Fundamentals of Materials Processing (Part- II)
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Lecture – 02
Continuum Mechanics

So, coming back to where we left, so we were looking at the two different definitions for stresses. The engineering stress; stress which is in general defined as force by area and if you take the original area, then what we get is engineering stress and if you take instantaneous area then what we get is the true stress.

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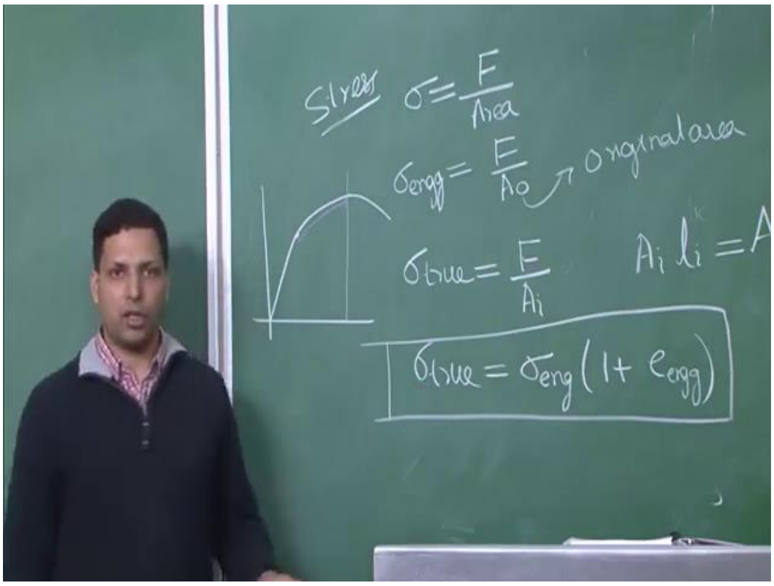

If we applied to this the fact that A i, l i is equal to because the volume remains constant during plastic deformation; we are talking about plastic deformation and therefore, A i, l i is equal to which is initial was instantaneous volume is equal to the original volume, we can apply to this and what we will get; it can be shown, so again we have a relation, so which relates sigma engineering to sigma true.

So, you can calculate sigma engineering very easily and from there you can obtain sigma true. One thing that we have to be careful is that these relations apply only up to the point where we get uniform deformation. We will talk more about uniform deformation in a material but briefly let me discuss what it is. So if you are doing a simple tensile test, if you remember the clot looks something like this. So up to the point where necking takes place, what we have the plastic deformation starts somewhere over here. So, from here all the way up to here the deformation that you have is uniform meaning the whole gage length or a whole area which is subjected to the deformation is deforming uniformly or has similar amount of deformation.

Beyond this what happens is that one particular region in this case neck; where the necking has found that particular region starts to deform more than the other region and therefore, you cannot describe the area or the change in length to the whole material or the whole sample and that is why these relations this one and that for the true strain cannot be valid beyond this point. So, these are the simple definitions for stress and strain, but stress and strain are not so simple and there can be stresses in multitude of directions, stress is not a scalar quantity; it is a tensor quantity not even a vector, it is a tensor quantity. So to get to that level, now let us look at some more inter cases about stress and strain.

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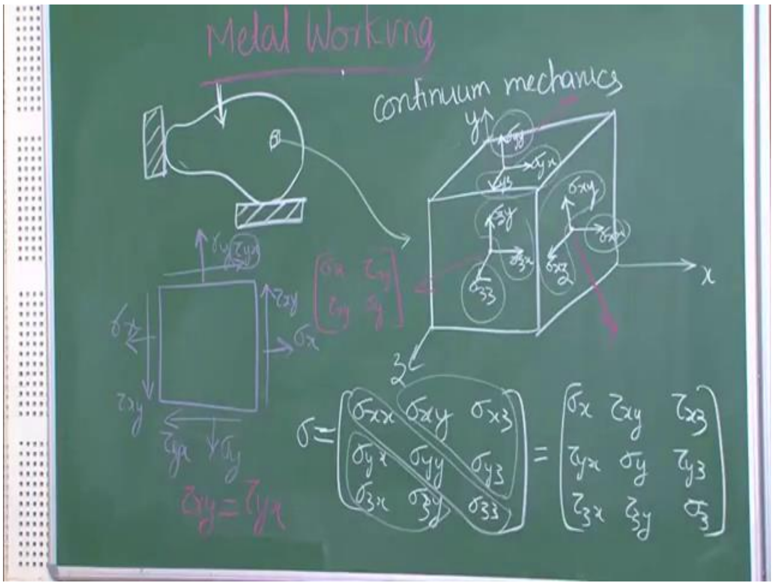

Let us say we have material of this shape where it is spent at those two regions and it is being deformed like this or the basically the force is being applied from here in this direction. Now each and every point, we will have the direction of stresses will be very different and there will be not just one component there if you try to resolve it in the different direction, there will be multitude of directions. So, to understand or to resolve this what we use is what is called as continuum mechanics which is nothing but just simply saying that in order to understand stresses and strains, we are assuming that this body is continuous that is your not looking at it as some of items. At each and every point you assume that it is uniformly solid or with uniform properties and then you describe what will be the stress and strain.

So, now let us say at one particular element over here it is actually very small, but I am for the sake of understanding or you say that this is how it looks. Now let us take element which can fill the whole space and cube is a very nice such element. So, we will take one cube over here and put it in larger view, now this is one element; a very very small (Refer Time: 04:54) small element from here and it may be having or it may be getting exposed to different forces, let us say one forcing this direction and something like this. It will be very difficult to understand this and therefore, what is done is that to resolve it in three different directions and in nine different components.

So, here this will be sigma x this will be called sigma xx, this will be called sigma xy, this will be called sigma xz. Similarly here it will have three components, this will be called sigma zz, this will be called sigma zx, this will be called sigma zy and similarly here it will have three components sigma yx, sigma yz and sigma yy. Now these nine components describe the complete stress state at this particular point in the system. Now similarly there can be different different variations or different values of these nine component at different points, but once you describe it in these nine components; you are able to see the complete picture at least at that particular point and that is why it is described or easier to understand in this direction or in this particular fashion.

Now here couple of things that you should observe, one that there are some what are called is normal stresses. So you see these are acting normally to the surface of this element, other stresses that you see over here are actually in the plane and these are also called as shear stresses. So, we are in a position to say that there are two different kinds of stresses normal stresses and shear stresses that act on to this. Another thing is that now we have these nine elements and since it is an tensor quantity, the sigma stress can actually be written like; so now this is stress has nine different components like this and the components which we were calling as normal stresses are actually the diagonal component of this and the stresses which we were calling as shear stresses are the non diagonal elements. So, there is symmetry to this all the stresses that are normal or coming in the diagonal and then shear stresses are the non diagonal elements.

This sigma xy which is also which I said is the shear stress is also represented as tau, so very often we will interchangeably use this a notation or may be also use something like this. So, I am putting it out here so that you do not get confuse later on, we may interchangeably used the notation tau xy or sigma xy and when we use this one we can use sigma xx or even sigma xx sigma x or sigma xx that does not matter, but the thing is that these are the shear stresses are many places represented as tau. Now this is the three dimensional version of it, you can also have a two dimensional version of it. So, let us say there stresses where all in one plane; in that case we would be representing it as; why we are looking at 2D is because many of times in most of this deformation, you would see that we are in a position to simplify the model or simplify the condition to a plane stress condition or in some cases plane strain condition.

Therefore, we can get read of many of these elements and they and make our life easier. So, let us say these are the x and y direction, so this is sigma x, this is sigma y and of course, there are shear components. So, this is tau x or tau yx which is also here tau yx and this is tau xy and this is also tau xy. Now can you say something about tau yx and tau xy, in general can we say anything about this; if a material is in equilibrium after deformation can we say or even if it is not getting deformed, but it is being under the under some imposes stresses tau xy and tau yx, can we say something about to a tau xy and tau yx; yes we can and we can say that it has to be tau xy has to be equal to tau yx; why because let us say it is not equal tau yx is greater than tau xy, then it will start to rotate, but we know that our body is in equilibrium and therefore, these two elements must be equal and therefore, in all these we will always have tau xy equal to tau yx and that also means that our matrix has been reduce to sigma x, sigma y, tau xy and this is also tau xy.

So, in effect we have only three elements and the same way we can also talk about tau xy, sigma xy equal to sigma yx and therefore, all the three elements that you see over here will be equal to the three elements over here. So, even in three dimension we do not have actually nine independent element, nine independent terms; we only have seven independent terms sorry six; 3 plus 3; 6 independent terms. The other 3 are equal to this other 3 and therefore in three dimension, we have six independent terms and in two dimension; we have three independent terms that we need to find out or we need to look at. Now that we have looked at the stresses, the next thing that we can look at is simple application of this is stresses in a particular condition.

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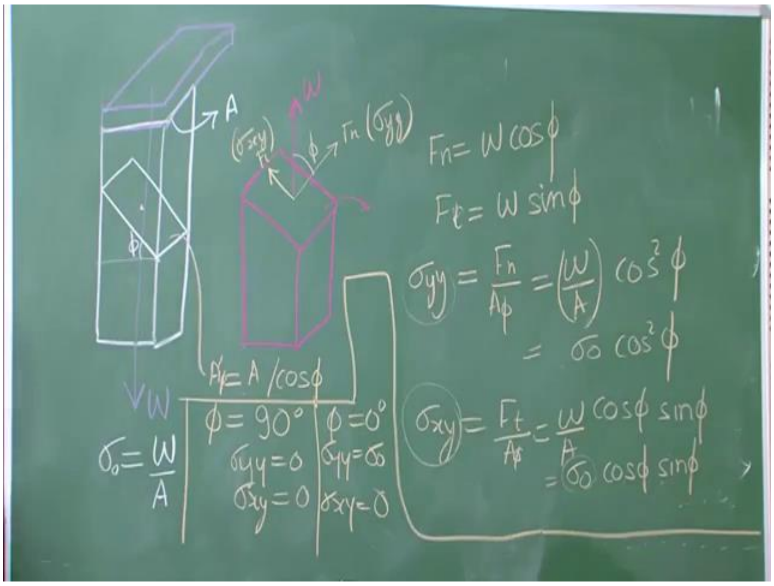

So, let me draw a simple configuration to explain how the same element if you look in different direction can have different values of stresses. So, let us say this is the white one that I have drawn is element on to which we intent to measure stresses and this is some weight that has been put on to it, let us say W is the total force; not just the weight, but weight times gravity, so this is the total force that is the acting on to this element. So, if you were to look at this particular area which let us say is equal to A; the cross sectional area of this element or this bar that we have over here, if we were to look at this then the sigma for this will be equal to W by A throughout anywhere, if you look at any cross section the stress in that cross section would be equal to W by A.

So, that is our let us call its sigma naught, but now let us say we are interested in a plane which is oriented a little differently. So, let us say we are interested in finding the stresses on a plane which is oriented like this; which is 5; at an angle 5 to what we have. So, it is inside the same material, but we are now interested in measuring it in a different orientation, you can call it change of reference frame or you can call it different point of view, different measuring it from a different direction so, but whatever it is we are interested in measuring the stresses over here

So, now let us draw this element over here separately and now here the force that is acting on to this is actually total force is like this and it is not normal to the surface. So, what do we need to do; we need to resolve it in F n; normal and F t; tangential. So, you say initially we had only normal component of the stress and because now we are measuring it in our different plane, we have inadvertently added a shear stress component to it. So, this there was only normal stress and now we have normal plus shear stress and this angle would be what this will be phi. Now you can say even see that F n is equal to W cos phi, F t is equal to W sin phi, area over here; this area is also now changed. So, if we were to take talk about this A prime; let us say and this is equal to A by or let us call it A phi.

So, a phi is equal to A by cos phi, so now if I want to find sigma yy which is in this direction and in this direction it will be sigma xy because it is a shear stress. So, if I want to find sigma yy which is normal stress, it will be equal to F n by A phi and we said we have given a value of sigma naught to W by A. So it becomes sigma naught y cos square phi, similarly for sigma xy which is now at the shear stress; there is Ft by A phi which is equal to W by A is equal to sigma naught. So, there are couple of things to note here, we started with the condition where is the force was normally, so this rectangular bar had only normal force acting on to it.

But depending on a configuration or the situation, you may have to look at forcing in the different direction and then resolve it. So, in that different direction which happens to be this at angle phi, we see that here the force is; are not normally any more just it does not have only the normal component, it also has a tangential which means now there are normal stresses as well as shear stresses and other thing we note is that there is a relation between the normal stresses and the origin stress that was acting. So, there is a way to transform access or transform the reference frame that is another thing that we understand from here. Similarly you can also get the shear stress component from this, from this original sigma naught.

So, we can transform and get not only the normal stresses, but we can also get the shear stresses. So, both normal stresses and shear stresses can be transformed, can be obtained by transforming from the original normal stresses that is one thing. Now let us put some values and see some something very strange over here. Now let us say that I put phi equal to 90 degrees, what does that mean. It means that phi is right now here; now it becomes like this and we are still looking at normal, that is normal in this direction and shear stresses like that, but what is the value that we get, if you put phi equal to 90; we know that it is 0. So, sigma yy becomes 0; if you put Fxy in over here again we put phi equal to 90 then again this becomes 0 and therefore, sigma xy is equal to 0. Now suddenly we see that all those stresses are being applied, but both normal stresses and shear stresses in this particular plane have reduced to 0. So, that some little strange, but it is not really that is strange, we will be able to explain it in few minutes when we look at the explanation using more circle.

You must have a you must remember more circle from your undergraduate days, but if not do not worry look at just the basics of these in the more circle, but before that you can again put another value which is phi equal to 0 degrees that is just like this, which is same to be same as that and what do we see that sigma yy, if you put phi equal to 0; it becomes 1 and sigma yy is equal to sigma naught and sigma xy, if you put phi equal to 0 in this which is again 0 and this again 0.

So, we get the original solution back when we put phi equal to 0 that is only normal stresses acting on over this. So, this particular example has have does under understand couple of things that depending on the orientation of the element, even if it has at the same point; see remember we are when we talk about element inside the component and our component is this rectangular bar inside it, we may be looking at our very small component over here, very just like a point.

So, in that point if we are looking at the stresses in this direction then it was just normal, if you just took this element and rotated it by phi; you get not only normal, but also shear stresses with a little different value and also up to a you may particular condition, you see that there is normal stresses also 0 and shear stresses also 0. So, these are just elements and you rotate them so the same stress state can be expressed in quite a bit different a ways that is what we are trying to express over here and when we talk about this example again which is 5 equal to 90 degree what we will talk about is mainly that there are principle stresses and that not both the principle stresses have actually reduce to 0.

So, this is a principle stress and this is a shear stress, but there is another principle stress component; we will define all those things, but I want to give you the answer before hand and that second principle stress is not really reduced to 0 that is still does exist over there. So, coming to understanding the stresses or the more circle a little bit more, let me talk about these elements just a little bit more about these elements. Now you saw that we started with a single element where there was no shear stress, but when you rotate it many at the same particular point even though we are at the same particular point, we were able to get just by rotating or in a looking at in different direction we were getting shear stresses.

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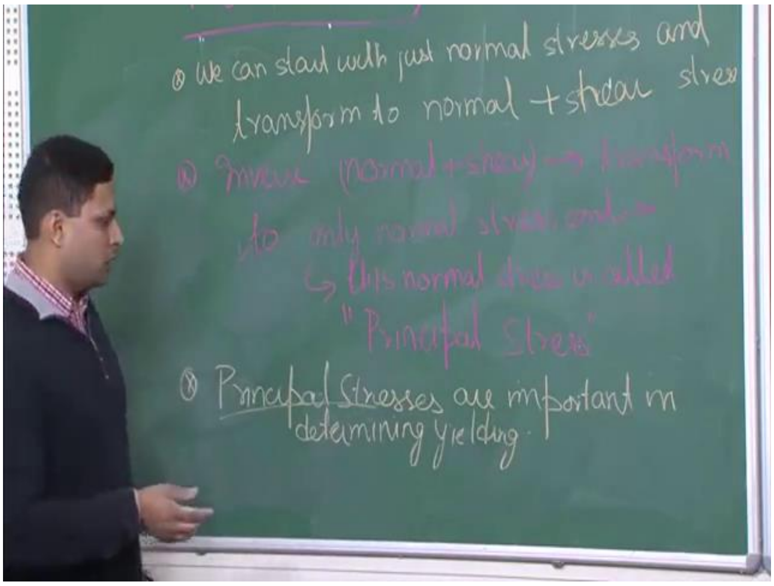

So, we can start with just normal stresses and transform to normal plus shear; values may be different, but the idea is that shear stress is does not arise basically shear stress is not in completely independent quantity that will that when you only apply shear stress shear, stress is will be generated. Even when you apply normal stresses, shear stresses are generated inside the material. So we saw this and inverse of this is true meaning, if you have normal plus shear; you can transform to only normal stress condition. So, even though we start with shear stresses along with normal stresses, but if you keep rotating you will get to a region and in fact in every condition you will always have a particular radiation, where you will get just the normal stresses. At that particular normal stress, this normal stress is called principle stresses.

So we talked about principles stresses earlier without giving definition or understanding but now I am explaining what is this principle stress. Again I warn you that do not get intermediated by all these terms; these are just starting points where we are using some equations. Later on when we get to the real examples, you would see that we what we use or much simpler form of these equations, but since we are talking about fundamentals, it is important that you get to understand all these basic fundamentals. So, these are called the principle stresses and why do we talk about principle stresses because principle stresses are important in determining yielding.

You may remember from your high school and even from your undergraduate days that when you do a simple tensile test on a dog bond, there gets a region where; what we called as yield point and we say that beyond is plastic deformation takes place. So, in a simple uniaxial deformation, it is very easy to determine or define where the plastic deformation starts, but as you saw; we were looking at complex state of stress where you have not only one component or stress there can be as much as six different components of stress acting on different points. In that particular condition, how would you say that a material would yield; would you say that sigma xx reaches this value, sigma xy reaches this value is that; that will not be so fusible, if you try to define each and every individual element and say; each of these would reach this value when yielding takes place.

In fact, you would see that the hydrostatic stresses are not even related to yielding. So, you can easily get rid of three of these terms and what you can look at even in the most general case, the yielding would depend only on three different values of stresses and that is why understanding the principles stresses becomes important. Principle stresses if you remember or the stresses in the condition where you are not talking about shear stresses. Either you talk about normal stresses and shear stresses in which case you have six different components or you talk only about principle stresses where you have then three components and in those three components what will be the yielding. So, discussions relating to the yielding of these in the next class.

Thank you.