Fundamentals of Materials Processing (Part- II)
Prof. Shashank Shekhar and Prof. Anshu Gaur
Department of Materials Science and Engineering
Indian Institute of Technology, Kanpur

Lecture – 17
Plane Strain Indentation

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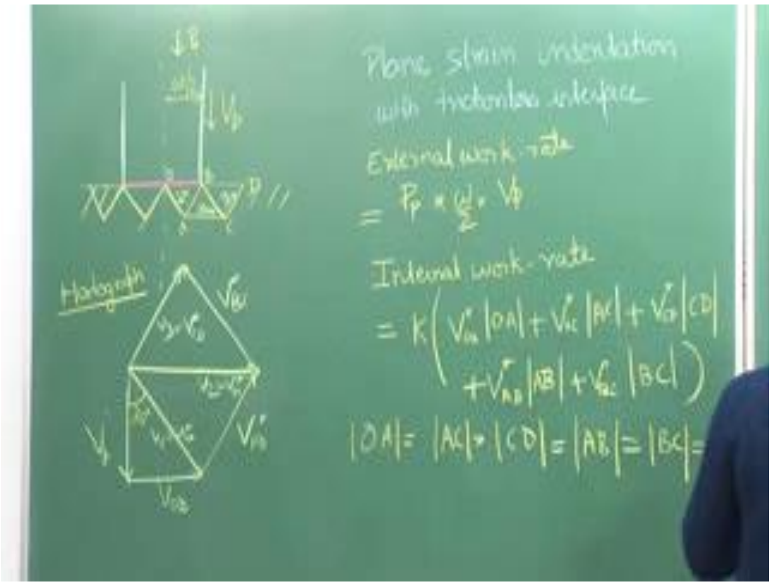

So, we will continue our discussion with where we left in the previous lecture. This is the condition of Plane Strain Indentation and there was additional context to it or additional condition constant to it which is with frictionless interface, where is the interface in this? When we look at this is our indenter, so let me emphasize the indenter with this blue colour, so this blue colour is our indenter. At this part of the material; this part of the drawing represents material and this indenter is coming down, if you remember and this is the field that we have assumed there is a additional one more triangle over here and we broke it into similar we use the symmetry to break down the problem.

So, now the problem is reduced to only this half, so we are concentrating only on this part and for this part, we got up to this point; this is the hodograph for the above condition, for the above assumed field of deformation. So, as you can see over here we have assumed our equilateral triangle, so all the sides are equal and the v 1, v 2, v 3 are at 60 degrees; which is what it comes to, so there are angle between these are 60 degrees and these our internal shear place, so this is also at 60 degrees making this one also equilateral triangle and if we know Vp, we can relate all the other velocities that we see over here with this velocity Vp.

So, from the geometry we can construct hodograph and from hodograph we can relate our various velocities to the Vp that as we saw in the previous lecture.

Now we will continue with this, so first question I have here is what do we mean by friction less interface, frictionless interface means this is our interface, it means there is no friction at this point. So, if you see keep indenting the material if it needs to flow laterally, it can flow laterally without any work being done, so that is what the frictionless interface over here means. Now what is the implication of that, the implication is now you see that there is a Vp and this with this is the velocity of the indenter which leads to velocity of the material V1 over here.

So, there is a discontinuity in velocity which leads to this O B; all though it is not a internal shear, but there is some this velocity discontinuity which is shown over here V; O B, but since this is frictionless therefore, no work is being done and therefore, I have not denoted it by the star; star are the once you remember their work is being done. So, these are the internal shear by default we have written it as star and these are the v 1, v 2, v 3 thermal movement velocity of the material, so the material is moving like this and since the outside is rigid so there must be some shearing taking place there or work being getting done over there.

So, in effect we have 5 different planes at which work is being done 1, 2, 3, 4, 5; these are the 5 different planes or regions where work is being done. So, next is again to write relation for external work, write relation for internal work and then find equation between them. So let us start with the simpler one which is external work, so let us say that the pressure here we assumed we have given it P p, is the pressure being applied. So, the external work rate will be equal to Pp times what will be the area; area will be only we have considered only this part W by 2.

So that is one dimension, what will be the other dimension. You remember we have stopped taking the other dimension assuming that the geometry remains constant throughout; it is a plane strain condition. So, the third dimension we can ignore and velocity; Vp. So, this was the external work rate; what about the internal work rate, internal work rate if you remember we have to summit over all the five place and K is the shear, stress would be the common factor shear strength and we have to summit over all these three, all these five different planes. So, it will be K times, so this gives us the relation for internal work rate.

Now what do we need to do, we need to relate all the velocities that we have over here with the velocity that we know which is Vp. Similarly relate all the dimensions OA with the dimension that we know, which is W by 2 and given our geometry, you would see that it will be very very straight forward. For example, OA, AC, CD what are these; these are these dimensions, now this is the equilateral triangle. So, all the three lengths are equal therefore, we can say that OA equal to AC equal to CD is equal to AB is equal to BC which is equal to how much this will be equal to this length, this length we have seen is equal to W by 2, so this comes out to W by 2.

Now what about the velocities, so for velocities we will go to this plot, again we see that the magnitude of all the velocities remain same why because this is again equilateral triangle. In equilateral triangle all this length represents the magnitude of the velocity in here and since the length are equal, so magnitude of the velocities are same and this will be related to Vp. So what is the angle over here, now if you look over here there will be this angle would actually come out to be 30 degrees and that would imply that if we take the velocities, the velocities would come out to be; so in terms of Vp it will come out to Vp by cos 30, which is equal to 2 by 2 Vp by root 3.

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So, now we have related all the velocities with the velocity we know which is Vp we have related all the dimensions which are these length to the dimension we know which is w by two and. So, we will insert it over here and we will see that our external work rate which is Pp into W by 2 into Vp is equal to; now this will become k times all these dimensions are equal. So this will become 5 times of whatever we get over here; the dimensions are equal to W by 2 and the velocities are equal to 2; Vp by root 3. So, we can cancel out our common terms and what we are left with is Pp by 2k which is the standard way we write it equal to 5 over root 3.

So, we see even though it look like a very complicated and involved problem, but whatever field we have assumed for it, we get a very simple and straight forward; we did not have to do much manipulation over here and we get a simple relation of Pp by 2K, which is 5 over root 3 or you can write it as 2.89 or remember this is for one particular field, which is this particular field. We could have taken different fields like we said that in the upper bound analysis, you assume a particular kind of deformation field or a collapse field and based on that you calculate the internal work rate and so if you take different kinds of field, you may find different values and we are interested in the optimum; you remember last time without about optima or best value of upper bound which is the lowest.

So, if you vary these things it has been shown that the lowest value that you will get will be equal to 2.57. So, this will be the lowest possible value of all the upper bound analysis that you can do taking different kinds of deformation field and this will be an effect your optimum or the best upper bound value best and optimum, so that is not all.

Now, that we have this P over 2K equal to 2.57 and there is another important or interesting information hidden over here. You remember I said that this indentation is very similar to what we do in hardness. So, in hardness also you are indenting a punch and based on the resistance, the material shows from that you measure hardness and here also you are punching the indenter or the indent that is coming into the material and based on that you get value of P over 2K.

So, this gives you a relation P will be; P can be related to the hardness K can be related to the material strength. So, this will give you relation between hardness and the yield strength, so hard is value that you obtain versus the yield strength of the material; let us see how. So what we will do here is, we will use one muses criterion and invoke k equal to Y over root 3; remember this is something that we did it at the very beginning of this course. So, you put it over here and you get; that is Y; Y is remember yield strength of the material, what is P; pressure required for indentation.

In many hardness measurements you directly use the pressure value for example in weaker hardness; you directly use the hardness value or the pressure value to quantify the hardness of the material So, this P can now be written as H v is approximately equal to 3Y and you may remember from your mechanical metallurgy course, this is our relation that we often use. So, your yield strength and hardness that you measure of the material are correlated, hardness is equal to 3Y or yield strength is one third of hardness for example, let us now that we have put this equation why not quickly do a simple example.

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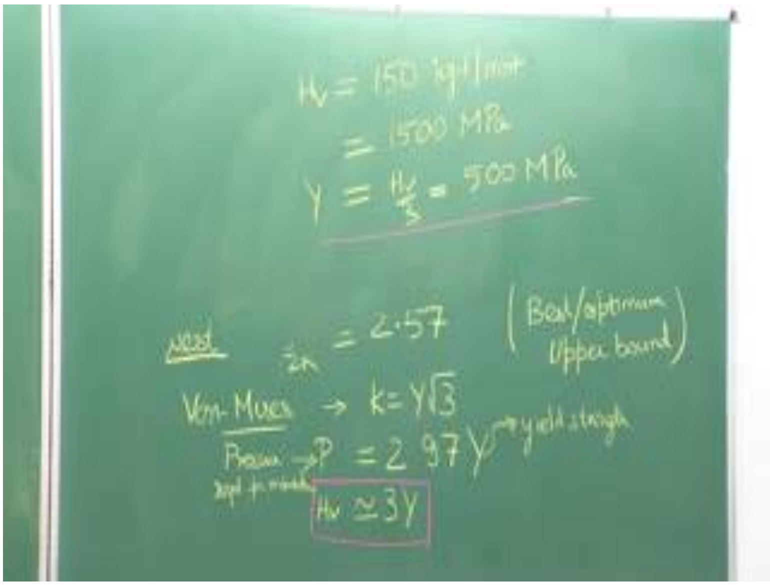

So, let us say it is given that harness value is equal to 150; usually hard hardness will be as number what there is a unit attached with it which usually is kg 4 per millilitre square. Now if you convert it to mega Pascal, you will see that it comes out to 1500 mega Pascal, so hardness has units of stress because it is pressure, so stress and pressure has similar units and it is usually when you write it in number; simple number it has the associated or implicit unit of kg force per millilitre square.

So, if you have hardness value given as 150; it means it is equivalent to 1500 mega Pascal and therefore, yield strength will be Hv by 3 equal to 500 mega Pascal. So, this is where you have a strong application of this relation; Hv equal to 3Y; if you know just the hardness value, which is a very simple way to make the measurement, you can extrapolate what will be the yield strength value of the material or on the other hand if we have just the yield strength value, you can say or expect what would be the hardness for this particular material.

So this is still another importance of this upper bound analysis, now let us get to even another example for use of upper bound analysis, which will be in our quite different set up this will be machining, even in machining you are actually deforming the material. Now if you are doing the deformation then you would want to know the relation or you should be able to apply upper bound analysis or at least this hodograph and we able to relate velocities etcetera, so that is what we will see over here.

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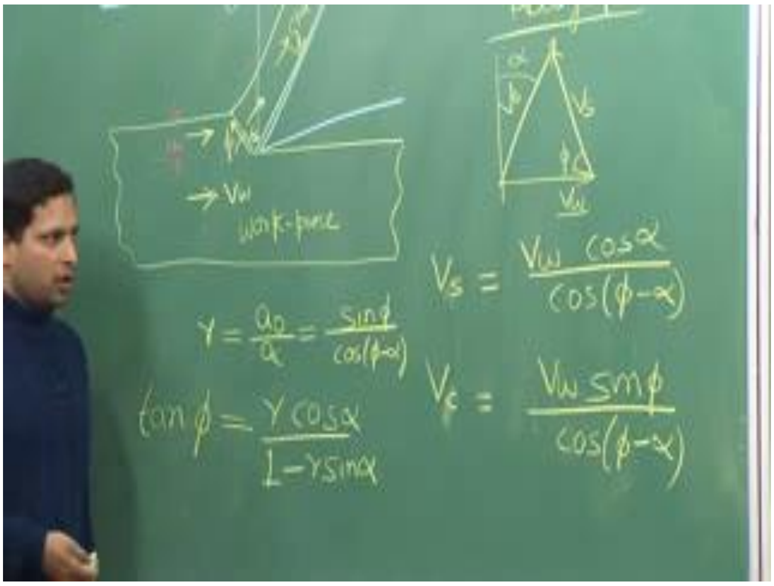

This is the plane strain machining, so this is the work piece and this is chip coming out and Vw represents the velocity at which the work piece is moving and somewhere here, will be your cutting tool which is usually characterized by parameter called rake angle, so this will be your rake angle alpha; so this is the set up showing machining. Over here what we are doing is taking of some amount of material. So this is the material that has been taken off, it is represented usually by A naught also called uncut chip thickness.

So, before cutting what is the amount that is that thickness that is being removed and after being removed what is the thickness of the chip because this is what is going into this remember this thickness is not same as this one, although the drawing did not come out as nicely as should have. If I draw it properly then these two are independent, so let me write over here there will be some thickness for this which will be called Ac and you would see that the construction is somewhat similar to the one that we drew at the very beginning of upper bound analysis and if there is a velocity.

So, this is the velocity of the uncut chip thickness and since the thickness here is different so the velocity of the chip would also be different, so let us call it Vc. So, a material is moving if i have to the first thing that I need to do is look for symmetry and as you can see here there is no nothing and symmetry here that can help me reduce my problem, so that part is gone. Second is to denote the moving parts, so here are the moving parts; this is moving here and when it crosses this line, it starts to move here. So, there must be a discontinuity here which is your shear is; where the internal shearing is taking place. So, this will be our let us call it Vs that is shearing velocity and there will be an implicit angle over here let us represent it by phi.

So when we are talking about deformation varying machining, these two angles become very important alpha and phi and another important factor becomes your ratio of chip thickness, the uncut chip thickness to the cut chip thickness which is A naught by Ac. Now let us get to the next step which is to draw the hodograph, at this stage you should be easily able to draw the hodograph for this one because this is example very easy to relate to the first problem that we discussed and if you have to draw, you have to have a origin; the original material is moving in this direction, it has a velocity Vw and let us say that for sake of argument Vc, which is the chip coming out is thinner in dimension then A naught which is usually what you would have that would mean that this will be moving at a much larger velocity; although in a different direction.

So, this will be your Vc and what is connecting Vw and Vc, it is this shearing velocity, so there will be Vs. Now what is known here is Vw and some other geometry parameters for example, what will be orientation of Vs; this is the Vs and this is at angle phi. So, this angle is phi what is the direction of Vc; Vc is at an angle alpha from normal. So, this these are some other facts we note that we know about this and we know all the angles and we know Vw, from here if you were given to calculate Vc and Vs, it should not be very difficult and as an exercise I will leave it to you to calculate Vc and Vs, I will just give you what would be the final form of the values and as you can see I will write it in terms of all the known parameters, which is Vw and phi and alpha.

So, this is actually shear and the second one is and like I said, it should be very straight forward. Now that you know this geometry, overall geometries known alpha and phi are known and we know Vw. So, we are calculating or we have to just represent Vs and Vc in terms of these. Now here if you look at it, we have this angle phi and I said there is another important parameter r, how do we get to know all these, how do we find out about this? In here from a geometry it looks like it is very straight forward, but it is not.

First let me write down what is the relation for phi, actually phi is given in terms of tan. So, tan phi is given by r cos alpha by 1 minus r sin alpha, where r is like what I said a naught by a c and it can also be written in terms of phi and alpha. So, in practice what you do is; you measure a naught and ac, you get r and then you put r in over here with the known alpha angle from there you get phi. Once you have phi alpha and of course, Vw then you can calculate Vs and Vc; however now that we have these step to this point, it is also useful to see how in machining we calculate some other important parameters when we are talking about deformation like strain and strain rate. So, let us get to, calculating the strain values in machining.

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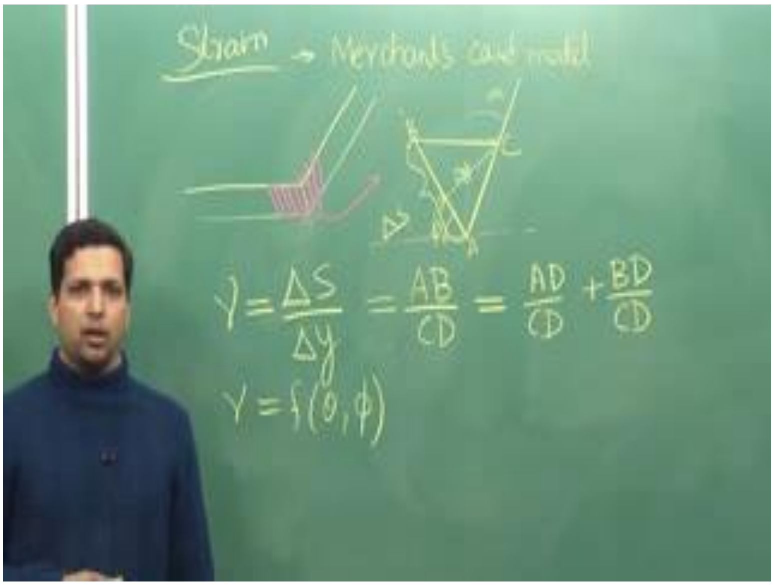

So our next aim is to be able to calculate strain in machining. For this there was a model proposed by merchant and it is called merchants card model. It is a simple or a simplified way to calculate strain, as you can see the way this deformation is taking place; it is not easy to calculate strain.

But if you look at this model it becomes things become much (Refer Time: 25:11) so let us say this is the uncut chip thickness and this is the direction in which the chip is flowing after being cut and over here what we will assume is that this uncut chip thickness is nothing, but stack of cards, so I will draw it as trapeziums actually rhombohedrals. So, these are the trapeziums where we have the cards start which are representing the cards. Now on to this side after deformation what is happening these cards are getting displaced, so you see what this deformation is doing an effect; it is displacing these cards by this amount, so if I had to explore this view it will look like this.

So, this is the displacement, so let us call it delta S; now this delta S is being displaced for a thickness of this size. So, this is our delta y with they have to calculate strain with respect to this. So if you look at the geometry, if you were to write the equation for strain; it will come out to gamma equal to delta S by delta y because this shear strain, so we are looking at the displacement or the for this displacement with respect to the thickness of this rectangle. So this is your delta S, which is the displacement with respect to the thickness which is delta y or if you write it in terms of the A B C D parameters then we can write it as AB by CD or further we can reduce it to AD by CD plus BD by CD.

So, this is the exploded view of the strain that will be taking place in the chip when we are doing the machining and the chip is getting formed. Now from here what are the other parameters again let us put down in the other parameters that we know over here. We know alpha; which is this one, we know phi which is this one. So now once you know all these, can you relate strain only in terms of the known angles phi and alpha? So, that will be an exercise that I will leave you with, that you should try on your own. You know this geometry, you have now this is the horizontal line, these two are horizontal lines and this is the perpendicular on to this and this represents delta y, this represents delta S and these are the points.

So based on this try to calculate gamma in terms of theta and phi, so I will leave you over here; try to do it on your own before you get to the next lecture and in the next lecture, you would see me solving this.

Thank you.