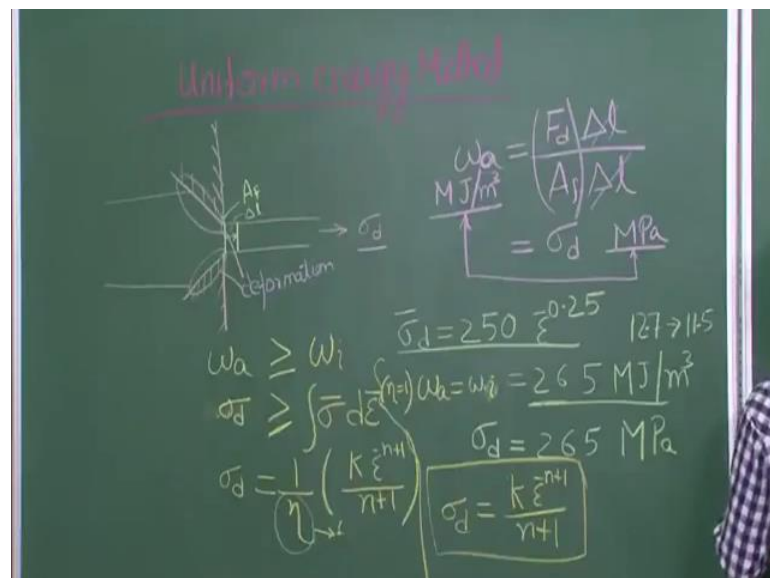


**Fundamentals of Materials Processing (Part- II)**  
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**Lecture – 13**  
**Wire Drawing**

So, in the previous class we saw we were working on uniform energy method and we saw that pressure extrusion can be equated to the actual work done per unit volume and I asked you to try the same exercise and try to relate  $\sigma_d$  to the actual work for wire drawing,  $\sigma_d$  is the stress required for drawing a wire.

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So, let us take a look at this, if you would have tried you would have seen that it is a very simple exercise.

So, this is how the setup will look like for wire drawing. So, this is the die and in here goes your original diameter wire and out comes the reduced diameter wire and it is here that you apply a stress  $\sigma_d$ . So, you when you are pulling it out the wire gets pulled in and it gets drawn after getting deformed. So, this is the region, these are the regions where the flow or the deformation is taking place and what you will note here is that when you here pulling out from a thinner diameter to a wire of a larger diameter and that is possible because this part is work hardened and therefore, this has a hirely ilustres

compared to this one and so you can pull it out and still not break this and in fact, that is what leads to another aspect that we need to understand about drawing, which is the maximum drawing reduction that you can get.

But before that our aim is to understand or relate  $\sigma_d$ . So, actual work per unit volume before that let us draw bit of how it will look like. So, let us say this is the area which is the final area and this is some length  $\Delta l$  by which you have moved this element. So, let us say a very infinitesimal element that has been drawn is coming out. So, you are applying a stress  $\sigma_d$  onto this and it moves by a distance  $\Delta l$  and the area of this is  $A_{final}$ .

Now if I want to write down what is the actual work see this is the actual work not the ideal work because you are applying some stress  $\sigma_d$ , which is which so far we are not assuming that friction is nonexistent or redundant work is nonexistent, we are assuming that  $\sigma_d$  exist in spite of all those and or accommodating all those additional deformations. So, here the actual work would be equal to  $Fd$ , which is the total work. So, let us say this is the  $\sigma_d$ . So,  $\sigma_d$  is nothing but  $f$  over  $A_f$ . So, this is the  $Fd$  or in terms of work per unit volume that is being done this is the total work fore stands distance and divided by volume which is  $A_f$  times  $\Delta l$  and this is what we know as  $\sigma_d$ , this two get cancelled out so actual work comes out to be equal to  $\sigma_d$ .

Another thing you should realize is that  $\sigma_d$  is actually stress, which are the different unit and work actual work is per unit volume is a different quantity and it has a different unit. So, this has usually the units of mega Pascal and this has usually the units of mega joule, and this is also true when we talked about a when we related actual work to the extrusion pressure.

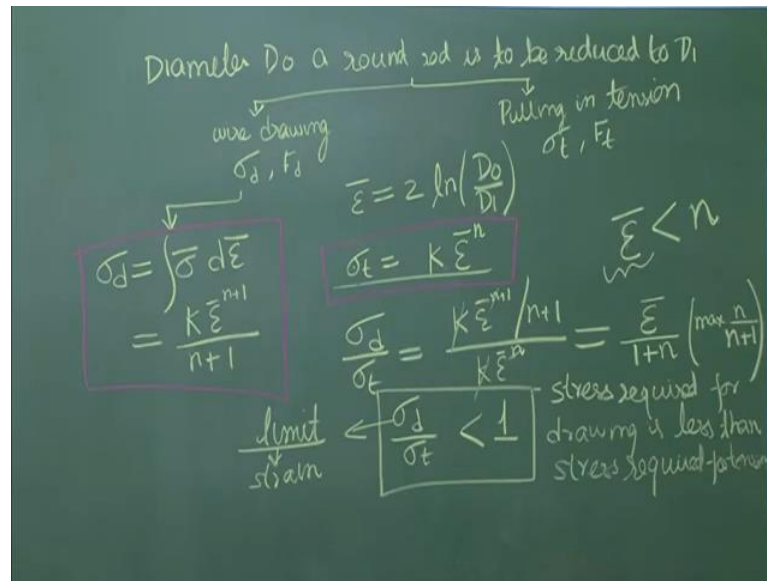
But what you will sees that dimensionally both of them are actually same one and the same thing and that is obvious from this also because we are seen we are used the equation here this is the total work divided by volume, which is work at your work per unit volume and it once you get once you cancelled out the common term, you get the units of stress. So, the dimensional dimensionally both of these are one and the same thing, although unit wise they represent different thing and this is even more clear when you look at the previous example where we had given that  $\sigma_d$  we had given a material

with flow behavior like this and we said that it is being drawn from 12.7 millimeter to 11.5 millimeter and over there we saw that work actual came out to it we solved this in the previous class mega joule per meter cube.

Now, if I give you the same material and the same condition and this time I tell you to find out what is  $\sigma_d$  or the drawing stress. You need not solve it again all you need to see it that this is the value. So, it will be the same because the question is same just that the unit would be mega Pascal. So, when you are applying or when you are reducing from 12.7 millimeter to 11.5 millimeter for a material which has a flow strength like this, this is the drawing stress that needs to be applied and this is the actual work per unit volume that is being carried out. Now when we said that  $\sigma_d$  is work actually is equal to  $\sigma_d$ , in the previous one also there is one more aspect or there is one more factor before we equate  $\sigma_d$  to the total work which is integral of  $\sigma_d$  times  $d$  epsilon. We said that work actual is greater than or equal to work ideal or work actual which we have already shown as  $\sigma_d$  is greater than equal to integral  $\sigma_d$  epsilon.

And if you want to put efficiency factor you can write it like this, assuming power law behavior it will come out to and this efficiency factor is what is accommodating your redundant deformation and frictional deformation and so if this is not present, this becomes basically those are not present then this means that  $\eta$  is equal to 1 and therefore,  $\sigma_d$  is equal to in this part what we are assuming is that there is as the name suggest uniform energy method we are assuming that there is no redundant deformation or any kind of frictional work going on and therefore, we are able to relate actual work which is  $\sigma_d$  to the ideal work which is integral of  $\sigma_{bar}$  times the epsilon bar. So, even in this problem that we solved the actual we saw that actual work is 26.5 mega Joule per meter cube or which is actually the ideal work given that efficiency is equal to 1 and in those condition  $\sigma_d$  will also come out to 26.5 mega Pascal. So, these values would remain same only the units would change. Now there is another very interesting aspect about wire drawing that will see in this next example.

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So, let us say a diameter  $D_0$  of round wire or a rod is to be reduced to  $D_1$ . So,  $D_0$  is your initial diameter and  $D_1$  is your final diameter, now you can do this by two ways: one is wire drawing and the other is by pulling in tension. So, let us say if you are pulling in tension there is a stress  $\sigma_t$  or if you want to calculate in terms of force, the force is  $F_t$ ; if you are using wire drawing let us say the stress you are applying is  $\sigma_d$  and the force you are applying is  $F_d$ . Now the question is, is there any relation that between these two, can we say something substantially about  $\sigma_d$  and  $\sigma_t$  or  $F_d$  and  $F_t$ ? Let us see from our earlier discussions we know that effective strain can be given by  $2 \ln D_0 / D_1$ .

Basically it should be  $A_0 / A_1$ , but since area is proportional to  $D^2$ . So, it comes out to  $2 \ln D_0 / D_1$ . Now if you are talking about  $\sigma_t$ . So, if this is the strain and we are pulling in tension, the stress required for this amount of work assuming power law behavior would be given by  $k \bar{\epsilon}^n$ . So, this is the stress that would be required if you are pulling in tension to reduce our wire from diameter  $D_0$  to  $D_1$ .

Now, let us look at what will be the stress required for reducing the rod diameter from  $D_0$  to  $D_1$  assuming wire drawing operation. Now we know that again is there is some assumption and that assumption is ideal work, so under ideal work  $\sigma_d$  is equal to integral of  $\sigma d\bar{\epsilon}$  which is equal to  $k \bar{\epsilon}^{n+1} / (n+1)$ .

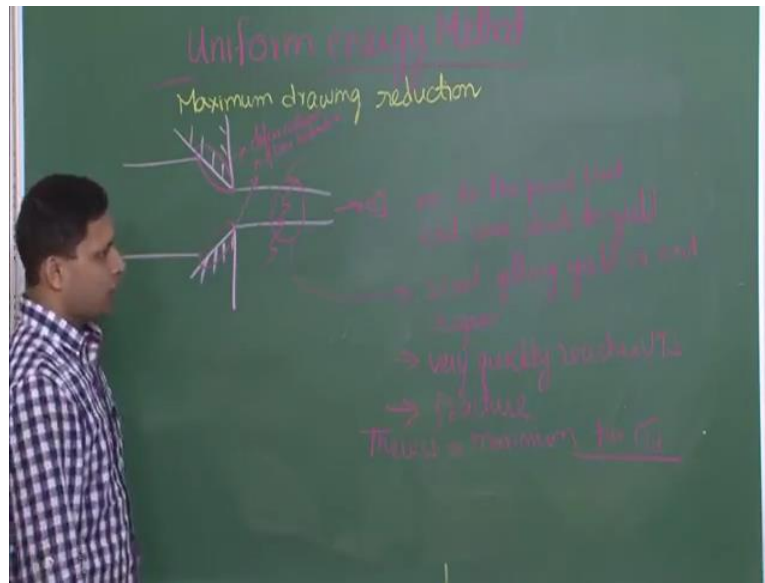
1 by n plus 1. So, first thing that you notice is that the values that have come out are actually different. So, the one must be larger and the other must be smaller, which means that they are one method would allow you to apply less stress and a still get the same amount of deformation, while the other method will require higher amount of stress. So, let us if we are able to find out, if we are able know it before hand it will be good for us because we can use it every time and get or work with much lower stresses to produce this kind of deformation . So, now, let us in order to compare let us divide one by the other, for let me put it like this. So, I have.

So, I get this. So, many of the terms will get cancelled and what I will get is  $\epsilon \bar{\pi} \frac{1}{1+n}$ . So, the ratio of  $\sigma_d$  to  $\sigma_t$  is  $\epsilon \bar{\pi} \frac{1}{1+n}$  or can we say anything about this is it greater than 1 less than less than 1 anything like this? Over here we need to use a fact that  $\epsilon \bar{\pi}$  that we looking at, it can it always is less than the value n. So, our uniform deformation are the point up to which we are looking for the uniform deformation is only true up to this point n and therefore, the maximum value that this here this ratio can take is n by n plus 1, therefore it means that  $\sigma_d$  by  $\sigma_t$  is less than 1 because this is n divided by n plus 1. So, the denominator is always greater than numerator and therefore, this value is this ratio  $\sigma_d$  by  $\sigma_t$  is always less than 1 and what that means, is that stress required for drawing is less than stress required for tension.

So, you see this is you can say a very amazing or interesting result it is says that, you can apply less stress than that required intention and it is still get this that kind of deformation. So, this is very also very useful for our drawing operation because it allows us to get much larger strain much larger area reduction because now you can see we are allow able to get much lesser stress value and get similar kind of deformation that we get in tension .

So, now, let us move to the next aspect for wire drawing, which is to find out if this is the drawing stress, is there a limit that we can apply the drawing stress and if this is there is a limit to, it this there a limit to the strain or area reduction that we can obtain. So, that is our next goal. We have seen several aspects of wire drawing so far and now we are moving to another interesting aspect of wire drawing, which is maximum to find out maximum draw ability.

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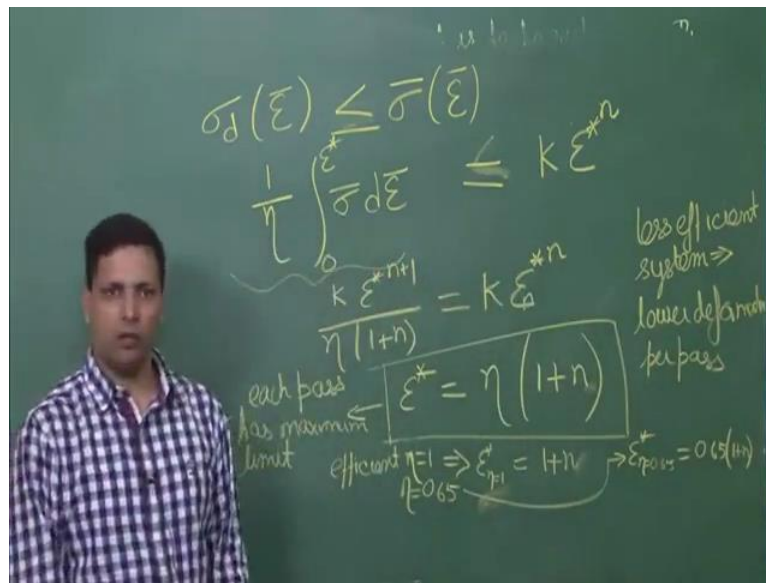
So, let me draw our wire drawing operation again. So, I should have let it be here never mind we can quickly draw it, so here is our wire drawing operation and like I said this is where your deformation or flow stress or flow behavior is taking place. So, these are the two regions where deformation or flow behavior is taking place and this is we are here applying  $\sigma_d$ ; now what we see is that you are applying stress to a thinner region and we will to pull out or a thicker region so that much we saw, but now let us say the  $\sigma_d$  increases to a point that the stream or this region start to yield. So, let us say  $\sigma_d$  increases to the point that exit wire starts to in yield. So, what will happen? What will happen is that you will start getting flow or yield in exit region, is that desirable? So, you are now getting extension or yielding in this region and eventually what will happen? If it keeps on if it reaches beyond and in most cases the stress or the work hardened material will already so much work hardened, that once it starts to yield it would have very quickly reached  $e_{UTs}$  value.

So, if it starts to yield and it very quickly reaches  $UTs$  then the next step is that it will fracture. So, the wire fracture from this region and it will not be drawing the original thick diameter wire and the whole set up will be will go a wire, that is what we do not want and that implies inherently that there is a maximum for  $\sigma_d$ . you cannot apply  $\sigma_d$  beyond this values. So, we that is what I said in the in over there that there is a limit and we see now that why there is a limit because it will lead to the failure of the wire drawing operation and then you will in industry these things are kept in succession. So,

there is one wire drawing operation feeding to a another wire drawing operation and so on, it means that the whole operation will get disrupted if any of the wire breaks down.

So, not only that whatever is the maximum you actually in industry you would keep that strain value or the stress value much lower than that maximum value because we do not want your operation to get disrupted at any stage, otherwise it will involve a lot of operation to restarted. So, now, let us find out mathematically, what is that maximum drawing stress that we must not theoretically cross?

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So, let us say that the material or this wire has been given a strain of epsilon therefore, at that epsilon you will have to apply some drawing stress. Now for this particular strain given the power log of behavior or any kind of flow behavior, you will also have equivalent flow stress value. So, what you what is should be the relation between these two? This drawing stress should be less than or equal to this flow stress. So, let me repeat this, this is the deformation that you are giving by the drawing operation and for that deformation you need to apply from drawing stress. So, I am writing as sigma d as the function of some strain and this is the flow stress for a given strain. So, if you know the flow stress flow behavior of the material, meaning you know the sigma is function of a strain then you can write this sigma bar or you can obtain this sigma bar value or the effective stress value. Now this drawing stress must be less than or e at the most it should be equal to this flow stress value.

and this we have already seen this drawing stress can be written as let us say that critical strain value is given by  $\epsilon^*$ , then this is this is the equation that we obtain just few minutes back that drawing stress is equal to  $1 - \eta$  times integral of  $\sigma$  bar times the  $\epsilon$  bar and  $\eta$  is there because we know there may be some redundant work, some fractional work and this should be less than or equal to let us say we have a power log behavior, for this is should be less than the maximum the drawing stress can go is equal to this value.

So, in order to solve this equation we will get ready of the unique quality and make it equal to understanding that this is the upper limit that we want. So, this is the upper limit, for at the upper limit case this would be equal to  $k \epsilon^n$  and then it is a just a matter of manipulating the equations and remember  $\epsilon^*$  is what?  $\epsilon^*$  is the value at which or at which or the critically strain beyond which the material should not go; if it goes beyond this it means we are applying more drawing stress than the material will have the flow stress behavior. So, this is the critical strain value and we will be able to see it or understand it much better graphically. So, will come to that, but for now what you will see is that  $\epsilon^*$  is equal to  $\eta + n$ .

So, if you know the efficiency of the system, you know the power log hardening behavior from that you know the  $n$  value, then you would know this is the maximum strain that should be imposed during wire drawing; if you try to go beyond this strain what will happened? The material will flow a fracture at the exit point at it will not be actually flowing or forming in the die region and this also means that each pass has a maximum limit, each pass of the deformation will have a maximum limit and if you want your wire to be reduce to diameter larger than this or to a diameter where your strain would be larger, what is the solution? You will have to go for multiple pass.

So, what do we know so far that the drawing stress must be lower than the flow stress of the material given a particular strain, and from that we are able to see that there is a maximum or critical strain below which the system should operate; if your allow you are strain to go beyond this it means a material will fracture at the exits region and not really flow the no flow will to be taking place in the die region other thing that you see  $\eta$  is actually the  $\eta$  value ranges from 0 to 1. So, 1 is when the material of the system is very very efficient. So, for a efficient system  $\eta$  is equal to 1, implies  $\epsilon^*$  and let me write as subscript  $\eta$  equal to 1 is equal to  $1 + n$ . So, this is the maximum strain in any



kind of system that you can get, if your value of  $n$  is 0.2 then the maximum strain that you can get give in our drawing operation will be 1.2.

If the system is not so efficient, let us say the usual value of efficiency is 0.65,  $\epsilon_{\text{star}}$  let me write it over here and again in the subscript left to me write 0.65 is equal to 0.65 of this value  $1 + n$ ; meaning the less efficient system is the less strain you would be able to give in 1 pass. So, the less efficient system implies lower deformation per pass, which in turned would imply that if you wanted some let us say from go to go from 12.6, 12.5 millimeter to 5.5 millimeter, you will need larger number of passes. So, the more efficient the system would be, you can do it lesser number of passes. So, that is my important message that we get from here, we will try to understand some of these concept using graph the stress strain plot the two stress, two strain plot, it will do that in the next class. So, I will leave you with this to mull over it, to think over it and try to understand the concept that we have discussed in this class. So, see you next class.

Thank you.