Fundamentals of Materials Processing (Part- II)

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Lecture – 15
Hodographs

Welcome back friends. So, we are in lecture number 15 and so far we have discussed stress strain analysis, plastic deformation, plastic instability and then we moved on to mechanics of metal forming. Over here we discussed the uniform energy method which is also the lower energy bound and then we went on to upper energy bound and I gave you just the introduction to upper bound analysis.

Today we will continue our understanding of this upper bound analysis and enable you to be able to calculate the maximum or the upper bound of the forces or pressure required for such deformation and like I said there is the methodology to this upper bound analysis; what is that methodology? An internal flow field is assumed, so you assumed that there is some internal flow field and it depends entirely up to you how you make that field as you will see when we progress through this lecture.

Energy consumed in deformation field is calculated using properties of the material; for example, the shear strength of the material you will be able to calculate the energy consumed and the rate of work. Rate of external work is also calculated based on the area at which this external force is applied. So you see there are two components; one there is internal consumption or the internal energy consumption and the external energy consumption. So, you have these two components and you equate the two and then you will be able to get a equation or a relation between the pressure or the force that has to be applied to do that kind of deformation and this pressure or deformation that whatever you calculate would be the upper bound that is the maximum that you will need to do to do that kind of deformation.

So, we will start with simple example and this example we will also introduce you to the concept of hodographs; like remember in the last lecture we said that you also need hodographs which are nothing, but velocity diagrams. So, this simple example will give you an idea on how to generate this hodographs.

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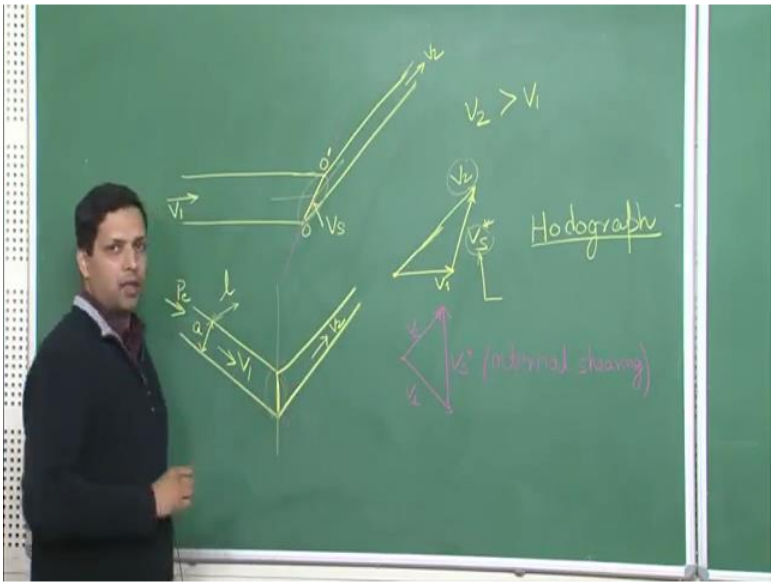

So, let us look at this simple deformation where a metal is coming in like this and then it gets deformed like this. So, you see first thing that you notice is that here the metal is much thicker in region; here the metal is much thinner in region.

So, if we say that this is velocity V 1 and this is velocity V 2, so which one should be one higher; obviously, velocity V 2 should be higher. So, you know that V 2 should be greater than V 1, now there is over here we have define through our construction or through our geometry; a sharp region where this deformation is taking place. This is the plane along which deformation takes place or where the shearing internal shearing is taking place. Now to draw the hodograph, what we will do; we will assume origin, so with respect to that origin; how was the material initially moving, it was moving like this V 1; this is our velocity V 1.

Now with respect it is the same old material, now this material in the final step is moving at a velocity V 2. Now remember since this as become thinner, the velocity V 2 is higher and it has to be parallel to this and also remember V 1 is along this, so it has to be parallel to this. So, we have the direction of V 1, we have the direction of V 2; now this is the initial velocity, this is the final velocity and there is this change in this velocity is occurring along this line and if you look at this; this will give you not only the magnitude also of the V 2, but you will also get the magnitude of this particular velocity which is call V s; which is where the shearing is taking place. So, you see have define as a sharp planed along which shearing is taking place, so this is the shearing which is causing. So, let say this material was moving here and suddenly when it crossing this line, it is start to move in this direction.

So, this particular plane is giving at some additional velocity or it is changing the velocity, so there must be vectorial velocity that is getting added to this initial velocity and which is represented by this, now what will be the orientation of this defector; this will be parallel to this line. So, let us say this is o, o prime, so this velocity V s is parallel to this planed o; o prime. So this is our V 1, this is our V 2 and this is V s where the shearing is taking place; now you see just because we know the orientation V 2, we know the orientation V s; we are now in the position to calculate the magnitude of V 2 and V s as long as we know what is V 1, so that is the advantage of hodographs.

So it is nothing very tricky or complicated; it is simple velocity diagrams. We have to look at the various components; how they are moving and then you will be able to get not only the magnitude, but also the velocity of the internal flow field that you have assumed. Now here this particular internal field was assumed by us that is where the deformation is taking place. We have not; other what could have been other thing possible over here, we could have assumed that this is a slow deformation taking place. We will not able to draw that the hodograph for that for now, but for the time being I am just trying to give an idea that there are other methods or other models that you could have constructed for this deformation.

Here the model that we have taken is that the deformation is taking place sharply along this line. We will get more complicated examples as you move on, but for now you should be able to appreciate the fact that this particular is also a model that has been generated by us. Remember one of the points or first point of the methodology for upper bound analysis was that you assume a deformation field and you calculate the energy rate for that. So, this is what we are doing; we are assuming the internal fielded of deformation or the internal field where collapse of the material is taking place and this is usually denoted by star.

So star denote the internal shearing velocity. So using this hodographs we are now in a position to get magnitude of V s star and as well as magnitude of V 2. Now let us say that this was a little bit different. So again there is no complication; just take a origin, so let us say I take the origin; this is my velocity V 1 and then with respect to this origin there is this velocity V 2 and again what we are assuming, that this is the plane along which deformation or collapse of the material is taking place.

Therefore the vector for the shearing velocity has to be drawn parallel to this line. So, again I will call it o; o prime, so I will draw it parallel which is in this case is vertical and so far I did not know the magnitude V 2, but now once these two vectors intersect; I know this the magnitude for V 2, I know this is the magnitude for V s which is again V s star. So, you see I have drawn two different cases and I have been able to draw the hodograph for both of them. So, that is the way to go about drawing the hodograph and like I said V s represents internal sharing and it is represented by the star. Now let us take this forward, now we have drawn the hodographs; how do we get to the calculating pressure or the forces from this particular deformation model that we have constructed.

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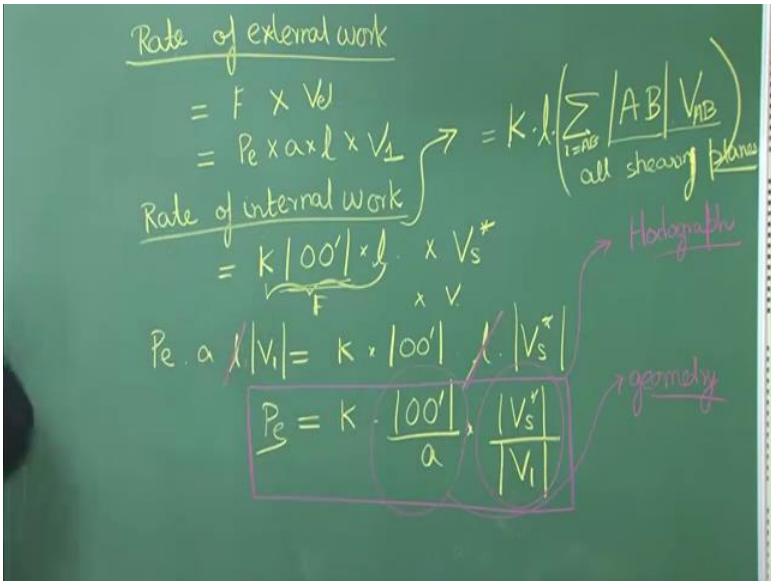

So, the first thing is we have to obtain a rate of external work; this rate is given by what usually the rate of the external work is given by force into velocity and what force and what velocity we are talking about. So let us say when this is moving; you must have applied some pressure or force. So, this the pressure or force we are talking about and let us look at in terms of pressure instated of force for the time being. So, this will become your P, so let us say this is the P that you are applying or to be to differentiate it; let us make it P e. So, let say let assume that is exclusion, so this is P e and when you are applying this pressure, the material is moving at a velocity V 1.

So, we know e, we know V 1 and if we know the cross section with which is a and there will be another dimension l. We assuming it is a plane strain condition. So, we will assume that this model or this geometry remains constant in the third dimension. So, in the third dimension; let us say there is the sum length l. So, now, force will be what P e into l times a that is force and this is our velocity V 1, this is the rate of the external work.

Now we need rate of; where is the internal work being done over here, the internal work is getting done in this region; you remember we said this, the internal shearing. So, this is their actual work is the inside the martial is taking place if this work was not taking place then actually no force or no energy would be consumed, whatever energy getting consumed is because internally this is the region where deformation is taking place; this is over here it is like this, over here it is like this. So, now this again we have to calculate the rate of internal work and we can again separated into terms like force into velocity which can be disassociated into shear strength times the area into which it is working.

So, you see this is the shear strength of material k, o; o prime is one dimension one is another dimension. So, this is shear strength times and area, so this force into velocity; so the units are same, so we are again talking in terms of force into velocity. So, it is indeed rate of work. So, we are calculating force times velocity or we are calculating rate of internal work. Now from what we have already discussed; if you want to calculate the pressure; what we need to do pressure or the force, what we need to do is equate rate of external work to rate of internal work, so let us put this two parts in one place.

So what do we see here; first thing is we notice is that l gets canceled and this will be true no matter what deformation field you assume, but you still make one assumption which is that in the third direction or in the perpendicular direction; the geometry remains constant that is in the third direction there is no change in the geometry. So, as long if you take that this l will always get canceled and because this will be true in general, we will not be showing this term in the future examples; in the next few example that we will consider, we will not take this one term; we will only take P e, a, V 1 over here and over here all the dimensions that we see on the plane of the board. The third direction we know will exit, but it will get automatically canceled out when we equate rate of internal work to rate of external work.

So, now we have over here like this and this to ensure that we talking over magnitude, we can put the models like this and we can say P e is equal to k times. So, this equating the rate of internal work to rate of external work, we are able to get a relation for P e; the pressure exclusion and it comes out like this, so it is terms of the shearing strength of the material. So, overall form of the equation will remain same; no matter what material you take, it will only give the only thing, you will have to change when you change material is the shear strength of the material and this term; where does this term; where we will obtain the value for this term, you can obtain this from the geometry. The geometry of the problem would allow you to get value for these terms, where would you get value for this V s star by V 1; we will get it through hodographs. So, let us come back to this one over here you can see you can get a relation between o prime and over here this is your a.

So, the relation between o prime and a; this you will obtain from the geometry of the problem, over here also o prime and a you can get the relation what should be the ratio between o prime and a; you can get that. On the other hand what will be the ratio between V s star and V 1 that you will get from this velocity diagram? So, here all these angles would; obviously, we known to you once you know; once you drawn it using this geometry. So, from this geometry you get this and from here you know all the angles and if you know the angles you will be able to get relation between V s star and V 1. Similarly over here, we will able to get relation between V s star and V 1 and therefore, you will again be able to calculate the ratio of V s star and v 1. So, you see that we are indeed able to calculate P e once; we know the geometry and we have drawn the hodographs related to it; so this is one example.

And to make it a little bit more complicated, we will get to that example, but before that let me tell you that if you had over here we are assuming that deformation or the internal shearing is taking place at only one plane; you remember this is only one plane. Now let us say that more than one part; one plane was involved in internal shearing then what will change over here, what will change over here is that you will have to calculate o o prime times this velocity on that particular plane for each and every plane like that or in other words this will become and since we have kept l over here I will keep using l for now.

So, will take l as a common factor outside and inside you will have A B which is the length and V A B; meaning the velocity on that particular plane. So, you will take summation of all such planes, the length of this plane; the length meaning that prerace that you are seeing on the geometry and shearing velocity on that plane. So these will be your terms, this is again your shear strength this is term that will ignore from now on, but now we are taking it as a common factor and there will be this summation across all planes.

So, this is the simple way to solve this particular kind of problem or to get the upper bound analysis. Now we will get to a little bit more complicated problem, so let me take another example which is of exclusion. By solving this problem, I will also take you through some important guidelines that you must follow to be able to solve this; first to get the hodographs and then eventually to solve the pressure values or the force values.

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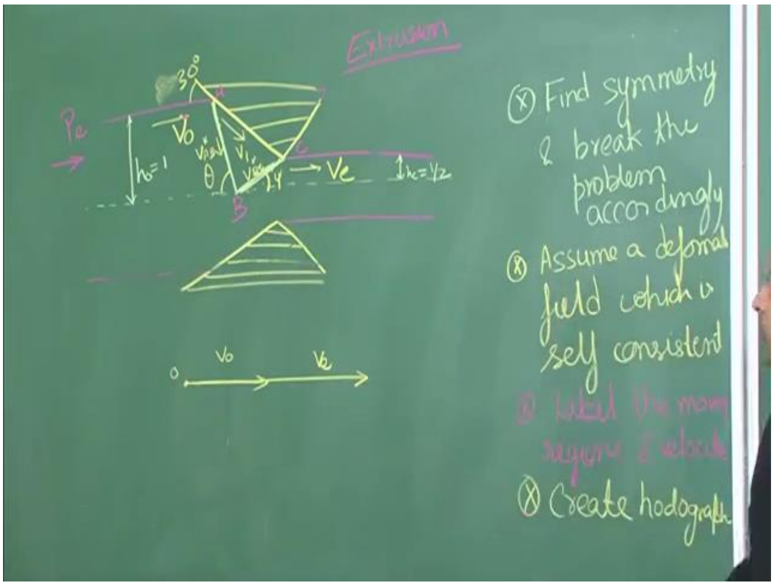

So, let say this is our die and let us say this is the material that is being inserted into this die, so this process is exclusion. So, again I will show pressure as P e; now the first thing whenever you are trying to solve problem like this is; to find symmetry and break the problem accordingly in this context what it means is; this is symmetric around which plane, it is symmetry around this plane. So, what we will do is we will only consider one half; now whatever happens in the one half will also be occurring in the other half.

So, all our solution would be valid even if we calculate only for one half. Now we cannot reduce this problem to any more simple geometry, we cannot have reduce it; we do not see any more symmetry over here, so we will continue with this. Now let us say that it is given that h naught over here is 1 and this h e is equal to 1 by 2. Now next step is to assume some deformation field which is self consistence. So what we can see here is that what is happening to the material, we can see the material is moving like this, it is then moving like this and then it is moving like this. Now this is the flow of the material and there must be a break in the velocity over here which means whenever there is a break, there must be some shearing going on.

So, there is some shearing going on along this some were around this region. Similarly there is must be some shearing going on around this region. Now we can assume that there is a deformation field like this, now this is totally user define deformation field; you see the material is moving like this so far that is good but how do we draw this discontinuity line or the plane we are internal shearing take place is depended upon the user and me as a user have defined that this is how the deformation is taking place or this is where the internal shearing is taking place.

There are even more complicated ways to have this kind of shearing field like I said it is user defined. So, you as a user can come up with something more complicated and now given that we have computational ability, we will be able to solve any of this, but what is the use of it; we will also come to that, but for now let us say that this is the user defined deformation field and this is one of the discontinuity line or where the shearing is taking place this is another discontinuity line where the shearing is taking place.

The third aspect is to define or label the moving regions and velocities. So, I have already labeled the moving regions, these are the moving regions and now I will also level points over here. So, this is let say point A and I will call this as point B and I will call this as point C. Now there are some geometries dependent upon the deformation field that I have define and there are some geometry depended upon die that has give to me. So, I will assume that the die geometry is already given to me for example, this is given to me as theta equal to 30 degree, I have already defined that this is h naught and this is h exit; which is half of h naught.

So, these are the geometries which are already provided to me and I have these angles rather than that; we also have these angles. So, let us say this is theta; I will not call this as theta, I will just put is as 30 degrees and I will call this as psi. So, now you see even for this simple deformation field that I have defined here, I could have defined it in million different ways, I could have taken this point over here, any were along this and angle theta and psi would change. Although the velocity or the movement of the particles or movement of the material is side remains the same as we have define earlier, but what is the discontinuity or what is the shearing velocity that part is continually changing as we select different points at which these two shearing angles converge.

So for now let us say these are some particular angles; theta and psi; some given theta psi values and the material is flowing like this and there is a discontinuity like this and the next step; once we have label the moving regions and the velocities. So, one thing I have not yet done which is labeling the velocity. So, this is V naught; velocity at which it is going in and this is velocity exit V e, this is V 1 so the material inside flowing here as V 1 and since there is a shearing going on. So, there must be some velocity over here also, we will call that V A B and since it is internal shearing, I will denote it as star. Again there is a shearing over here along this plane and I will denoted by V star B C. So, these are the velocities that are over there.

So, now I will label the moving regions and velocities, next step for me is to create the hodograph. Again for creating the hodograph, you remember what we have to do; we have to take a origin. So, this is our origin; now from the origin the first velocity that we know of is V naught. So, there is some velocity V naught; right now we do not know the; we know the magnitude only of the v note. So, let us say this magnitude is known to you, but we do not know the magnitude of for example, V e, but we know that it must be larger than V naught. So, this is V naught, this is V e and so we will come back to this particular part of this lecture or we will keep solving it in the next lecture. So, we will come back on this in the next class.

Thanks.