Fundamentals of Materials Processing (Part- II)

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Lecture - 16
Upper-Bound Analysis

We will continue where we left.

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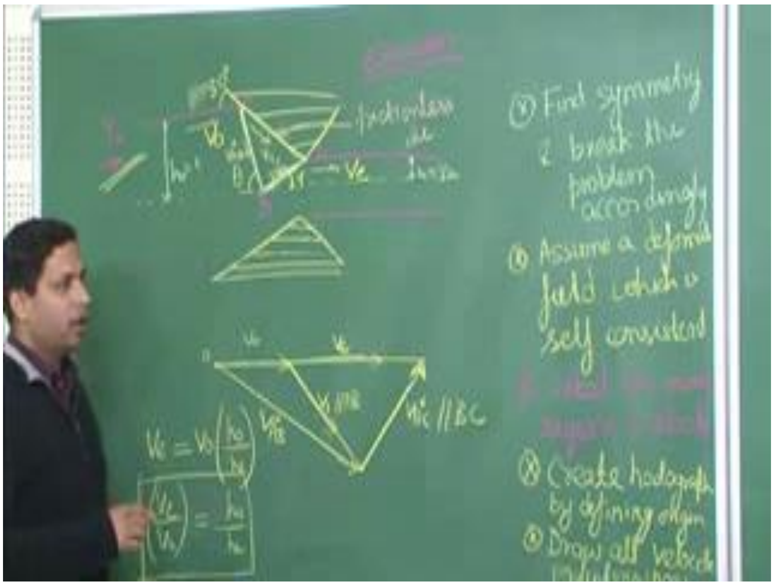

So, we saw that we next step is to create hodograph which, let me be explicit by first defining origin. So, I have defined origin with respect to that I will draw the velocities first of the moving elements V naught V 1 V e. So, how is V 1 over here? V 1 is like this. Then draw all the velocities including the shearing velocities. So, what we have done here is V 1 which was the moving element V naught which is the starting element and this is V e. Where is the shearing taking place? Shearing is taking place over here live V AB.

Now, V AB is connecting V naught and V 1. So, you must draw V AB from this origin at this angle whatever is this angle now these two must interested to over here. So, this gives me now magnitude of V AB magnitude of V 1. Next we have V BC, which is another which is the next shearing velocity or the shearing internal shearing taking place. And this is connecting V 1 to V exit, so V 1 from here you have to connect to V exit. Or this angle is not coming out write let me this has to be parallel to this line BC. And we did not know the magnitude of V, we adjust drew the V along the write orientation. So, we will extend it until they meet and therefore now we also get the magnitude of V e.

So, now we just by knowing the V naught and from the geometry we were able to get the orientation of V 1, V AB, V BC and V e we are now also in a position to get the magnitude of this internal velocities as well as some of these other velocities including the V e. As the cross check if you are given some exact numbers for 30 degrees theta and psi you would expect that V e is equal to V naught times h naught by h e, because if h e is half of h naught then V exit must be twice. So, this ratio h naught by achieve should be the same as V e by V naught. And this part you can cross check once you have drawn this hodograph. You would be able to see that V e over V naught is actually coming out to be h naught by h e.

I am not showing it here with from this particular diagram, you may not be able to see because we do not have the exact values of theta and psi. And more over the drawing that I have drawn here or made here is not is exact, it is just rough sketch. So, although I said that we one should be parallel to V AB, but V 1 is actually parallel to AB, but in the real drawing it may not have come out to be parallel to be AB. Similarly, V BC may not have come parallel to BC.

If you draw it exactly the way that we are suppose to then you would see that this ratio is indeed hell ride, so this will whole true. And that will be cross check that your hodograph is drawn right. Now, once you have the hodograph and you have the geometry you are now what we can do, we can get our pressure the upper limit for this value Pe. How do we do that, we get to this in this part.

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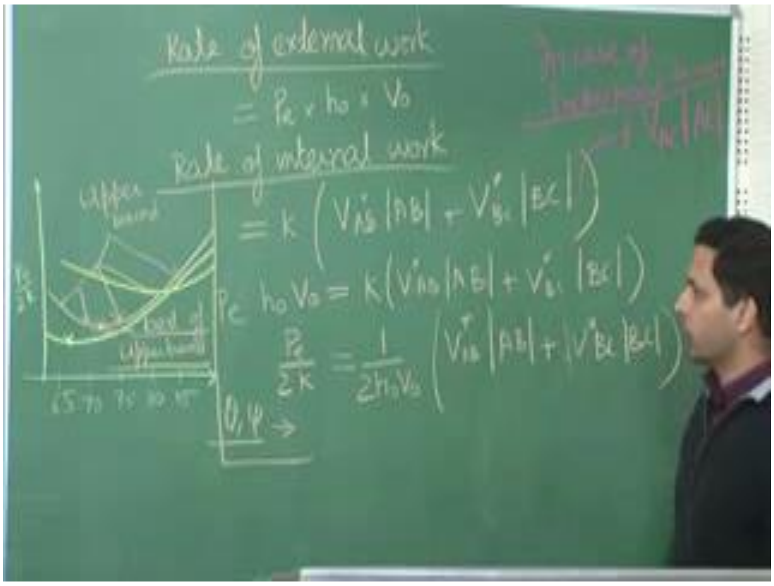

So, we have rate of external work. Again what will be the rate of external work? Simply Pe times h naught and I am not including l which is the third dimension you remember that will get cancelled automatically. So, is let me come back to this board. So, this pressure is acting on this area h naught times the third dimension l that is the force and this is the velocity that is being generated because of this. So, force times velocity that is the rate of external work. Now what we need is rate of internal work. This is remember k which is the shear a strength of the material and we are not writing the third dimension l because we know that it will get cancelled out, but what we need to write or the different planes; what are the different planes where shearing is taking place.

Two of these we know very clearly; this one and this one, what about this one? AC is initially taking place over here. Now if we assume that this is friction less die, which is one of the common assumptions in most of these models, then no shearing is taking place along this line. And the only shearing is taking place are along this and this line. So, assuming that there is friction less die we will have only these two. So, let me write down those two and it will come out too. So, this is this is the summation that we talked about in the previous example we saw we were able to use generalized equation where we use a summation. So, it is the expansion of the summation that across the different shearing planes you have to multiply the velocity times the length of the plane.

Now, we will equate this and this because that is how we get the extrusion pressure the upper limit for the extrusion pressure. So we have, or from here we can say this is the standard format for writing this Pe by 2 k this is saying that now you are making it in effect independent of the material property; you have taken the material property also on this side pressure also on this side so whatever we get over here is only geometry dependent.

Even the velocities that you see over here I mentioned earlier also and let me repeat, but this is only the magnitude of course we are only multiplying the magnitude of the velocity with the magnitude of the length of the plane. And you do this for all the shearing planes. Now, here depending on what particular value of theta and psi that you take over there; so depending on theta and psi you will get different values of Pe over 2 k. As an example in this particular case if you had taken let me schematically draw how the Pe over 2 k might have look like, if you had changed just the angle theta over here. So, let say theta and psi are actually dependent, so if you had change theta from let say 65 degrees to 70 to 75 to 80 to 85 what you would have seen is that Pe over 2 k would have varied something like this. So, this p over 2 k you can now clearly see is dependent on your definition of the deformation field. There is another thing or another important point hidden in this particular plot and it is this point. So, at this particular point what you see is that you get the lowest value of Pe over 2 k.

Now you remember all these are upper bound values. Upper bound values mean that you are calculating the maximum possible or whatever value you get is the maximum that you may need. So, even this is the maximum of the pressure that will be required for this kind of deformation. This is in effect the best upper bound value, because you are getting the lowest of the upper bound and that will give your smaller window of the range of pressure of forces required. So, this is best of upper bound values. So, we are able to deduce or understand two things from here, so let me write it down over here.

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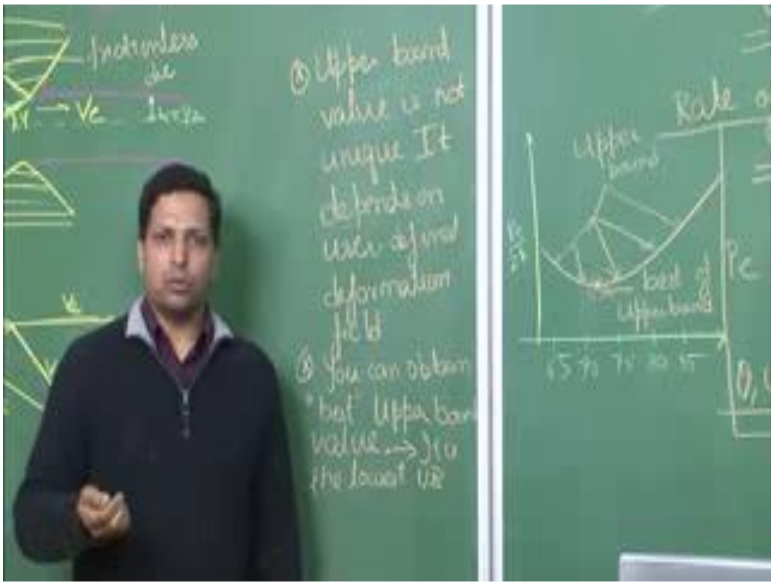

So, these are the guidelines, so if you follow these guidelines you would be able to create the hodograph and hence solve the problem. What we learnt from this one is that upper bound value is not unique. Unlike in the uniform energy method you get a unique value, only one particular geometry will give you one particular value. But for the upper bound the value is not unique, it depends on user defined deformation field. Another important point that we understand is, that from this range of values you can obtain best upper yield value; I am writing it in quotes, best in the sense that it gives you the lowest value of the upper bound meaning you do not need to go any higher than that. And what is that best value, it is the lowest one.

So, for example, let say that your value one particular set of deformation filed tells you that your upper bound is 7 mega Pascal other one says 7.5, even third one says 8, then what is the best upper bound value. You know that all of them have been done in a way that all of them are actually upper bound, so why not take the lowest one because that will give you a more or a much better range, because let say the lower bound or our uniform energy method tells you that your lower bound is 6.5. So now, you know that your actual pressure requirement will be between 6.5 mega Pascal to 7 mega Pascal.

So, this will give you a small window, and therefore you will be able to operate between these regions why would you like to use a much higher pressure then what is required. So, that is what we need when we say you can obtain best upper bound value which is the lowest of all the range. And this brings us to another point that I raised earlier, was that this is one just one particular kind of deformation field. You can have lot more deformation field, but why should you go about all those deformation filed different kinds of deformation filed. It is because you want to get the best value of upper bound. Let say this is one range of upper bound, another one this you upper bound like this. So, you know that this is even lower than this, so this is even better and so on.

Let say if still the other one gives you something like this, so this is higher you are not interested in this, you will just keep on looking at the lower and lowest. And lowest value the lowest of these upper bound values are what you are interested in. And that is why you can or you need to keep coming up with deformation field which will give you the lowest value lower and lower value of upper bound.

Now having said that; I said that there are other deformation fields that can be generated over here, although the material is moving like this; so what are some of those other different or different deformation field that can be drawn for this?

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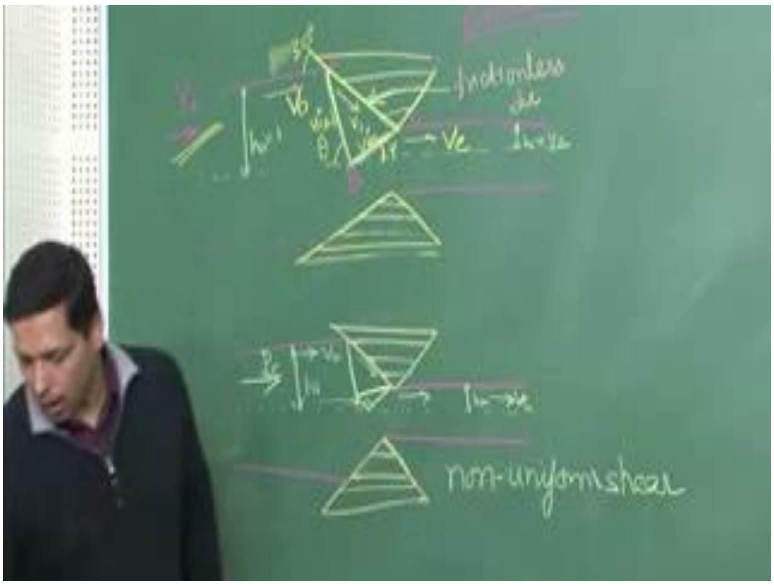

Now, let us forget everything and try to draw still another deformation field for this. So, I will draw similar kind of die. So, let say everything is similar we have a die like this. And we will also assume that the material is flowing; the same width of materials flowing through this. So, again we can still use that guideline that is to reduce your problem looking at the symmetry. Again we know that this is the symmetry plane whatever happens on the upper side has to happen on the lower side. So, why not talk about only the upper part. Now here we will move on to generating at deformation field which is a little bit different and in fact even call it even more complicated then what we have seen over here.

So, you know that the material is flowing like this and like this. So, we assume that when this material flows from here to here at all points there are one continuous or one uniform kind of shearing velocity. But that may not be always true you may have something like this, what does this mean? This means is that material flowing up to this point they get a shearing velocity like this, but materials below this point when they are moving through this so the initial is like this to final is like this, but the shearing velocity at this point is different from here. So, this is called non-uniform shear.

And you can follow the same processes for this. You can generate hodograph and from the hodograph you can again get the equate rate of external work to rate of internal work and get relation for Pe, given write this is h naught this is h e and you will have V naught and V e. And now since you have defined two different slip lines or two different shear zones so you will have to have shear different shear velocities over here and there will also be another shear velocity over here. So, those parameters will have to be put in to that equation. And because of the hodograph and the geometry that you know the geometry you will be able to define all those and hence get the magnitude and direction for all those and eventually you will be able to get Pe in terms of k.

Another aspect that we need to consider here is that we said that this part is frictional less; what if this was not friction less, what will be the difference if our in our problem this part was not friction less. Actually, the difference will not be by large. All we will have to do is that we will have to add another shearing plane because this will become equivalent to a shearing plane, this is not internal shearing but there is still some shearing taking place over there and over here we will add V AC times length of AC.

So, there will be three terms inside this bracket, in case of friction present between die and work. So, when there is friction present between the die and the work piece which is along that AC what it will change is that there will be another term over here, because another internal work not in necessarily internal but another work is getting done over here and that is with velocity V AC along plane AC. So, that is another thing that you should be aware off.

Now let us move on to still another example. This one will give us some idea also about the indentation relation of hardness to yield strength. You must have seen that hardness values are given as three times the yield strength or yield strength is given as one third of the hardness value. So, where does that come from, you would be able to see from this example.

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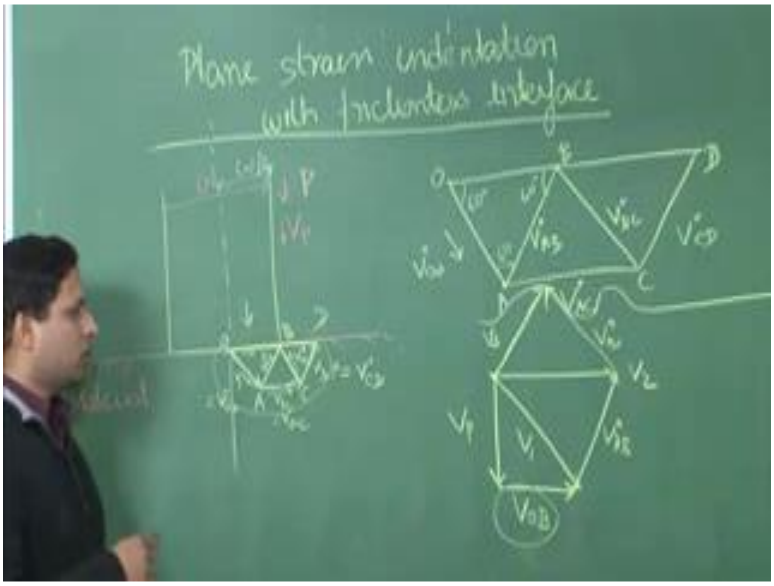

What is the geometry, this geometry is called Plane Stream Indentation with Friction Less Discontinuity. So, what is the geometry here, let us draw the geometry. Let say this is the indenter, this indenter does not have any geometry at the end of it, it is a flat indenter. And let say there is a surface over here, so this is surface over here. And let say this width is w, and there is a pressure being applied P and this was moving with velocity V P. So, this indenter is moving inside this material, this is your material or work piece.

So very similar to what you do in hardness test, where you have an intender which punches inside the material only that we do not have any particular geometry for this indenter. For example, in because you have this diamond kind of shape or in Brinell you have that is spherical shape, but here we are taking very flat shape for the indenter and it is punching inside the material. So, first thing guide according to the guideline what is the first thing that we need to do, we need to reduce now a problem with respect to geometry. So, again here what we see the geometry it is equivalent or symmetric around this central plane. So, let us draw centre line over here, so this becomes w by 2.

Now, whatever is happening on this side is exactly what is happening on the other side, so we can concentrate only on one side of the problem. Now what is happening? Once this is moving in this direction, so how is the material moving, first; then second step is to draw the velocity of the various moving parts and level them. So, let say the material moving like this then moving like this and then moving outline this. This is self consistent; meaning material is moving and coming out that is what we would expect. You cannot draw it as if the material is moving in and going some very inside it. That is not possible, that is what we meant then we said it has to be self consistent.

Now if these are the velocities of the moving particles or the moving material then there is a discontinuity over here, so there must be well discontinuity like this, there must be a discontinuity like this. So, now let us label them. Let us put this as origin, I will put this A, B, C and let us put this as D. Now this is one triangle, this is another triangle and so on. And this is of V 1, V 2, V 3, and because there is shearing taking place over here so this is V AB, there is again shearing taking place over here so this is V BC.

So, now we have not only levelled it as the guidelines is we have also drawn different velocities on to this. So, we have the velocity of the moving parts and also we are the velocity of the shearing parts. What is happening at this inter phase? Outside this if you remember if you go back and look at the lecture notes the material is suppose to be rigid. So, there is rigid material here and the material is moving here. So, the there is shearing also taking place at this place, then shearing to also taking place along this plane, there is shearing also taking place along this plane. So, we have shearing here, here, here, and we have obviously had shearing because these are the internal shearing taking place then they have velocity discontinuity. So, we are representing these two as star, but these are also now as good as star. So, this V 1 is equal to V star OA, V 2 is equal to V star AC, V 3 is equal to V star CD.

Now next what do we have to see over here, let me take this part and draw it in a little bit larger region. So this is the velocity, not the velocity diagram but the geometry of the problem and this is the assumed deformation field. Now, what are the different variables for this particular deformation field? Yes remember I said that this is not only one deformation field there can be even other kind of deformation field, the only restriction or constant is that it has to be self consistent. So, even in self consistent model you can come up with various other deformation fields. Now let say we are selected this deformation field, even in this deformation field you can have various variations in the geometry. For example, you can have this as isosceles triangle or you can have these all these has different angles. Or for the simplest one you can take all these as 60 degrees. So, this is the equilateral triangle. This one is equilateral, this is also equilateral triangle and this is also equilateral triangle.

And to simplify our problem in this particular case will go ahead with the model where we will say that all these are 60 degrees and we have equilateral triangle. Now, what is the other implication when we say that it is equilateral triangle? So, we had V star OA V star AC; what this means is that all the velocities must be at 60 degree angle to each other. Let us first draw origin, from the origin the material is moving like this V P, and from V P there is a change in the velocity when we go from V P to V 1 therefore there must be our discontinuity velocity which we will represent as V OB. Now we have this is V P this represents our V 1; this is at 60 degrees and at 60 degree to V 1 we will have V 2 and at even 60 degrees to this we will have V 3. Connecting V 1 and V 2 we have this V star AB which is also at 60 degrees. So, this is V star AB and connecting V 2 and V 3 we have V star BC.

So, we have this velocity diagram over here. Now once we have the velocity diagram what can you say about the magnitude of these velocities, what can you say about the magnitude of the length OA, OB, OC? I will leave you with this problem to think about it and welcome back to this particular problem again when we meet in the next class. So what I want you to think about in the mean time is first to understand what is the relation between the different lengths of the plane that we are talking about; different velocities what is the relation, how are the related to V P; and if there is any amount of work being done along the plane OB.

So, think about these three problems and if you are able to solve it you go head and also try to calculate what will be the P in terms of k. If you are able to get that up to that point then you are going ahead or you are moving in the right direction and you are absolutely in face with the class. So, we will meet in the next class until then I hope that you try or you give a look at this problem.

Thank you.