Fundamentals of Materials Processing (Part- II)
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Lecture – 05
Yield Stress Criterion

So, we will continue our discussion on yielding, so yielding is the value of stress at which the material starts to deform plastically.

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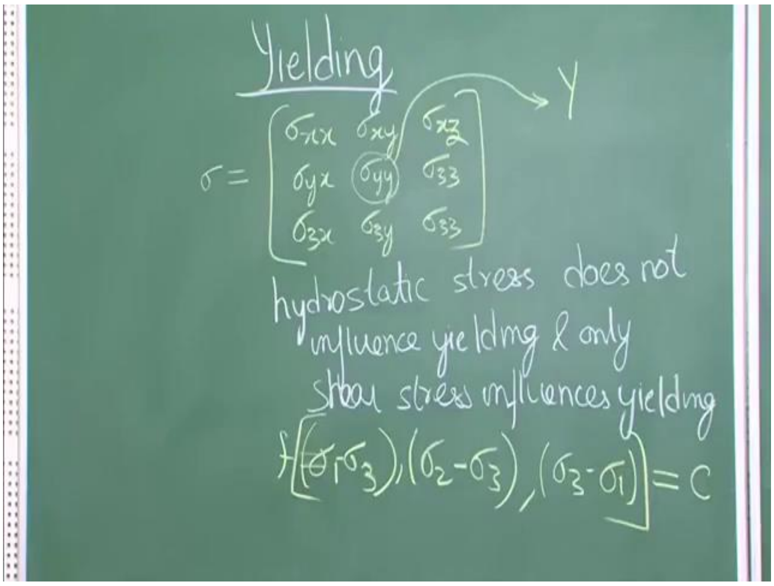

So let us look again at the stress, we are talking about yielding. Now we know that stress, now we can write it as a tensor quantity. So, it is sigma xx, sigma yx, sigma yy, sigma zz, sigma zx, sigma zy and sigma zz. So, this is the stress; this is the complete representation of the stress tensor. Now when we are talking about simple tensile kind of condition; deformation condition there only one quantity exists. So, this is sigma y and we know from our simple tensile test then that when this value reaches y; which is the yield strength of the material then we say that the material is such started to deform.

But how do we talk about yielding or the initiation of plastic deformation when more than one element or not element, but more than one of the components of this matrix is not non zero. So in this particular case only this was non zero, everything else was zero; what about if more than one term is non zero. So, let us say this is non zero and there are some more two or three terms which are non zero, then how do we define that deformation has started or begun. So, either you can think of a very simplistic model where you would say each and every value have give a, that each of these value should reach a certain point and then we will be in a position to say that yielding has begun, but that is not corroborated by our experimental results. Experimental results like we showed has shown as that it is hydro static stress, so the hydro static stress does not influence yielding and only shear stress components, only shear stress; not the component but the overall shear stress is what influences yielding.

If we take this statement into account then we can say that this function, which is function of shear stress. Let us not write it as like this; sigma 1 minus sigma 3 are the function of sigma 2 minus sigma 3 and sigma 3 minus sigma 1; when this function reaches some critical value c, we do not know what this function is right now, but all we know is that it has to be a function of shear stresses and if we look at the shear stresses in terms of principle stresses then sigma minus sigma 3, sigma 2 minus sigma 3 and sigma 3 minus sigma 1 then they all represent shear stresses; the three shear stress components. So now we have a form of the equation like this which is in terms of shear stresses and when this function reaches a critical value c then we will say that this is plastically deforming or yielding.

Now, the next task is to find out what this function is, what is this function which has to reach this critical value. We know it is the function of shear stresses, but we still do not know the exact function and to get to that exact function, we will have to put some constraints, get a formulation and then verify empherically whether this particular criterion holds true or not and it may be that certain materials will follow one kind of criterion, other materials will follow other kind of criterion. So coming back to this certain assumptions, so what are those certain assumptions or constraints that we need to put so that we can formulate some criterions.

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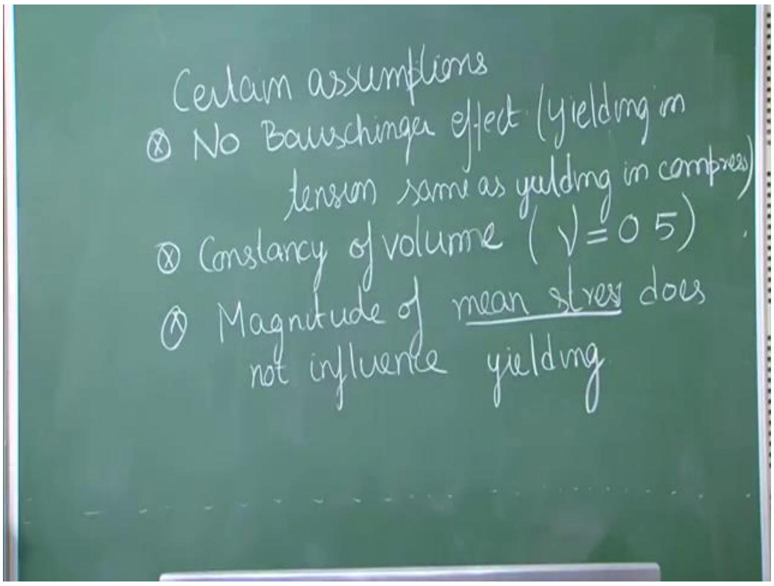

First and foremost is; No Bauschinger condition or No Bauschinger effect; what is Bauschinger effect, Bauschinger effect is when you have different behavior in tensile and different behavior in compressive. So, what we are saying is that it should not be there meaning behavior should be similar whether we are talking about compressive or tensile and that would mean that the relation that we will get to; they will be symmetric with respect to plus or minus sign because plus and minus sign are what will reflect whether it is tensile or compressive stress.

So, it has to be no Bauschinger effect or yielding in tension same as yielding in compression. What is the other assumption; it is non compressibility or constancy of volume, so we are assuming it is plastic deformation under material which does not compress upon deformation. Therefore, if you look at the total change in volume, it should be 0 and from that you can easily show that cozens ratio nu should come out to be 0.5 and will actually look at one example to understand this.

So this is another constraint and the other is that magnitude of mean stress does not influence yielding. Now we have already stated it in a much more general way but to formulate this relation, we are taking one particular or one much more defined; getting a defined approach and saying that magnitude of mean stress does not influence yielding. We have already said that hydro static stress should not influence yielding and now we are putting it in a more specific term that mean stress should not influence yielding.

So, now if we are to take these into consideration and the fact that we already know that it is a function of shear stresses, then we have two more things to save and before we put those constraints in a formal way, let us look at it in what the mohr circle will have to say about this, about the constraint that we have put, about the fact that magnitude of mean stress does not influence yielding or the fact that the hydro static stress does not influence yielding

So, let us look at a simple mohr circle to explain what we have just written, so all the jumbo mungo that we have talked about can be much simply represented as we have done earlier also by a simple mohr circle.

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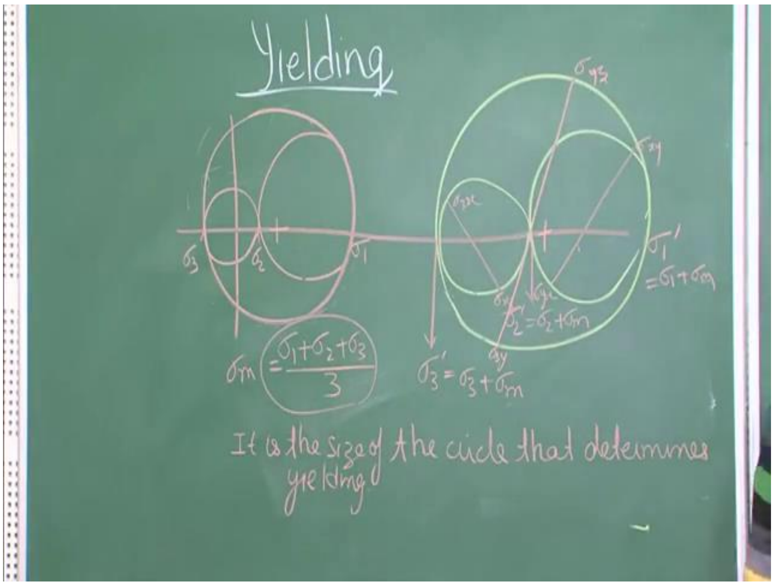

So, let us say this is a mohr circle representing the yielding condition, so let us say it is at this particular condition that it is yielding and there are multi axle stresses, so we are drawing three circles. Now there is sigma 1 over here, there is sigma 2 over here and there is sigma 3 over here, now what is the hydro static stress it is the sigma xx, sigma yy and sigma zz and the mean of these we can find out very easily for example, if we are talking about stresses in this direction, the mean of this will be OK.

So, this is the element where we have this particular stress condition. Now if this part same element, were also imposed some hydro static stress with additional amount this. Let us say this is sigma m equal to this, so now each of this stresses is added by this amount sigma m and let us say now it has moved to over here. So, size of the circle remain same, so even though the drawing may not be able to represent it or may not be able to convey it, but remember this circle is same in diameter as this, this circle is same in diameter as this, this circle is same in diameter as this. So, just keep in mind that we are trying to draw it over on the board, so it may not come out accurately, but that is the fact that the size of the circle remains same. Only thing that has changed is the overall placement of the circle, so the circle was placed over here and now it is placed over here; everything else remains same.

So, now similarly here so now this is your mu sigma 1 prime which is equal to sigma 1 plus sigma m. Similarly this is sigma 2 prime which is equal to sigma 2 plus sigma m and this is sigma 3 prime which is equal to sigma 3 plus sigma m. Now what all those things that we said over there means or in those words what it means is that if yielding is taking place over here, it is also taking place for this one. If yielding is not taking place here, yielding will not take place even for this condition.

So, just imagine here let us say yielding is not taking place and sigma 1, sigma 2, sigma 3 these are some values. Now you have increased those values by a significant fraction, you can keep on increasing it no matter what, I mean until the continuum mechanic holds until that stress you can keep increasing it; which would mean that you can keep shifting this circle all the way to on the positive x axis or on the other hand, if you want you can keep shifting it in the negative direction where you have compressive stresses, so you can keep shifting it in the negative x direction.

But yielding will not take place, if yielding is not taking place over here. So, now, absolute magnitude of sigma 1, sigma 2 and sigma 3 does not make or influence the yielding. What is only going to influence the yielding is the size of the circle, now the size of the circle as you can see as we have already mentioned is same here and here or no matter wherever you take it on the x axis. So, as long as the size of the circle is same whatever is the yielding state here; it will remain the same for there and therefore, if you are actually changing something which is causing that size change in the size of the diameter of the circle that is the only thing that can actually change the state of yielding.

So, let us say we start with a very small circle there yielding is not taking place keep increasing it, keep increasing it and there may be some critical size; let us say this critical size at which yielding takes place. So, as soon as the circle size reaches this value it the material starts to deform plastically and anything beyond that the material starts to deform and it has yielded. So what we can say from here is that, it is the size of the circle that determines yielding. Remember if you keep moving it, if you keep the placement different and keep the size same nothing is changing in terms of yielding, although absolute value of hydro static stresses has increased, we have added some quantity sigma m to all the principle stresses or it may even be some particular angle, so in this particular angle you may have sigma xy, sigma yx, sigma xz, sigma zx and similarly sigma yz and sigma zy.

So there may be stresses; all those stresses would have actually increased in value. The normal stresses have increased in value, but the yielding condition has not changed. So, that is the very very important understanding and now based on this, we will look at two different yielding criterions. So, like we said earlier that yielding is defined now by thus difference between the principle stresses because why are we taking the difference between the principle stresses because difference between the principle stresses are nothing, but shear stresses.

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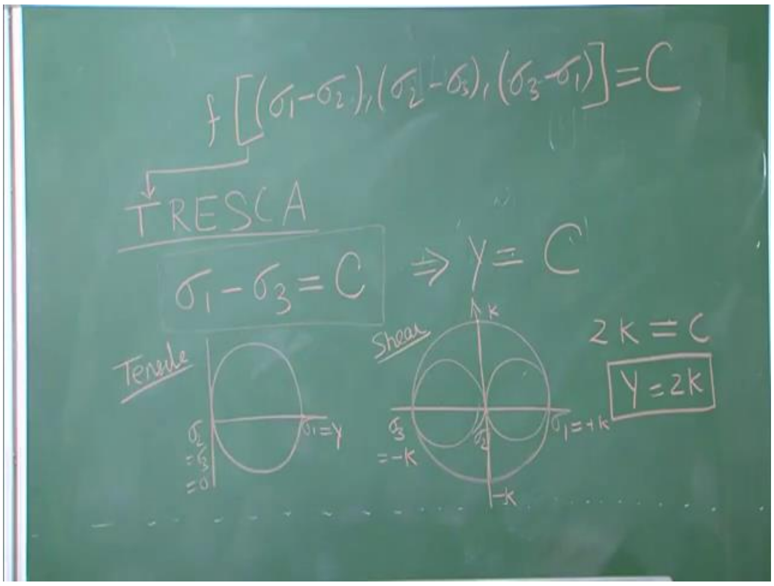

So, we have said f sigma 1 minus sigma 2 should reach some critical value. This is what we have done so far now people have come up with two different criterions, one of them is called TRESCA and like I said, you can come up with any number of criterions and maybe different materials will follow different criterions as long as they follow some of those constraints and the fact that the mean stresses should not influence yielding. So, what is TRESCA criterion what the TRESCA criterion says is that; when sigma 1, minus sigma 3 reaches a critical value c that yielding will start to take place, so this is the defining equation for TRESCA criterion.

As of now we do not know c, we do not know how to relate it with other known properties of the material. So, for example, you can do simple tensile test and get yield strength of the material, you can do simple shear stress and you can get the shear strength of the material which is given by the symbol k. So, how do we relate c with those known quantities of the material and what will be that c value. So, for to do that what will do is simply take a tensile condition, meaning let us say we have put the material in tensile test and over there we will find out the value of sigma 1 and sigma 3 in terms of y and put it over here.

Now if in terms of ; when we are doing the tensile test and the material has just begun to yield then at that condition what will be the sigma 1 value. So if we draw the mohr circle, this is how it will look like. So if it has just started; this is sigma 1, this will be your sigma 2 equal to sigma 3 which will be equal to 0 and if it has just began to yield then this value must be equal to y. So, now let us put this over here and we see that y is equal to c, sigma 1 is y, sigma 3 is 0, so you put this over here and this implies that c is equal to y.

The other is you take a shear stress test of the material, now what we are trying to do is we are trying to relate the value of c in terms of the shear strength. So, y is our yield strength and k will be our shear strength. So, now we are doing a shear stress of the material, under green we draw the mohr circle like this. So, this is sigma 1 and this is sigma 2, this is sigma 3 and if the material is just about to yield then this value must be k because this is the shear stress and this is symmetric, so this must be equal to k, this must be equal to minus k.

So, the radius is k which means sigma 1 which is also equal to the radius should equal to plus k and sigma 3 which is equal to minus k which is also equal to radius, but in the negative direction, so this is minus k. So if we put this over here; what we get is k minus, minus k equal to c which is 2 k equal to c. So when using TRESCA criterion, we get not only the value of c in terms of y and in terms of k, we also get what is the relation between y and k which is y equal to 2 k. So, this is one criterion to define plasticity when the onset of plasticity takes place in the material.

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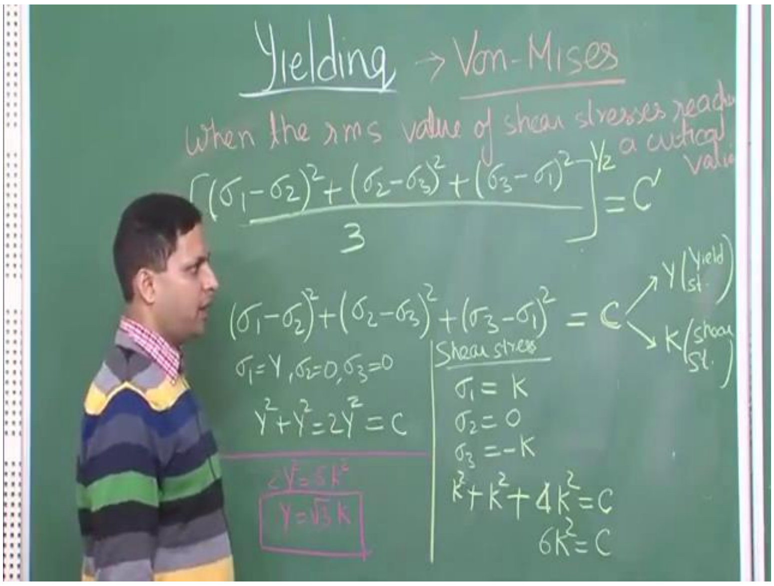

Now, we will move on to another criterion, which is called the Von Mises criterion, so this is the Von Mises criterion. So, one was the TRESCA, now the second one we are looking at is the Von Mises criterion. The TRESCA criterion as we saw is very simple, it is just saying sigma 1 minus sigma 3 equal to c, but in the Von Mises it is a little bit more involved. What it is saying is that when the rms; that is root mean square value of shear stresses reaches a critical value. So, we looked at TRESCA and this is the Von Mises criterion, so what the Von Mises criterion is saying that when the root mean square value of shear stresses reach a critical value.

So, now let us put it mathematically, so if you put it mathematically this is how we will represent. So this particular quantity which is on the left hand side, it should reach some critical value c, so this is c is a constant. So, no matter if as long as this it is the same material, no matter what is the stress condition but if you take out this particular function value then that function value should reach this constant c and once it reaches c, you can say that the formation has begun or plastic shift plastic deformation begun, so this is the onset of plasticity as defined by Von Mises.

Now, what you will again see is that we have taken the quantities in terms of sigma 1 minus sigma 2; sigma 2 minus sigma 3 and sigma 3 minus sigma 1 and not as absolute quantity sigma 1, 2, 3. Again that is related to the fact that hydro static stresses do not influence yielding, what influences yielding are shear stresses and if you remember again from the mohr circle these are what; these are nothing, but shear stresses, so these are the three different shear stress components.

In the TRESCA, it is concerned with only one kind of shear stress which is the largest shear stress because we are looking at sigma 1 minus sigma 3. But in Von Mises, we are saying that all the shear stresses make a difference and we have formulated the relation based on that. Now you can; let us put it; for now put this c not a c, but c prime and there is a reason behind it and the reason is that I can simplify this further and I have forgotten this 1 by 2 sign where root mean square, so this is under root all everything. Now here we can may simplify it to certain extent and say that since 3 is a constant and we can take square on both side, so this can be further simplified into this one. So, this is just the same thing, but rewritten in a little different way. So, this left hand side value again should reach some predefined constant c and then we will be able to say that the material has started to yield.

Now what is next, here we have the form of the equation, but what we do not have is the value of c. More importantly like in the previous case, we would like to relate c to yield strength of the material and the shear strength of the material. So, let us look at the value of c and to do that again all you need to do is take tensile stress condition and if you take tensile stress condition, what happens; your sigma 1 is equal to y sigma 2 is equal to 0 and sigma 3 is equal to 0. So we have these values, you insert it in over here and what we get is y square plus y square which is equal to 2 y square is equal to this constant c.

So, we have found that value of c in terms of the yield strength of the material, so yield strength of the material you can find out by very simple test. So, now see the significance of this equation, yield strength is a value which can be very easily obtained by a simple tensile test, but in a real deformation, you may be exposing or you may the material may be imposed two different kinds of multi axial stresses.

Now, all those various combinations of multi axial stresses you can find whether that particular element will start to yield or not, just by putting the values of sigma 1, 2, 3 from that particular element into this equation and now this value of c is already known which is equal to 2 y square. So, you are able to relate all of those to the yield strength of the material.

Now, similarly you can do a shear stress study of the same material so that probably you are interested in finding c not in terms of y, but probably in terms of k which is the shear strength. So just let me reiterate it is the shear strength, y is our yield strength, so now here what we will have is that sigma 1 like in previous case, it will be equal to k, sigma 1 will be equal to 0, sigma 3 will again be equal to minus k. Again you put it over here and what you see is that this is sigma 1, minus sigma 2 which is equal to k square, this is sigma 2 minus sigma 3. So, this is k square again and this is sigma 3 minus sigma 1, so it is minus 2 k or 4 k square is equal to c, so 6 k square is equal to c.

Now we have two relations; one in terms of yield strength, the other in terms of shear strength. Now we can combine these two to get a relation between yield strength and shear strength. So, what it is saying is that under these conditions you have some relation between yield strength and shear strength and what is that. So, your 2 y square is equal to 6 k square which means y is equal to root 3 k. So, here if you are using material where you are applying this Von Mises criterion then it says that in this particular condition or this is valid for those where y is equal is follows this kind of relation, y is yield strength is root three time the shear strength.

On the other hand when you are using the TRESCA then we see that y is equal to 2 k, it is a minor difference, but still significant one is 2k and this is root 3; which is 1.732. So, here it is y equal to 1.732 k and there it is saying y is equal to 2k. So, these are two of the most widely used criterions to be able to describe the onset of plasticity under different multi axial stresses and the best part of it is that you are able to define the onset of plasticity in terms of the property which is very easy to extract, like the yield strength or the shear strength and in both the cases, you can see we are able to relate c with y and k and no matter what is the different kind of stresses, what are the multi axial stresses, it may have all the nine different components or present in it or it may have just four, in all those cases, you would be able to get that for that particular stress condition that material will be deforming or not.

So, now we have looked at two examples for the yield criterion, it is time we look at one example and what I will do is, I will leave you with the question today.

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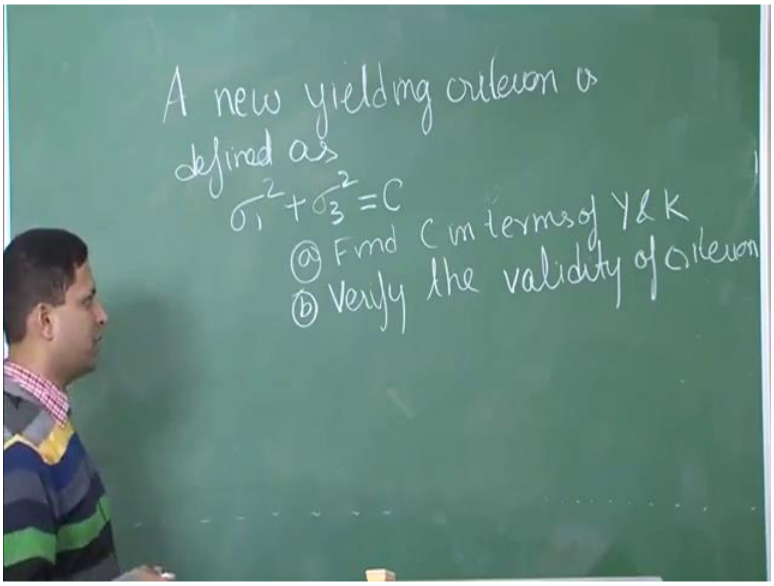

So, the question is that a new yielding criterion is defined, so those are the two and those are not the only two, you can come up with as many as possible as I said but then you will have to cross check; first that it is that it verifies with or it accepts all the exemption that we have made and plus that it is empirically valid. So, a new yielding criterion is defined as sigma 1 square plus sigma 3 square is equal to c, so this is the yielding criterion that I am defining for you.

Now what you have to do is first find c in terms of y and k, y is the yield strength and k is the shear strength second; verify the validity of the credited criterion. So, before you get to the solution, I would request you to spend some time and try to solve this; this is a very straight forward problem and you should be able to do it in 5 to 10 minutes. So, I will leave you with this question and will come back to the solution in the next class.

Thank you.