Fundamentals of Materials Processing (Part- II)

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Lecture – 03
Stress Invariants

Welcome back. And we were discussing about stresses and strains in individual elements of the component. We saw that even though you may start with element has only normal stresses, but if you rotate the element you can have normal as well as shear stresses. Now we said that inverse is also possible where you can have a combination of normal and shear stresses, but you can rotate it to a position where you have now only normal stresses. And these are called, remember what we talked about these are called principle stresses.

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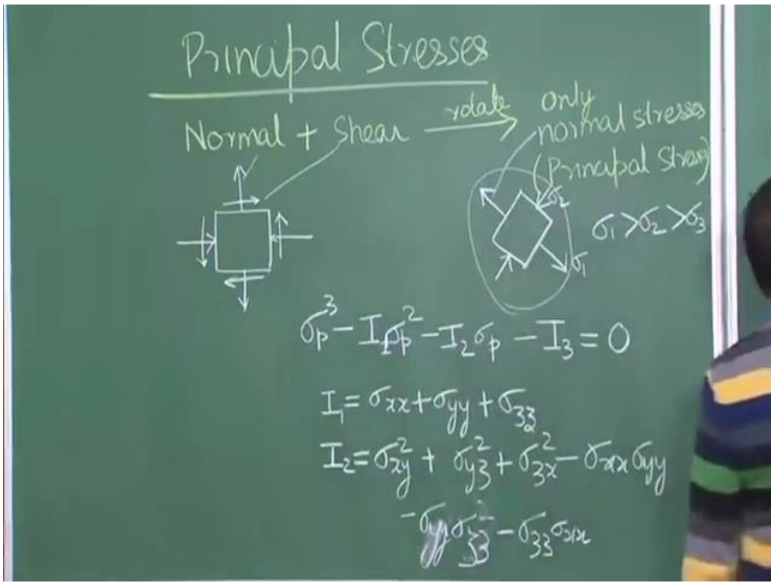

So, we were talking about principle stresses; a condition on an element where you have normal plus shear and if you rotate particular angle and that you will be able to find when we discuss about Mohr’s circle. So, when you rotate you will get to proposition where have only normal stresses; and this is what we call as principle stresses. So, if you what to understand it pictorially you can say that we start let us take the element in a; this is a small element (Refer Time: 01:48) small element inside the component. So, we have stresses like this. Remember this is trying to rotate it clock wise this these two are rotate trying rotate it anti clock wise and these two must be the magnitude of this must be equal to magnitude of this because the element is in equilibrium.

So, here we have both normal and shear stresses. Now let say for a particular rotation in this case let say after rotation of 45 degrees we end up having stresses like this. Then here you can see there you have only normal stresses, you do not have any shear stresses. And in that case whatever the values of these two normal stresses; there are two normal stresses we are talking in 2 D; two dimension. So, here you will have two principle stresses: one will be call sigma 1 other will be called sigma 2. And the convention that we follow is that you will give value sigma 1 to the one which is highest than sigma 2 which is second lowest. And in case we are talking in three dimensions you will have three principle stresses and it will be called sigma 3.

So, we see these that there can be up to three principle stresses and in two dimensions it will be two principle stresses. And even they principle stresses exists there will be no shear stresses inside that particular component. So, it is that that particular angle and you will be able to see when we use what is called as Mohr's circle construction that you can see that there will exist a particular rotation for which you will have no shear stresses and only normal stresses. And we will also see that in general there you cannot say that there will also be a position where there will only be shear stresses and there will be non normal stresses. In particular cases it can be, but in general that will not be true.

Now once you know; what are this principle stresses it is now time to look at some equation. Again, do not get work down by these equations, these equations I am showing over here just for sake of completeness. So, how do we find this principle stresses? Let say you have a particular condition where you are able to get sigma x sigma y sigma z sigma xy sigma yx and sigma zx and all those parameters or all those stresses, the 6 components then you would be able to find these principle stresses using this equation. Where this is the unknown quantity or the roots of this will give you the value: I 1 is actually call the stress invariant or I 1. I 2 stress invariant 2 times sigma p minus I 3. So, you see this is the cubic equation, where sigma p is your unknown under roots the when you solve this the values that you get will give you sigma 1 sigma 2 and sigma 3. And if we are talking about two dimensions then this will get reduced automatically to or quadratic equation and you will get only two values. And here I 1, I 2, I 3 are the values where or the invariants such called as the stress invariants are where you will put in the values or from the non parameters or the non stress values.

So, I 1 is given as sigma xx plus sigma yy plus sigma zz. Sigma x I 2 stress invariant sigma 2; sorry I 2 is given as sigma xy square plus sigma yz square plus sigma zx square minus sigma xx sigma yy, sorry this is minus and again minus sigma xx sigma yz square, this is not a z square this is xx. So, we had xx and yy, so here we will have yy and zz sorry, please make that correction. So, this is yy I went ahead for I 3 it is I 2, so I 2 we have minus sigma xx sigma yy minus sigma yy sigma zz and minus sigma zz sigma xx: this is I 2.

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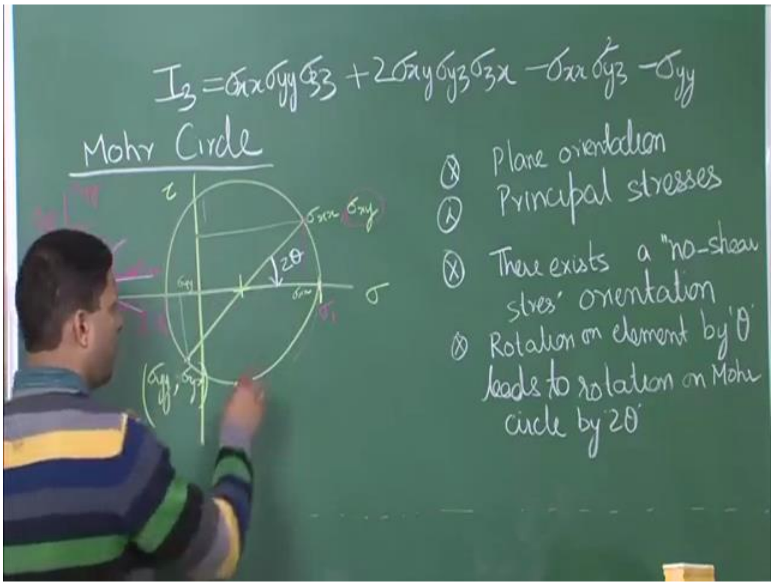

And you have I 3 which is given as sigma zz plus 2 sigma xy. So, we have I 1, I 2, I 3; now what you will sees that this is your I 1 now if you let for particular condition you will know sigma xx sigma yy sigma zz and you will also know sigma xy sigma yz and sigma zx. Once you know all those values you can just insert it here to get I 1, by you would think that if you start with the different rotation you may get different values of sigma xx sigma yy and sigma zz in that case you would get different values of I 1, I 2, I 3 and even insert it over here you should get different principle stresses, although the elementary means the same and the sigma p or the principle stresses should come out to be the same value; but that is why we call it stress invariant.

No matter what particular orientation of the element you take, when you take these values for that particular element in that particular orientation the some of these would come out to a value which is constant. So, no matter how much you rotate it as long as you are at the same element this value will not change. So, these combination the three you can vary but the some of these combinations will or waves come out be constant and that is why it is called invariant. Similarly, for I 2 and similarly for I 3; and well you put I also those three over here you can see you will get same principle stresses for that particular element.

Now this is trying to find out principle stresses using this equation, but we have another formalism or construction what is called as Mohr's circle construction. This is a very interesting way to find out the values of the values of principles stresses. So, let say we take two axis where your x axis represents normal stresses and your y axis represents shear stresses. Now, you let say you take biaxial stress condition where you have a particular stress condition which will show like this. Let say now you have sigma xx which is given by some positive value and you have some sigma yy value which is negative.

So, you plot this sigma xx and on that particular plane you also have a shear value, so you will be able to plot a shear value for this. And similarly you will have a shear value for sigma yy and right now we are taking at two dimensional case. So, if I were to draw this; so this is our xx in x we have positive, so this is tensile, here we have compressive. So, this is element we are talking about, this is sigma xx, this is sigma yy, and this is sigma xy. So, this is trying to rotate it clock wise, and equal in magnitude but opposite in direction what we have is sigma yx. This is trying to rotate it anti clock wise. So, this is sigma xx on the x axis and on the y axis we have sigma xy. Over here we have sigma yy and on the y axis we have minus sigma xy or which is equal to sigma yx. So, this is your sigma yy value this is sigma xx value.

Now when you plot these two points over here, right now do not think that I have drawn a circle so the circle comes later. Once you have these two points and taking this to be a diameter now if you draw circle this is what is called a Mohr's circle for this particular element. And let us say this is the center, so now all you need to do to find the principle stresses is look at the values on the x axis. Over here this gives you sigma 1 and since we are talking in to two dimension we have only two principle stresses sigma 1 and sigma 2.

So, once you were in a position to put these two points on this two dimensional map you have x and y coordinates, you have x and y coordinates at this place you are able to draw a circle assuming these two points to be the diameter. And once you have this circle, the circle wherever it intersects the x axis those two points are what gives you the principle stresses.

So, this is very very simple geometric construction and technique to find out the principle stresses, you do not have to go through all that complicated mathematics. So, that is why said do not get intimidated by looking at the equations, there are simpler ways, but for the sake of completeness we have put in all those equations. And there are simpler ways like this one to find out the principle stresses, and in many a cases when you are dealing with our some of the deformation technique we do not even need to find each and every components like even principle stresses most of the time we are simplified it further as you will see later on so that you can deal with just one number, you do not have to deal with all the 7 parameters or even in case of two dimension you do not have deal with 4 parameters.

Another thing that you should realized from this is that when you have a clock wise shear movement then that is given has positive value. Here the shear is positive, if you look at sigma xy and sigma xy over here it is clock wise and clock wise is related to positive value. On the other hand sigma yx is anti clock wise and that is related to negative value. So you see sigma yx is same in magnitude as sigma xy, but it is on the positive y axis and this is on the negative y axis. For the tensile in compression or the normal stresses when the when we are talking about tensile stresses it is positive. So, sigma 1 as you can see in this particular case comes out to be positive and sigma 2 in this case comes out to be negative. Similarly, sigma xx which is tensile is plotted as positive and sigma yy which is compression is plotted as negative value over here.

So, these are some of the things that you should understand that go along with Mohr’s circle. And to put it in perspective or to put it formalize it let us write down what are the things that we can find out as result of Mohr’s circle. So, we can find out plane orientation. We can find out principle stresses from this, which also see one important thing that I have been telling you so far is that there exists a particular orientation. So, you see we had this orientation to begin with, but if you rotated by some angle which will also imply a rotation on this particular plane will get to a particular orientation where you have only normal stresses. Remember this is why we are calling these as principle stresses.

So, when we rotate it by a particular angle we do get to a position where we have only normal stresses and we have no shear stresses. So, there exists a no shear stress orientation. But how much do we need to rotate, when we rotate some by some angle let say if I see over here that this has to be rotated by 2 theta and there is the reason why I am writing it as 2 theta not as theta how much should the actual element we rotated by. Now we have gone into the equation and so I will simple put it here inwards that if you want to go to 2 theta in the Mohr's circle then you have to rotate your element by half of that angle, so you will have to rotate it by theta. So, this is going clock wise. So, here you will go clock wise, so this is your theta.

In other words when you rotate on the element by angle theta on the Mohr's circle it gets rotated by 2 theta. So, there is rotation on element by theta leads to rotation on Mohr's circle by 2 theta. And just now I notice that I mistakenly have called this as anti clock wise actually this no this is clock wise, so right I was right so do not worry about that. So, these are the some of the things that we can extract from Mohr's circle. Now another thing that you would see is that we said that there exists a no shear stress orientation. Do we see a no normal stress orientation over here? What will be the condition where you will have to have where you will have no normal stress? So, one of these would have to be here and the other will have to be over here, but if you take this as the diameter it will be a completely new circle.

So, if you are talking about the same element. Remember this one circle represents one element, here we can rotated and it means rotating the same element or at the same point inside the component, but if we draw new circle over here it would represent a new element or a different element on the component. So, we cannot take these two points for this same element, but they may of course exists some elements where this and this will be your stresses; sigma xx and sigma yy. However, for this particular element that they does not exists a particular orientation where you can get only shear stresses and non normal stresses. We will get to a simple Mohr's circle for that two for certain cases where you can get elements like that.

Now before we move on let us look at one of the example; not example but we talked about a case where we rotated earlier we were rotating the element by certain angle and we saw that at 90 degree there did not exists either a normal stress or a shear stress remember. So, we were talking about it in the previous lecture.

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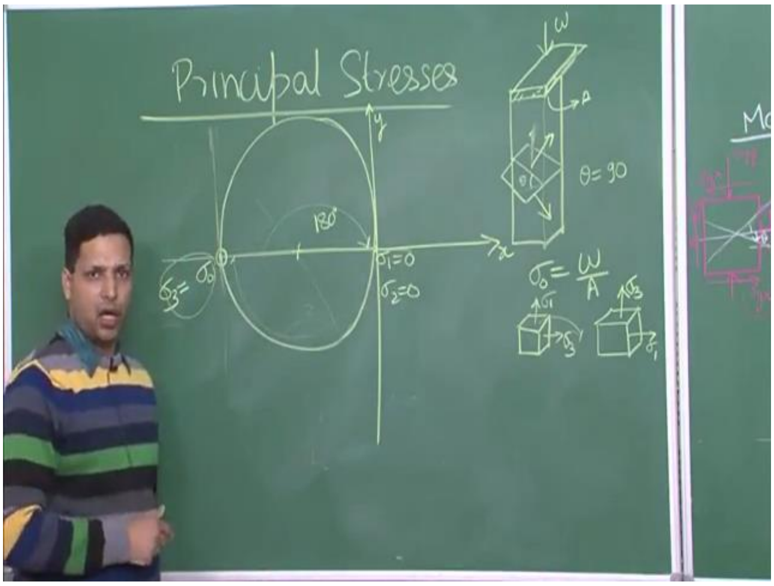

Now if we were to look at such particular condition which it is nothing but a simple tensile or compressive behavior; so a Mohr's circle which represents just one principle stress or basically where to begin within the normal element we have only one principle stress. So, you remember in this particular case, since we are we were talking about; so there was some weight that was going on to this, so this is compressive.

Let me be consistent and let me draw just the compressive part. So, if I have to draw the compressive it means the principle stress would be on the negative side. So, my circle is on the left side of my y axis because we are talking about compressive stresses. This is your y axis, this is positive x axis and on the negative x axis we have the stress. And when we look at the element just like that, that is the normal condition we have stresses acting on it where sigma w is the force and a is the normal area; so stress equal to w by a.

So, this was the stress acting on it, and there are no other stresses in perpendicular directions. So, this is represented as this sigma naught which is our sigma actually 3. Why I am calling 3, because sigma 1 which is 0 is higher or larger in magnitude than sigma sigma 3 this is the negative quantity. So, this becomes our sigma 1 which is equal to 0 and sigma 2 is of course also 0. So, we have sigma 1 equal to 0 sigma 2 equal to 0 and sigma 3 equal to sigma naught which is the negative quantity. Now in this particular case when we rotate it by 90 degrees, we rotate it we have taken a different plane from this normal, so this was theta and we said that if you take theta equal to 90 degrees which means over here if this is the circle you are rotating a line by 2 theta; which means you are doing nothing but you are rotating it by 180 degrees.

And therefore, this sigma that we were trying to calculate is 0, shear stress is 0, but that does not mean that does not exist any principle stress. If you look at 90 degrees from this that is there is normal stress like this and there this is this is an element there is also normal stress like this. So, if you at look at the normal stress at 90 degrees from it they still exists this value.

So, we are looking in fact, if you look at it carefully what we are saying is that our original element which existed like this has been rotated by 90 degrees. So, what was sigma 1 this was sigma 3, now where the sigma 1 should have existed now there exists sigma 1, where sigma 3 should exists there exists sigma 1. So, this is what happened when we rotated this by 90 degrees. So, this is the example of Mohr’s circle where we have now seen a general condition, where you have sigma xx sigma xy sigma yy and sigma yx and there we have looked at a simple compressive stress.

Now, let me how will it be different when you are talking about uniaxial tensile condition, how should this Mohr’s circle look like. You must have got enough pretty good idea by now, because what we drew here was uniaxial compression.

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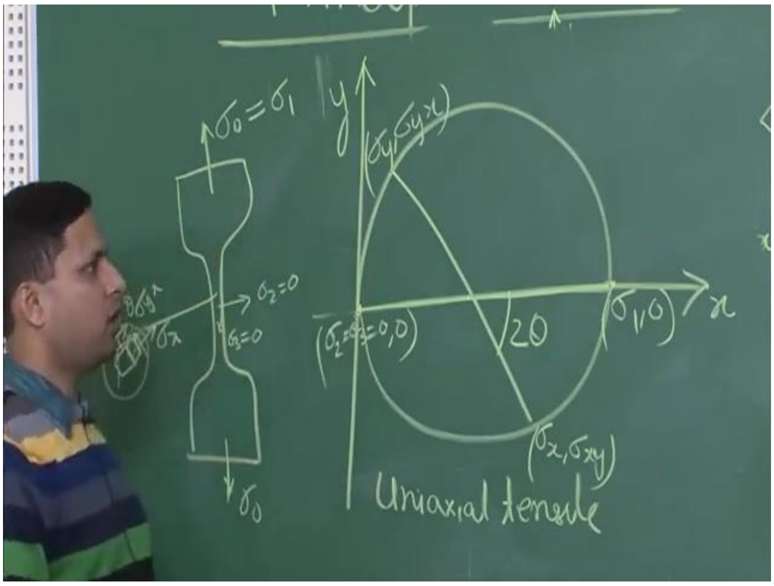

So, if you want to draw a uniaxial tensile stress condition; Mohr’s circle for uniaxial tensile condition it will be like this. It is the same circle but now it exists in the positive x axis. On y axis it is symmetric, so it does not matter because y axis it has; y axis why does it have to be symmetric because tau xy has to be equal to tau yx, in magnitude and sign they have to be opposite one will be clock wise the other will be anti clock wise.

If we look at the tensile condition, so these two points represent the principle stresses. So, let us say I have; so this sigma naught is actually equal to sigma 1 and sigma 2 and sigma 3 in the perpendicular directions are all equal to 0. So, this becomes sigma 1, this becomes sigma 2 equal to sigma 3 equal to 0. And this particular case, in this particular orientation if you take any element inside this gauge area that element is only exposed to normal stress, and there are no shear stresses which is what is reflected by this diameter. One principle stress the other principle stress and these too do not have or the y axis component for both of them is 0. So, they does not exists any shear stress.

So, this is the condition for uniaxial tensile. However, if you take an element over here and look at it in a different orientation; so instead of now looking at it in the same orientation as the stresses now let me take at orientation of element in little bit different direction. So, now you see this is a little bit rotated. I rotate it by clock wise by some angle let us say theta, what will that imply this the stresses on that particular element will now we given by a diameter which is now rotated from this original one by angle 2 theta.

So, this will be your sigma x comma sigma xy, this will be your sigma y coma sigma yx. And just from looking at this I can say that sigma xy is negative, it means if I am calling this as sigma x then sigma x y is negative it implies it must been anti clock wise direction. So, this x sigma xy must be like this.

On the other hand sigma yx is positive, so if this is my sigma y sigma xy sorry the sigma yx sigma y and this is sigma yx sigma yx must be clock wise because sigma yx is positive. And both my sigma y and sigma x are positive so both of them are in tensile condition. This sigma x as you see is positive; this is sigma y which is again positive. So, we are able to get are extract so much information from this Mohr's circle just by looking at it. And by simple construction we are able to not only get the stress conditions or the stress values are different orientation, but also their magnitude their sign and we can also work it backwards when we have a condition of normal stresses plus mix stress or for plus shear stresses we are able to get back to the principle stresses.

So, we will look at one more example of Mohr's circle when we meet in the next lecture. So, will end at this point and will come back next time.

Thanks.