I put in it will be t to power 4 and hence we would get pi by 3 VIv cube times t to power 4. So this is the extended volume at any time t for the situation where nucleation rate is a constant and the growth rate is a constant.

And remember growth rate is a constant for situations like interface control growth. For example an example of an interface growth control system could be alpha iron at high temperature when cool down transforms to gamma iron. So alpha iron which is a body centered cubic phase changes to FCC, where the growth kinetics would simply be control by atoms jumping from the parent phase the alpha phase to the to the nucleus which is a gamma phase or the phase centered cubic phase.

So now this extended volume now I can substitute in this expression to give me the rate or the fraction transformed at any time t to be written as 1-exponential and I substitute the extended volume so the thing that will cancel out is basically the system volume so we do not want to be dependent on the system volume. And this would simply be - pi by 3 VI not no V will not be there V is cancelled out I times the growth velocity small v cube times t to power 4.

If I lump all of these together since all of these are constants the nucleation rate as well as the growth rate I can lump it into some constant parameter k. Then f beta would simply be 1-exponential (-kt to power 4). So this is telling me how the fraction of beta is transforming as a function of time and important thing to note here is that the time has an exponent of 4. This is characteristic of a transformation in which the nucleation rate is a constant and a growth rate is a constant. What would happen in a case where let us say the growth rate was parabol and we keep the nucleation rate constant.

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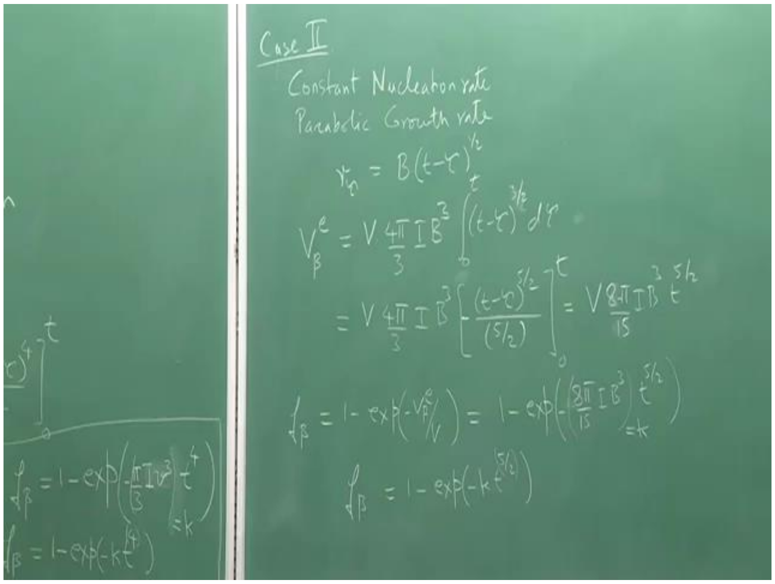

So we have a second case if I call this as let us say Case 1, then let me call this as Case 2. Constant nucleation rate, parabolic growth rate. It will be interesting to analyze this, so what we have is that r tau the size of the nucleus that was formed at time tau would be given by a constant B times (t-tau) to power half.

And what that means is that in this case I have to substitute for r tau in the integral and extended volume V beta e there is simply V times 4 pi by 3 IB cube integral 0 to t (t-tau) to power half d tau. This will not be half this will be cube, so this will become 3 by 2. If we integrate this we would get V times 4 pi by 3 IB cube (t-tau) to power 5 by 2 upon 5 by 2 with the integration limits 0 to t. And there is a minus note the minus sign as well.

After putting in the integration limits we would get V 4 pi by 3 IB cube times well actually this 5 by 2, this would become 8 this would become 15. So 8 pi by 15 IB cube t to power 5 by 2 and hence the kinetics of transformation fraction of beta transformed as a function of time which is given by 1-exponential times the extended volume upon V, exponential of that is 1-exponential (-(8 pi by 15 IB cube) times t to power 5 by 2).

This the terms in this are all constant B is a constant, I is a constant and if I call this as that parameter k. then the fraction beta transformed for this particular situation of parabolic growth f beta is given by 1-exponential (-kt to power 5 by 2). So here you note the exponent here. In the previous case of constant nucleation rate and constant growth we had an exponent of 4, in this situation where we had a constant nucleation rate but the parabolic growth the exponent is not 4 but it is 5 by 2.

Similarly we can consider other cases I will consider couple of other cases in the next lecture where how the exponent changes when I change my nucleation rate or the growth rate kinetics how the overall kinetics changes or the exponent in the Avrami relationship changes would be explored and I will stop here.