Fundamentals of Materials Processing (Part- II)
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Lecture – 18
Strain calculation Models and Friction

I hope you have given it a little bit of try. It is important that you keep trying even if you are not able to do it in the first time, but if you try you would know what is your short coming in understanding and that is why I always try to leave you with some examples or some problem to solve it on your own. So, always try and if not then you can come back and see the solution at that time you would be able to understand what are the things, where you were getting stuck.

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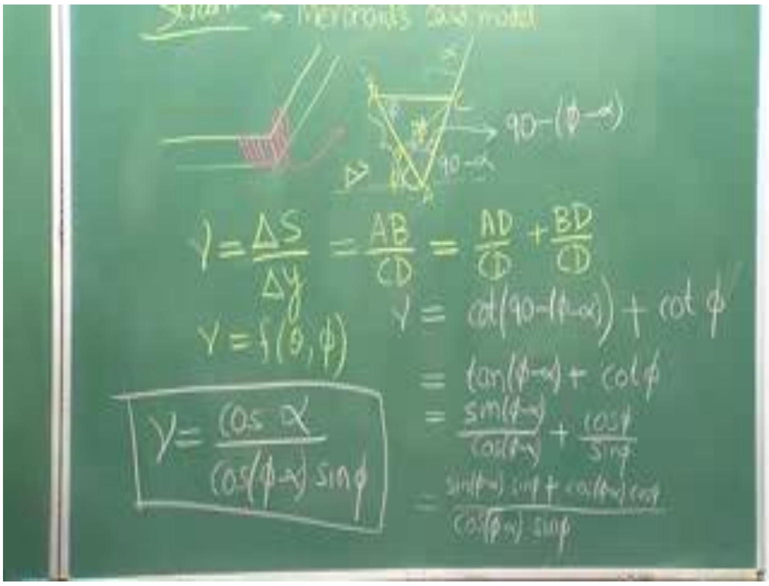

So now, let us come back here we talked about the geometry; so this is phi angle this is alpha angle, now if this is phi and these are two horizontal lines this becomes phi. If this is alpha this becomes 90 minus alpha and if this is phi this becomes 90 minus phi. And if this is 90 minus phi this, of course this is alpha. So, this inside angle is it will becomes 90 minus phi minus alpha this angle. And now let us look in terms of AD by CD, what is AD by CD? This is nothing but cot of this angle. So, this becomes; what is BD by CD? BD by CD which is nothing but cot of this angle phi, so this is your gamma now gamma can be written as since this is 90 minus this we will write it as tan plus cot phi. So, we have achieved our first goal which is to write strain which is the shear strain in terms of only phi and alpha.

Our next aim here would be to simplify it further if it is possible. So, let see if how we can simplify it. First we just make turn it into sin and cos, so tan becomes cos phi by sin phi and if we add it this becomes; and this becomes sin phi minus alpha sin phi plus cos phi minus alpha cos phi. Now do you see these terms does it represent anything, it represents cos a minus b. You remember when we have cos a minus b it becomes cos a cos b plus sin a sin b. So, this is nothing but cos a minus b. What are the a and b? You can take any of them as a and other as b because cos a is an even function. Therefore, you can write phi minus alpha minus phi, and therefore the numerator becomes; so this is the further simplified form of this shear strain. And we can use this to find what is the total amount of strain that is being imposed onto the when that dip when the chip is coming out.

So, in the chip this is the amount of strain that has been imposed when you deform it along this line. So, we have assumed that all the deformation is talking place along this line that is our deform that that is our assumed deformation field. And because of that and assuming and talking this card model, we see that the materials or the cards are getting stacked which means they are getting displaced which is the deformation and that displacement or deformation is given by this shear strain where that is over here.

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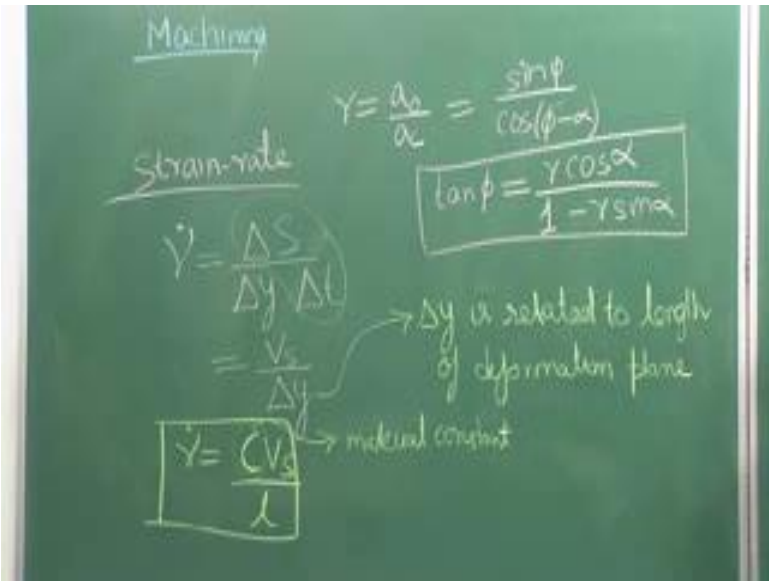

Now, this is just one thing but another factor that we had was r which is a naught by ac. That too is not very difficult to obtain from here, if you look at the geometry over here you would see that there is this is your common tangent common dimension, this is your a naught which is perpendicular to this and this is ac, so if you draw the perpendicular like this over here you draw the perpendicular like this over here you can show very easily that a naught by ac would become; again I am leaving you do to do this on your own. And as I said just use this geometry that I have drawn over here dropping perpendiculars from this point onto these lines. And you know where are the alpha and phi you should be able to get r equal to a naught by ac which is equal to sin phi by cos phi minus alpha. And from this you can manipulate to get tan phi.

So, the equation that we mentioned over here there of tan phi is it comes from the geometry itself. So, with that we have covered the strain part of machining. And we started with using upper bound analysis where we used photograph to relate the velocities for the v shear and v chip and then we moved on to calculating strain. Now another part of the deformation since we are talking about machining another part of deformation that we can easily calculate over here is strain rate.

Strain rate is what this it is the rate of change of strain. So, if you were to write it in terms of delta strain rate can be written as; so this is strain dot which means strain rate this can be written as we know that delta s by delta y is the strain in terms of card model and the time it takes to do the displacement can also be added into this equation. So, this becomes delta s delta y by delta t. So, this is your strain rate.

Now, over here if you look at it delta s by delta t what does this represent, this is the displacement rate of the cards, so the rate at which the cards are getting displaced in this direction. And what is the velocity that represents the rate of displacement in that direction it is equal to v s. You see that strain rate as now got transformed into an equation where we have shear velocity and another infinite decimal element delta y. And experiments have shown that this delta y is related to the length of the deformation plane.

What is that length of the deformation plane, it is this one. Let us give it a name, let us call it l; so this is the length l. And delta y has it has been found, as see for example, if you can you can keep reducing the size of the card and in that case your delta y will keep changing, but at the same time l will also change. And therefore, it has been found that delta y is actually always related to the length of the deformation plane. And therefore, this equation can be written as CV s by l; where c is a material constant that depends on what material you are using it is not a universal constant, it will change on whether you are machining aluminum or you are machining steel or you are machining titanium and so on.

But now you have a formal relation for a strain rate; it is c times v s over l. So, if you know the shear velocity which you can get from your geometry and the work rate or velocity of the work piece and the l, again which you can get from geometry and c which is the material dependent parameter; it will not change as long as your material is same. So, you will be able to calculate this gamma dot which is the strain rate. So, now we have in the machining part we have also covered strain and the strain rate. And in fact, now we have covered most of the important aspects related to the mechanics, the fundamentals of stress strain, and plasticity all related to our metal processing. Another very important aspect of metal processing or any kind of manufacturing process that involved metal processes is friction. Because, friction will change the mechanics very very out aggressively, and therefore we must understand what is the role of friction.

So, our next important topic is friction. So, let us start with first, why is friction important friction. As I said it changes the mechanics it also reduces efficiency because whenever you have friction in the system you will need larger force, larger work that needs to be done.

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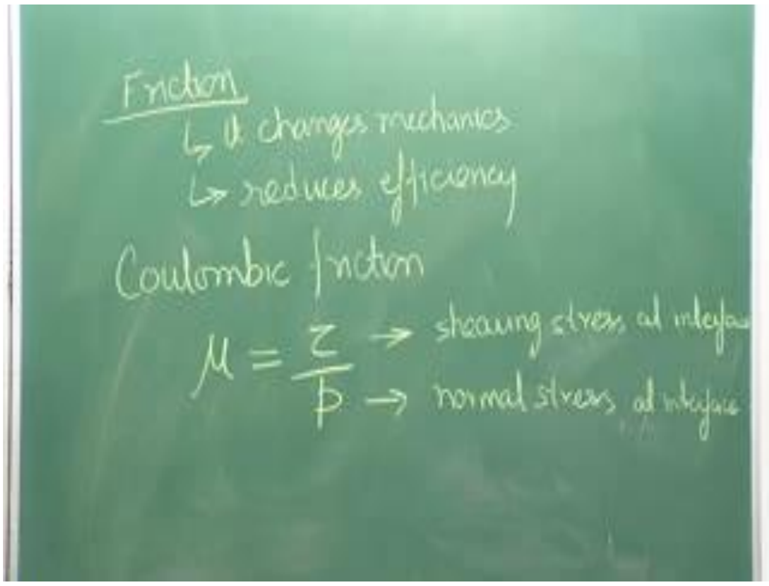

And therefore, friction becomes important. So, this is our last topic for this module which is metal processing. And why is it important? Like I said it changes, it reduces efficiency. These are some of the important factors which lead us to delve or understand friction properly, because if you know understand friction then you can manage it and you will be able to reduce or improve your efficiency and not incur much loss. And that is why it is important to understand friction.

One important way to describe friction is what is called as Coulombic friction. What is coulombic friction? This is actually a factor or a ratio between the shear stress acting on a particular plane divided by normal pressure acting on that particular point. So, this ratio is called tau over p. And whenever we are assuming coulombic friction we assume that mu remains constant throughout the surface. So, tau over here is our shearing stress at interface, p is the normal stress. These frictions would particularly be important when we have two different pieces coming together and trying to impugn onto each other. For example, when you are trying to forge in those kinds of conditions there will be interface between the die and the work piece. And we between this die and the work piece there will be friction which will decide or determine that this friction will determine how much force or energy is required for the deformation.

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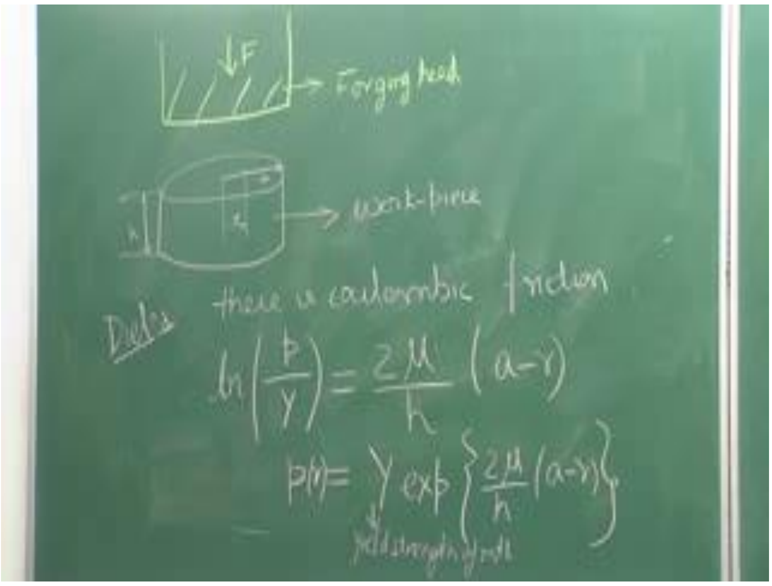

So, we will see for our simple example; let say we have a cylindrical piece like this and this is under some forging head. So, this will come down with some force or pressure and this is a work piece that we want to deform and give some shape to it. So, let say it as some height h, and since it is cylindrical it as a diameter a. Let us assume that there is coulombic, when I say there is coulombic friction I mean that this ratio value mu remains constant throughout the interface. With this assumption you can easily show that and it has been derived in dieter; if you go to dieter the mechanical metallurgy book over there you can see the derivation that this l n p over y; where p is a function of this r. So, we are talking about at some particular distance r which is represented over here.

So, at some distance r the pressure value, the normal per pressure value is given by this relation, but this can be derived and it has been shown in this particular book. And we can write it as; so this p which is a function of r is given by this relation, where y is yield strength of the materials. First thing that we need to understand here is that what this equation is saying is that pressure is not constant throughout. See whenever we are pressing we inherently assume until we know about friction that pressure remains constant. That same amount of pressure is being applied at each and every point, but that is not the case. This equation is saying that actually this is changing with r. Since it as a cylindrical geometry it is changing with r, probably if you had a square geometry you would see a different relation. But the idea is that pressure is different at different points, that are a very important implication of this friction; that pressure is not constant which means deformation will not be uniform which means that your final product may have continuous variation in the properties. So, that is one thing.

Now, this pressure if we draw it how will the distribution look like; if you plot this equation you would see that there is maxima actually at the centre.

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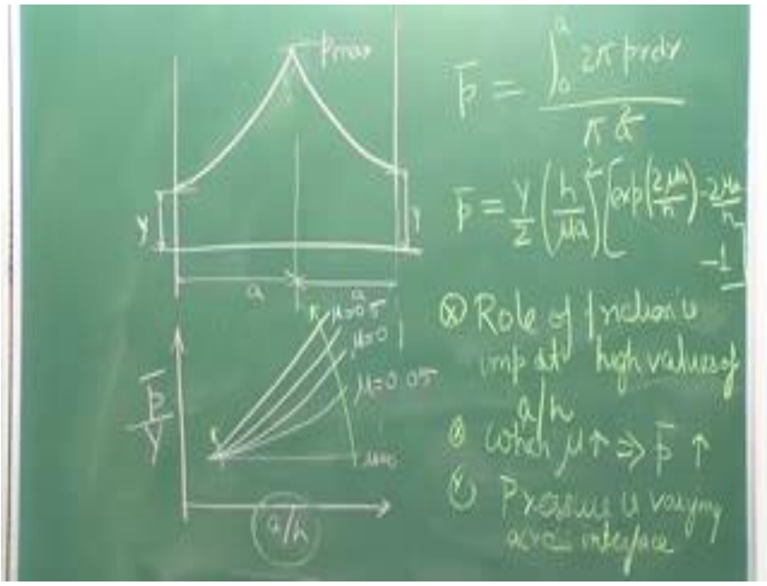

So this is a, this is a the pressure variation would actually come out to be; let me draw it a little bit more accurately it is not a straight line should come out to be, and from here if you put r equal to a that is on the ends if you put r equal to a you would see that it becomes e to the power 0 which means pressure is equal to y. So, you know that at this stream pressure is equal to yield strength y.

So, when you are deforming the minimum pressure at any particular point a is equal to yield strength. So that so far that is good because, if it were less than yield strength at any particular point then that particular point is probably not getting deformed, but that is not happening. And at each and every point your pressure is higher than the yield strength and every point is getting deformed. And in the centre you have maxima. So, this is how the pressure distribution would look like.

However, we are not happy with just how the pressure distribution looks like, particularly when you are doing some deformation what you are more interested is what is the average pressure that is acting. Let us accept that it is not constant, it is not uniform, even though it is varying what will be the average value of pressure that is being applied, because when you do the mechanics you want to take some average value you have to rough calculation, you do not want to calculate at each and every point what is the total deformation. In that case what you will be interested is finding out what will be the average pressure.

So, how do you calculate the average pressure? Simple just integrate it 0 to a divided by pi a square. And you can show that average pressure is equal to y by 2. Again I am not deriving it because all these derivations are given in dieter and you can go through those in more detail over there; and there is another minus 1 over. So, this is the average pressure, now let us also try to plot this and see how the pressure variation looks like, and this time I will be plotting it in terms of a by h. That is our reason why I am doing this and I will normalize the pressure for different materials, so I will divide it by y. So, it is independent of what material we are looking at and the plots will look like this. So, there are some very important information hidden in this curve and this curve.

First let me get back to this equation; original equation that we had over here. Now if you look over here, if you put mu equal to 0 what it says it that p r is equal to y; it does not depend on the radius r. So, pressure becomes constant which is what our instinct says. So, when we are not assuming friction the pressure is actually uniform there is no variation in the pressure. So, whenever you have friction you have to understand the variation so that you can find out how the pressure is varying.

Second what you see here is that again when mu is equal to 0 the pressure variation does not depend on what is the ratio of a over h. However, as the friction increases your pressure variation changes. Not only that it also says that role of friction is important but at particularly at high values of a by h. So, what do you see at high value of a by h, even if you change the friction by a small amount your pressure is changing a lot.

However, if your a by h value is very small which means that you have a very very thin contact area with respect to the total height then variation in pressure is not very large. So, here if you go from 0 to 0.5 the pressure variation is not very large. And therefore, in effect friction is not so important not so dominant over there, but when a by h is large meaning you have a much disked kind of shaped where a is very large meaning it is height is less and area is large, so the contact area is large with respect to h then over there you can see that if you change the mu by even a small amount your pressure is varying a lot. So, the role of friction is important at higher values of a by h.

Some other minor or smaller conclusions that when mu increases implies average pressure increases. Pressure is varying across interface. Now if you remember in the upper bound analysis we used pressure to calculate or relate it with internal properties, but what we see here is that pressure is also dependent on the location on the interface; at some places pressure is low, at some places pressure is high like what is given by here and what is shown in this board.

So, at this particular point pressure is low at this point pressure is high, so we cannot take one value of pressure when we are talking about upper bound analysis, there will be have to be a continuous change in pressure so your overall mechanics changes. So, now you can clearly see how the mechanics has changed when your pressure value has changed or when your friction has increased from 0; it is on from frictionless condition you have moved to a frictional condition.

And therefore, again if you revisit our nano Indian or sorry our indentation problem we assumed there was no friction over here and then we drew a deformation field like this; and we said that this particular region there is no friction and therefore we will not count the work being done here. Let say there was friction then it is not just about adding the work being done over here, it is also that you will have to have a variable pressure over here. So, remember you will have to have pressure variation like this. Now, your pressure is not one quantity but it is a field, and therefore you will have to calculate depending on this variation in the pressure you will have to calculate the overall mechanics. So, things become complicated very quickly when you add friction into this.

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Another important point when we are talking about not point, but when we are talking about friction we have looked at one coulombic friction. There is still another kind of friction which is called sticking friction. In fact, when I am writing onto the board what is working here is sticking friction. This sticking friction in metal working is used particularly in hot deformation condition; meaning no sliding takes place.

Now compare it with the writing of the chalk, that particular point of the chalk that particular powder remains stuck over there it is not sliding, if it were sliding then I would not have been able to write anything. So, writing on the board is actually similar to the sticking friction that we have. And it is similar when we are talking about hot deformation, there will be no sliding when you are trying to deform at higher temperatures.

So, we will get delve into more detail into this when we meet for the next lecture. In the meantime I would encourage you again to go and look through this sticking friction versus coulombic friction. So, see you next time.

Thank you.