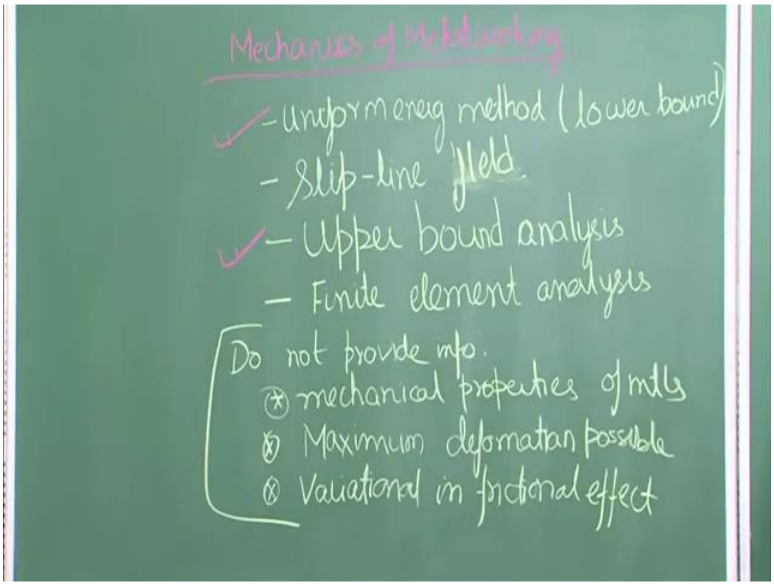
Fundamentals of Materials Processing (Part- II)
Prof. Shashank Shekhar and Prof. Anshu Gaur
Department of Materials Science and Engineering
Indian Institute of Technology, Kanpur

Lecture – 12
Mechanics of Metal Working

So, friends by now you would have realized that the fundamentals of all the deformation process remain same. The parameters which we need to understand in all these are same like the strain, strain rate and temperature and according to that the flow stress of the material will change and that we have discussed thoroughly in last several lectures, it is now time to move on to the next important fundamentals which is to understand what will be the total load road requirement, total power requirement or pressure requirement, total stress requirement when we are trying to do deformation process.

In this process of trying to understand this mechanics, we will also the get to know some more process or some of these processes that we have discussed in little more detail like we will see particularly wire drawing process and the rolling process and extrusion process in a little bit greater depth.

(Refer Slide Time: 01:13)



So our next target is to understand mechanics; what we are trying to understand like I said are the load requirements, the force calculations, the stress calculations and so on. There are different ways or different techniques that have been developed and I will list out some of those. One of them is uniform energy method which is also called as lower bound meaning; whatever load stress requirements that you calculate using this method that is the minimum that will be required and you will see why it is the minimum.

Then there is something called as slip line field, where we assume some particular place along which the shear is taking place and according to that you calculate the stress requirements for the overall deformation; slip line field sorry. Then we have what is called as upper bound analysis, though we will not prove, but as the names suggest upper bound analysis puts the upper cap on that load requirement or the stress requirement. So, if you know upper bound stress or the stress calculation using this method, then you know that is the maximum load or stress or pressure or power requirements for that particular kind deformation and therefore, to be able to give; you can say to have a holistic view.

What we will do is; we will go with upper bound analysis sorry; the uniform energy method which is the lower bound and the upper bound analysis. So, we will discuss these two in our course so that you will know what is the lower bound and the upper bound. So, here the actual values must lie somewhere in between and of course, there are some more integrate and more accurate methods, but which require computational techniques. So, something like finite element analysis where you can distribute the whole element in two d arrays or three d arrays and calculate the stress strain at each and every of these for each and every element of these and then accordingly calculate the total load requirements.

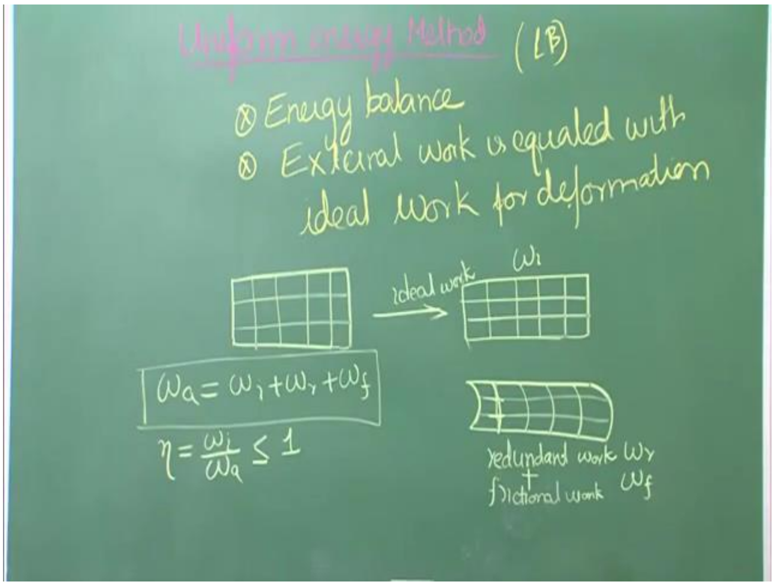
So as the name suggests, this is more computational in nature and above three are analytical in nature. So, we will, but the two that we are looking at are also analytical in nature. So, let us start with uniform energy method before I get to uniform energy method; there is one more thing in general that you must be aware of when we are discussing the different techniques and it is that these techniques do not expect that when we are doing this analysis, we are able to get all different kinds of information.

Now we are only limited to calculating total load stress and pressure power requirements like this, but it does not; these analysis do not provide information or something like these. It does not tell you the mechanical properties of the material, for that you will have to understand the mechanical behaviour or you will take a full pressure course on mechanical behaviour of material. It does not tell you what will be the maximum deformation. So, sometime you may see that the deformation should be 0.3, but it is not telling if they actually the material could be exposed to that kind of deformation, so those things are beyond the purview of this analysis.

Variation in frictional forces, so there may be a variation in the frictional forces; a frictional not the frictional forces, but frictional effect. So, you will have to assume some kind of a frictional field and accordingly you can calculate the total effect or the load requirements or the total change in a mechanics, but this method will not tell you what kind of frictional effects does exist, what are the different kinds of frictional variation that exist in that particular material; over that particular process. So you must be aware of the limitation in somewhere you can say; these are the limitation do not expect everything to come out of this kind of mechanics analysis; this method can only give you the load requirements stress requirements and so on.

So, now coming back what we were discussing, what we wanted to discuss which is uniform energy method or also called as lower bound.

(Refer Slide Time: 07:10)



Why is it lower bound? By the name itself you should have realise by now that what it is saying is that the ideal work needed to do the deformation is what is calculated using uniform energy method.

So, this is in fact, nothing, but energy balance. I will explain a little bit more; external work that is done for the process is equated with ideal work for deformation. Now what is ideal work, let us say you have a bar like this and I am just for the sake of better visualization I am dividing into several grids. Now let us say I want to deform it and make it thinner, so when you make it thinner; it will also expand in one direction. So, let us assume it two dimensional in nature that is third dimensional remains constant. So, after deformation; this is, so the 4 lines; so it remain four lines and the two lines still remain two lines, only they are closer and the vertical lines are now further apart.

So, this is the deformation that I want, so this would be ideal work; however, in reality what happens is not always ideal in nature and what you may get is something like this. So now what you see; each of the elements have moreover less acquired the same shape that we desired, but at the individual level, there is some additional amount of deformation and whenever there is additional amount of deformation; it means additional work much have been carried out. So, there is some redundant work done being done here, there is additional energy getting consumed in doing this additional deformation; which was not required; I do not want this additional deformation, I want only this much deformation. So, they there is some additional redundant deformation which is not required and that is also implies that you need additional work or additional energy; that is one thing.

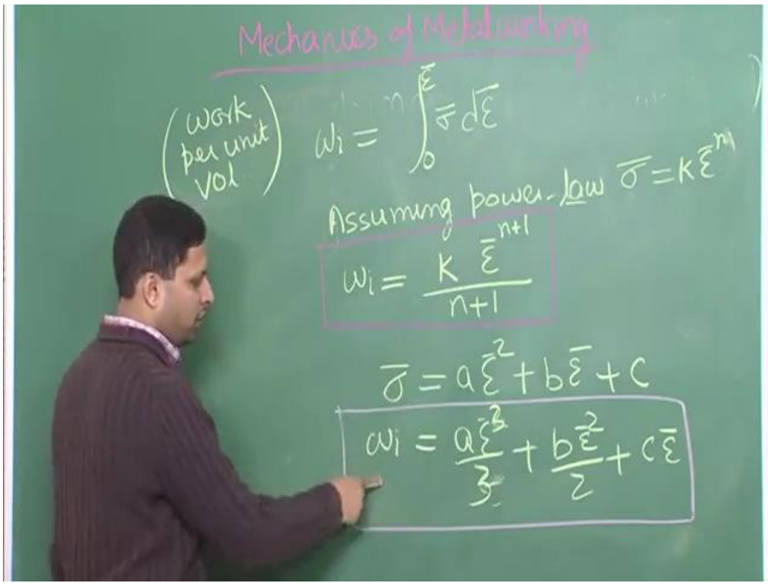
Then there is also frictional work, let us say that you put two dies on top and bottom to do the deformation. So, you are assuming that this layer would slide easily against the die on top of it, to get the deformation like this, but it may not be the fact; it may not be what actually happens. In reality what will happen is that the surface of the work piece will have to work against a frictional force of the die and therefore, there is again some additional work required. So, there are these two main additional work required which increase the total amount of work required.

So, if you were to put it; this is your W i, which is ideal work this is W r, redundant work and this is W f frictional work and the actual work is the sum of all these three. So, you see we wanted only this much, but we have do additional work and therefore, the total energy requirement, work requirement increases and this sufficiency of the ratio of ideal work to actual work that is called efficiency W ideal by W actual and it will be less than 1 because actual work requires more energy or more work than ideal work; therefore, W i by W a will always b less than 1; I should say less than equal to 1; when will they be equal to 1; when W r and W f are 0.

So, there is no redundant work, no frictional work in that case our ideal work would be same as actual work. So, this is important concept which will be utilized to calculate the stress requirements for uniform energy method. So, we are only devoting to the work ideal; that is how much would be the energy or the force requirement if nothing else was being done, if no redundant work was being done, if no frictional work was being done.

So, let us look at one example, not example but let us formulate a simple equation for this ideal work.

(Refer Slide Time: 13:07)



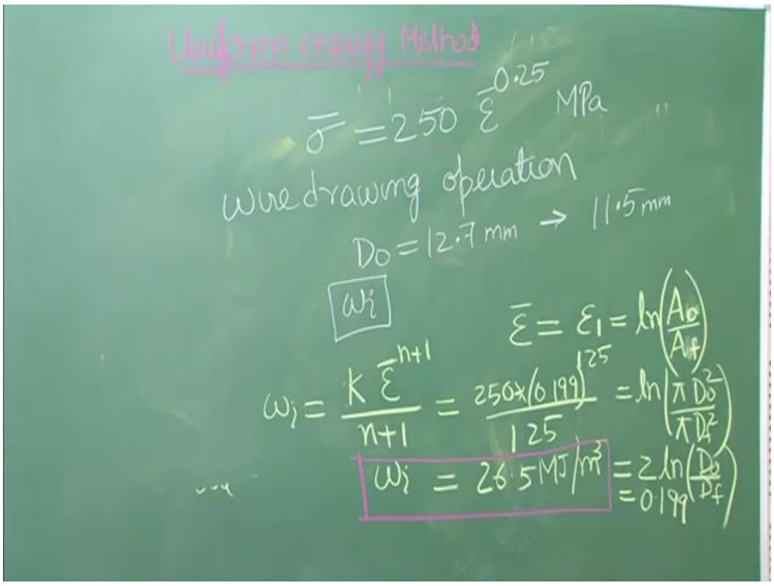
Now W ideal as we said, it can be easily given by just the amount of work that is required for deformation. So, if the flow stress of the material is sigma actual deformation required is d epsilon bar and you have to change the strain from 0 to epsilon bar then ideal work which is work per unit volume. So you should be aware of the unit, it is not the total work, but work per unit volume. So, the ideal work is nothing, but integration of flow stress; equivalent flow stress times D f equivalent d epsilon integrated over the whole strain range.

Now assuming power law behaviour; what is power law behaviour it was this kind of behaviour. So assuming that our material follows this kind of flow behaviour, so the flow stress is given by this equation where k and n are constant; we have already seen those equations earlier then what you need to do is; just put it in here and you would be able to show that ideal work is nothing, but it will boil down to this equation; ideal work per unit volume. So keep that unit in your mind, this is ideal work per unit volume assuming a power law behaviour can be given by this where let us say this is the total strain that was to be given to the material or for that whatever element that we are talking about; in that element if this is the total strain that was to be given. So, the work per unit volume in that is and that element in that component in whatever unit we are looking at; this equation gives you the total work done per unit volume and this is not only limited to power law behaviour, you could have a flow stress behaviour described by something like this and in that case, what you would get. So, you will have to multiple or you will have to integrate sigma d epsilon over the range 0; 2 epsilon bar and what you get is.

So, the only point I am trying to make here is that do not take this equation as you can say hard and fast rule, it depends on what is the flow behaviour are the fundamental equation is this, you have to integrate flow stress times d epsilon bar and if the flow stress was given by this, you will come to the work per unit volume equation something like this.

So, now it is time to solve a simple equation; every time we have some equation, we move on to why we try to solve equation or solve example based on that.

(Refer Slide Time: 17:00)



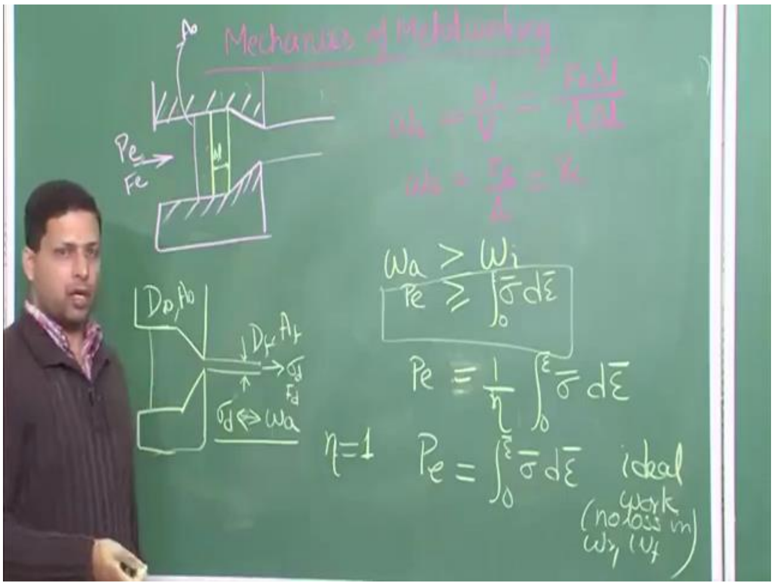
So, let us say a material strain hardening behaviour is given by 250 epsilon bar to the power 0.25. So, k is 250 mega Pascal as you would see I am writing that unit over here and n is 0.25. So, this is the flow stress behaviour that you are given and it is said that in wire drawing operation; wire drawing operation is where you thin down the wire by doing deformation on it, we will again drawn the details of it in a moment in another example that we will come very quickly, we will come to it very soon, but for now it is suffice to say that wire drawing operation is where you take a wire thick wire and the process is such that, it reduces the diameter of the wire. So, in this case when you are doing the wire drawing operation, the diameter where it is reduced from 12.7 millimetre to 11.5 millimetre.

Now, you remember what you have to do is calculate work ideal; that is total work per unit volume. The first thing that you may think of is to calculate the strain or basically what you will think is that W i is given over there for (Refer Time: 18:43) behaviour. So, we know k, we know n and only thing that we need to calculate is strain through that much is right; however, how do you calculate strain. Again this is something you have to be always careful about; we have to calculate the effective strain and effective strain if you remember for wire drawing operation is equal to epsilon 1 which is equal to l n A final by A naught; sorry it is A naught by A final. Now A naught is what or A is given phi d square. So, this comes out to phi D naught square; by phi D f square or you would see it comes down to 2 times ln D naught by D f. The point I am going through each and every step is again that just because I have given in the question, in the problem D naught and D f do not assume that epsilon 1 will become ln D naught by D f, if you go by the first principles; you get that epsilon bar is equal to 2 times ln of D naught by d f.

So, now you put in the values over here and you would see that it comes out 2.199 and we will put the same values over here and you would see. So, it will come down to 26.5 mega joule per meter cube. So, that is the amount of work that is being done when you do a simple wire drawing operation on a material with this kind of flow behaviour. So, this is the way to calculate work ideal and to calculate the strain. So, you have to again like I said you have always careful in how you calculate strains.

Next what we will look at is to relate in extrusion or in wire drawing process, the pressure that is required for extrusion or the stress that is required for wire drawing to the total or the ideal work that is required; that is needed.

(Refer Slide Time: 21:38)



So, let me start with extrusion, so us let say this is our die and this is the material that is being push through. So, you are applying some pressure P e; meaning p extrusion; so extrusion pressure. So, extrusion pressure is what a force divided by the area.

So, let us say the area over here is A naught, now let us consider a small element over here; the length of this small element is delta I, how much force is being applied on this a small element; it is F e, what is the area of this small element; it is circular let us assume that it is circular in nature. So, the area of this small element is cross sectional area is A naught and if you now considered that you have moved from this region to this region, so this is the actual work that you would have done. So, let us say whatever is the P or F e that is the we have we are not relating it with any inherent property or inherent behaviour.

We are just saying that there is you know from some where there is some pressure of force; that is being applied and when you are applying that force, you are doing some work; what is that work you have moving this infinitesimally small element from this point to this point, so there is work done. Now if I want to calculate work done per unit volume and this will be the actual work because I am relating it to the actual pressure of force required. So, the work done per unit volume will be total work by volume; work is what force F e times delta l and volume is what, this is the area cross sectional and this total work has been done in this overall volume. So, A times delta l or work actual is nothing, but F e by A actually this is A naught and F e by A naught is; we said it is p e, so this is p extrusion.

So, the actual work per unit volume is equal to the actual pressure that is being applied. Now we know that W a is greater than or equal to W i, therefore P e is greater than or equal to integral sigma bar d epsilon bar 0; 2 epsilon. So, now we are using this relation; we have derived a relation between P e and sigma bar or basically the ideal work. We already know that W actual should be greater than W ideal and here W actual is nothing, but the actual pressure that you are applying and this is your ideal work. So, P e has to be greater than this or in other words you could have said P e is greater than or actually is equal to 1 over eta; eta is efficiency because in terms of ideal work if you want to put then W ideal by eta is the actual work which is equal to P; so we can write it like this.

So, this is a very again important result; what are the different possibilities, the different possibilities are eta is equal to 1 meaning we are saying that it is ideal work that is all there is no redundant work, there is no friction work; in that case P e is equal to 0 to epsilon bar, sigma bar, d epsilon bar; in case of ideal work or meaning no loss in W r or W f. So, no loss during redundant work or frictional work then in that case we get eta equal to 1 and therefore, P or the extrusion pressure is nothing but integral of this quantity, it is a very useful relation that you will see later on.

In a very similar way, I would want you to do the similar kind of derivation or analysis for the wire drawing operation. So let us say this is my initial diameter D naught, this is the D final and this is sigma D, which is associated with F d and let us say the this is D f. So, there is also area A f A naught. So, do a similar analysis to find the relation between sigma d and work actual.

So I will leave you with this thought, we will actually do it in the next class, but I would hope that you would give me it a try because it will give you a good feel of the numbers or the way that we are getting to this equation and it will also be help full in solving some further problems. So thank you and try to solve this problem.

Thanks.