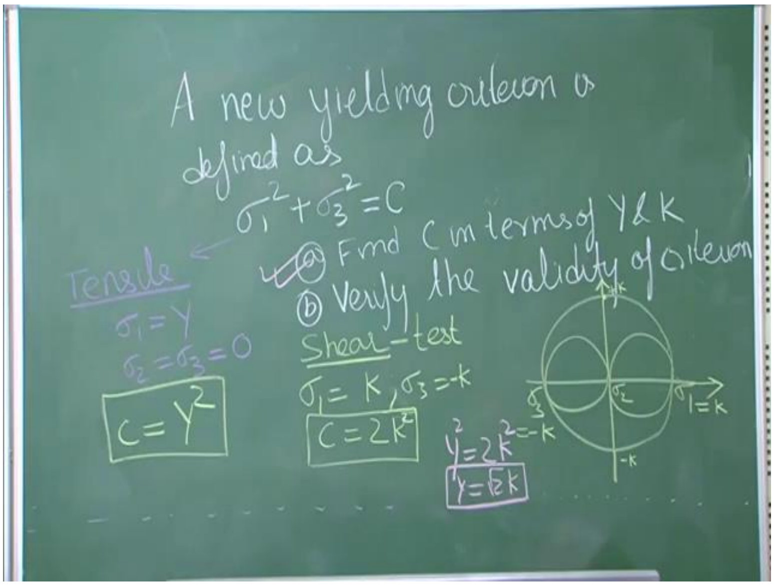
Fundamentals of Materials Processing (Part- II)
Prof. Shashank Shekhar and Prof. Anshu Gaur
Department of Materials Science and Engineering
Indian Institute of Technology, Kanpur

Lecture – 06
Effective Stress and Strain

So, I left you with this particular question on yielding criterion and what we asked or we gave to you is a newly defined yield criterion, sigma 1 square plus sigma 3 square equal to c, sigma 1 and sigma 3 as you know are principal stresses, c is a constant. So, the question was assuming of course, that it is valid find c in terms of y and k that is yield strength and shear strength and in the second part to verify whether this criterion is valid at all or not.

(Refer Slide Time: 00:54)



So, this is again like I said earlier also that it is not very difficult and it is a straight forward question, what we need to do is if you want to find c in terms of y, you have to assume tensile condition or tensile test; assuming that we are doing a tensile test on the material and you know the yield strength of the material as y at the condition where the stress reaches y in that particular case sigma 1 is equal to y, sigma 2 is equal to sigma 3 is equal to 0 and when you put it over there, what you get is c equal to y square.

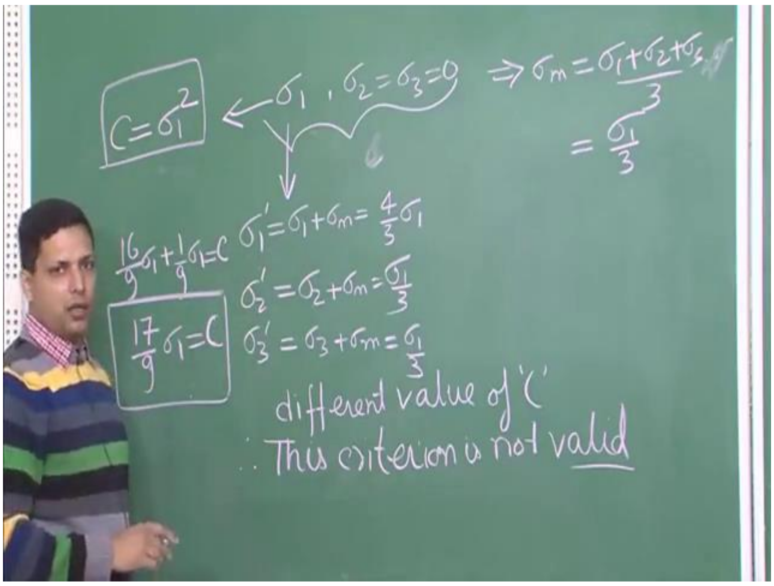
So, if you put sigma 1 equal to y over here, sigma 3 equal to 0 then you straight forward get c equal to y square. Now let us assume that you are doing the shear test and again you know the shear strength of the material which is k, now here sigma 1 is equal to k, remember the mohr circle which we drew, so let me draw it again for the sake of understanding. So, this element let us say it has started to yield under shear stress, so this is your k plus k this is minus k and since k is the radius, so sigma 1 is also equal to k sigma 3 is equal to minus k and this is what we are doing over here, we are just putting sigma 1 equal to k, sigma 3 equal to minus k.

So, this is equal to k square plus k square equal to c, so it is equal to c is equal to 2 k square. So, the first part was even is in fact very very straight forward nothing as long as you understand how to draw the mohr circle for tensile condition and shear condition, you can put in the values of sigma 1, sigma 2, sigma 3 and then put it in over here and you will get c, so c is equal to y square and c is equal to 2 k square.

Now, if you put them together what you will get is also a relation between y square; y and k. So let us put it like this; since c both of them are c which is the same value, we can say y square is equal to 2 k square or y is equal to root 2 k. So, this is again giving us a relation between y and k, so that takes care of the first part of the problem.

Now, comes the second part which is a little bit more involved but still very straight forward. You have to verify the validity of the criterion; you remember the one of the assumptions when we talk about the yield criterion is that the mean stresses should not influence yielding.

(Refer Slide Time: 04:25)



Which means if I take the mean stresses; let us say I take some sigma 1 and in a condition where sigma 2 and sigma 3 are just 0 to begin with, let us say this is one particular condition to begin with, here the mean stresses would be equal to which is equal to sigma 1 by 3.

Now let us say that we add this mean stress to all the three principal stresses, so now sigma 1 becomes sigma 1 prime, which is equal to sigma 1 plus sigma m which is equal to sigma 1 plus sigma m which is equal to sigma 1 plus sigma 1 by 3, so it is 4 by 3; sigma 1, sigma 2 becomes sigma 2 prime, which is sigma 2 plus sigma m which is sigma 1 by 3 because initially sigma 2 is 0. Similarly, sigma 3 prime is equal to sigma 3 plus sigma m, which is equal to sigma 1 by 3. So, what we have done is we took the initial condition like this and we have added hydro static stress or mean stress to each of the components and from that we are able to now get news principle stresses or in effect, we have just moved the circle by the side.

Now, if the criterion is valid then if we apply the yielding criterion on this and the yielding criterion on this particular circle which are of the same size, we should get same value of c because either way if the yielding has taken place, c should be less than that; if yielding has not taken place then c should be the value of this sigma 1 square plus sigma 3 square should be less than a critical value and if the yielding has taken place, sigma 1 square plus sigma 3 square should be higher than this critical value c and if it is just at the yielding then it should come out to equal to c in both the cases.

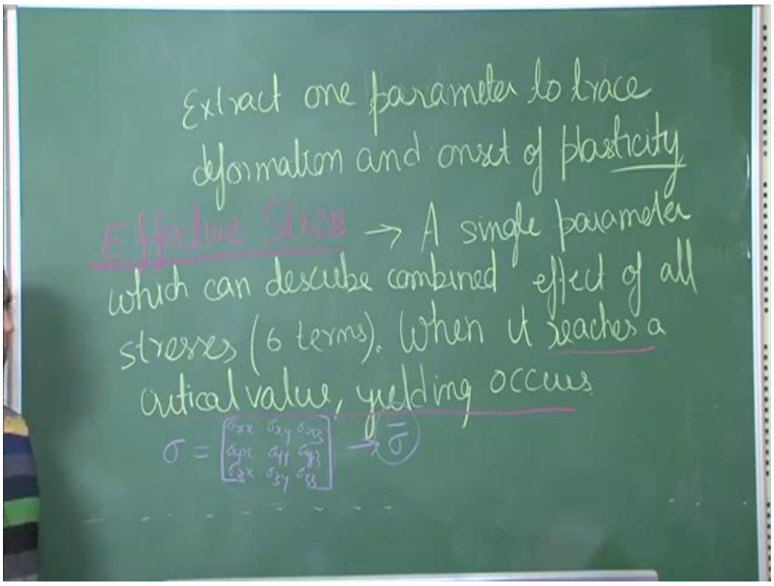
So, now let us look at this particular condition over here we have already seen that c is equal to from over there, if we look at simple tensile condition where we have sigma 1 and therefore when we put in the first equation which is sigma 1 square plus sigma 3 square equal to c, you see that c is equal to sigma 1 square. So, that is the value of c whatever be the value of sigma y, most likely we will put y over here, but I am even trying to put the value over here we are just saying some value sigma 1.

Now will do the same thing over here, now here we have sigma 1 square, this is sigma 3 square. So, this becomes 16 by 9; sigma 1 plus 1 by 9; sigma 1 equal to c, so c has now become 17 by 9 sigma 1 is equal to c . So, now what do we see that when you apply the same yield criterion before applying the mean stress and after applying the mean stress, you get different value of c; in effect what it is saying is that the material will deform differently or under different condition before applying mean stress and material will deform differently or the onset will take place at different value, when you have applied mean stress. But we know this is not true, we know that mean stresses do not influence yielding therefore, this criterion is not valid.

So let me repeat; why we are saying that this is not a valid criterion. We have taken; you have applied the yield criterion onto the first condition where we have not applied any mean stress. Second condition we have again applied mean stress and again we applied the same and we use the same yield criterion, we get different value of c and different value of c means that here the yielding will take place at different value, here the yielding will take place at different value; meaning yielding is getting onset of yielding is getting influenced by hydro static stress which is not possible, our assumptions categorically stated that hydro static stresses cannot influence yielding and therefore, this cannot be true and therefore, this criterion is not valid. So, that is the logic why we are rejecting this.

So again as long as you are able to see what we are doing here, you can realize that it is again very straight forward and does not involve a great deal of mathematics to solve this. So, now we have dealt up to good extent in the different kinds of yielding criterion and we are now in a position to say no matter what is the axial; what kind of multi axial stresses are being applied, whether the material will yield or not. But as a mathematician or as a person dealing with mechanics of materials; I am still not happy.

(Refer Slide Time: 10:41)



We need something more; what is that we need more; we want to be able to use or extract one parameter to trace deformation and onset of plasticity, so this is what we want further. We have solved good amount of problem to certain extent, we are now in a position to not get worked down by too many terms in stresses, we can still deal with it, I just one value as long as the value of that criterion defined by that criterion c reaches a value or when the criterion reaches that critical value c, we can say plasticity has taken place or has onset, but we also want to be able to extract or represent the whole stresses in one term and that is called effective stress.

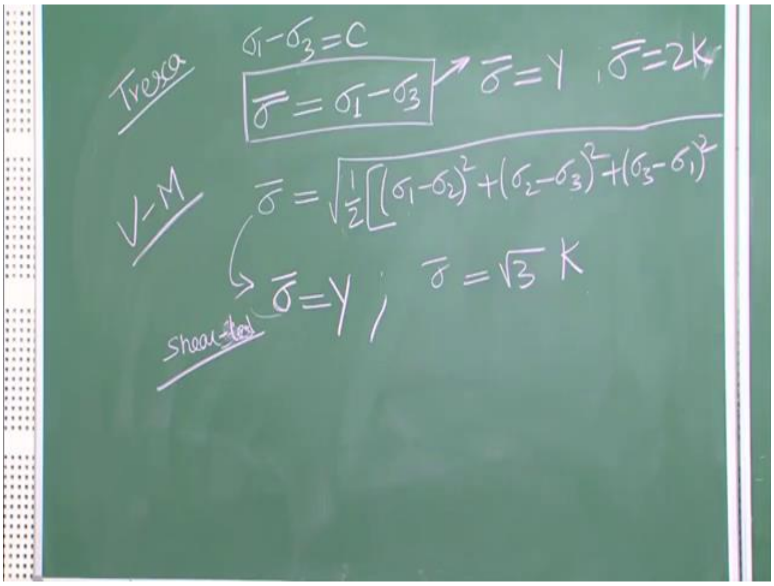
So this is our next goal, we want to be able to define or be able to represent all the various terms in the stress tensor by some singular quantity called effective stress.

Let me define what exactly is effective stress; it is single parameter which can describe combined effect; all stresses. So, for example, in our tensor we have 6 independent terms and we can say that when it reaches a critical value when yielding occurs. So what we are saying is that, if you have stress tensor given by this; again you remember sigma xx, sigma xy, sigma xz, sigma yx, sigma yy, sigma yz sigma zx, sigma zy and sigma zz, so this is yz.

Now this has 9 terms, out of which 6 are independent. What we want is that to be able to represent the different terms or the combined effect of these terms by one single parameter and this is what we want to represent as sigma effective. So, we want to get this value sigma effective. So, that we do not have to deal with so many different terms. So far what we were doing was just to find out the onset of yielding; now we have expanded our horizon. Now we want to say whatever behavior, whether it is onset of yielding or not onset of yielding; everything we should be able to define with this one single term and that is what we are calling as effective stress sigma y.

Now here one thing gives us a hint on how to get this value and it is this, when it reaches a critical value yielding occurs. Now that sound familiar, we just did something similar to that; we were able to get yield criterion and based on that we said that that when equation after or that when formulae reaches a critical value then yielding takes place. So, our relation for sigma effective lies in that definition of the yielding criterion, we will see how in a moment.

(Refer Slide Time: 15:32)



So now let us say again, we are assuming that the material follows Tresca criterion. So, we have to make an assumption, you remember I said that when we are talking about elasticity then we have a very perfect relation; Hookes Law defines the behavior very perfectly but when we are in the plastic regime, we do not have a exact relation; we have approximate relations and these criterions are part of that. So, we have to assume or have an understanding from somewhere else, what particular criterion the material follows and in this particular case let us say the material is following Tresca criterion and we knew;

we know from our; just in the previous lecture we saw that this Tresca criterion defines the yielding condition to be like this.

So now you see these are stress terms, so this c value is also nothing, but has a unit of stress and therefore, we can define sigma effective as sigma 1 minus sigma 3. So, now instead of this being a critical value, we are saying just find out whatever the value is and that is the effective effect of all the different stresses and in this particular case we have already said that it is only the difference between the two which influences the plastic behavior. So, this can also now which has a unit of stresses, can also be this term sigma 1 x sigma 1 minus sigma 3 can also be defined as the effective stress. So, this is a definition in when we are assuming the material follows Tresca criterion.

Similarly, if we say that the material follows Von Mises and over there we had that long relation. We can now with a little bit of manipulation, we can say so that the units have on the RHS also have the units of stress, we can put it in this form and also put some factors; this factor you would see has a interesting position here. So, these are as you can see squared terms, but they all are under square root which means that the overall value will have the units of stresses and the criterion that we defined was very similar to this where this whole and everything inside this should have reached some critical value. Now instead of talking about that one just critical value where yielding occurred, we are saying what happens before that, after that can be used this as effective stress to define the overall behavior and that is what we have done. So, instead of saying a particular c value right now we are saying that this is representing the effective stress, so this is our effective stress.

Now over here let us say we have sigma bar equal to y, which is effective stress is equal to y. In this particular case, it can happen when can it happen for example, in the Tresca criterion let us say when we are looking at uniaxial tensile condition. So, when we are looking at uniaxial tensile condition, we know that sigma 1 minus sigma 3 over here at the just at the onset of yielding sigma 1 is equal to y and sigma 3 is equal to 0 and therefore, sigma bar is equal to y.

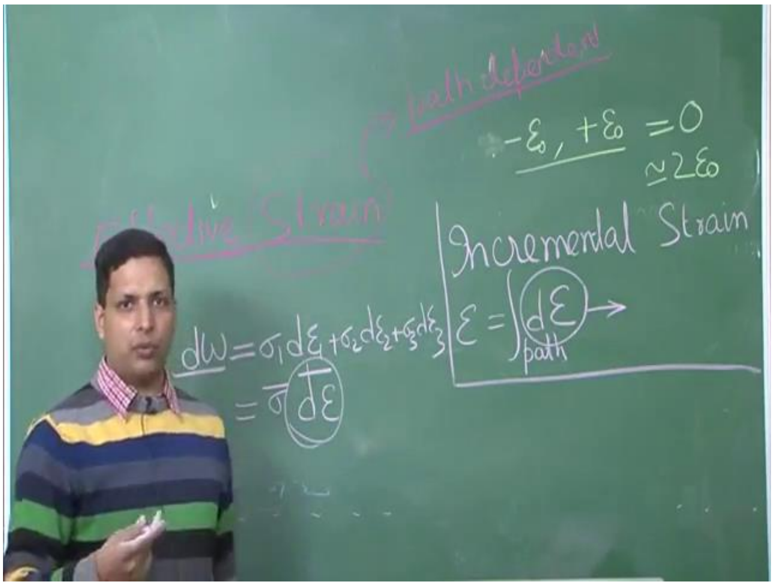
Now, similarly here we are if we are talking about uniaxial tensile conditions sigma 1 is again; y sigma 2 and 3 at 0 and therefore, it becomes 2 sigma at 1 square. So, 2 by 2 and it becomes square root under sigma and therefore, it again becomes sigma bar equal to y.

So, in both of these we see that it does indeed follow the uniaxial tensile condition and that is one reason why this factor 1 by 2 was put over here. So if this allows us or this as allowed us so that the sigma bar is similar to our sigma true stress, so sigma true and sigma bar are synonymous at this point.

Now, let us talk about shear stress condition or shearing, so basically we are talking about shearing shear test. So, in the shear test sigma 1 is equal 2k and sigma 3 is equal to minus k and therefore, this becomes sigma bar equal to 2k. So, in this particular case sigma bar is equal to 2k. Now similarly if you put the shear test condition over here, we have sigma 1 equal to plus k sigma 3 equal to minus k and sigma 2 equal to 0 and what you will see is that sigma bar comes out to be root 3k.

So, this is just for your information that these are the values that you will get when you use Tresca or assume Tresca criterion condition and when you use Von Mises condition. But the important thing is that we are in a position to define effective stress of course, assuming a particular kind of criterion; Von Mises or Tresca or we you may even have your own yield criterion, so based on that you can define effective stress. Now if you have effective stress then we also want to have effective strain, if you have effective stress and effective strain then things will become so much easier. Now instead of dealing with multiaxial stress, multiaxial strain; it will all boil down to just two numbers effective stress and effective strain and you will be able to define the whole deformation geometry or the deformation condition by just these two numbers, so, that brings us to the next term which is effective strain.

(Refer Slide Time: 22:17)



Now, it is much easier said than done why because strain is path dependent; remember you cannot simply add or subtract strains. Just give you a very simple and straight forward example let us say; I deform a material under compressive strain. So, where I take the strain all the way to since it is compressive so that strain was taking all the way to minus epsilon, sum value naught and then I leave the material. So, it has been plastically deformed and the deformation has been taken to minus epsilon naught.

Now, I take this material and do another kind of deformation where it is given plus epsilon naught; a tensile kind of strain where it is deformed to plus epsilon naught. If you were to vectorially or even scalarly add these two quantities, what you will get is that the total strain is 0, but is it really 0; if you do two deformation then the total strain does it come back to 0; no absolutely not in fact, if anything it is closer to 2 epsilon naught. So, our strains that is why we say are path dependent and our aim to get a effective strain is not so straight forward or at least we have to make some more assumptions, so what are those assumptions.

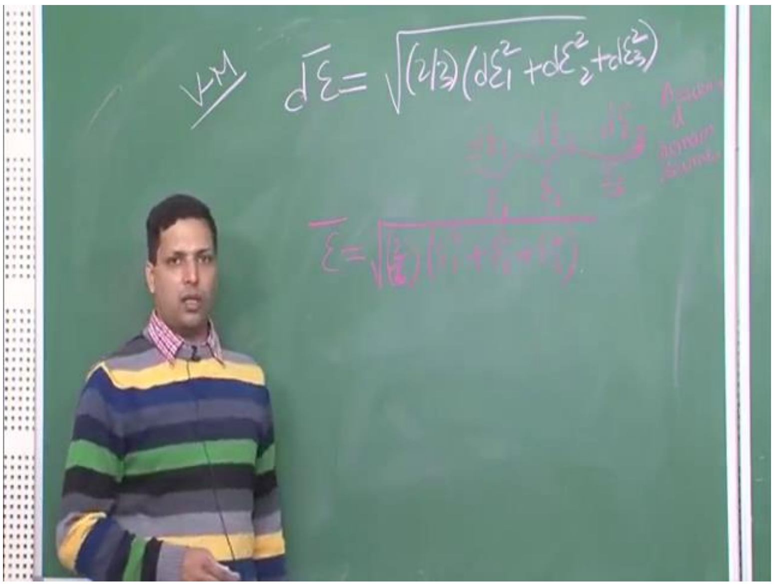
First of all what we will need to do is, we will have to use incremental strain and if you use the incremental strain then in that case; strain is based on the path that you are taking sum of these incremental strain and from this you would be easy able to show very easily that minus epsilon naught plus epsilon naught will come out to 2 epsilon naught and this is the incremental strain and you remember when the strains are incremental then we can do a lot more mathematical transformations, we can do axis changes, we can deal with them as a matrix as a tensor quantity but when the strain becomes much larger then we cannot do all those things. So, the strains have to be small in value which is; it has to be incremental strain. Now this incremental strain has to be defined, how do we know what is; if we know the I mean we already know what is incremental strain; but in terms of effective strain, we need to define it and effective strain is defined as such.

So if we have sum work done; a small incremental work done sigma 1 d, epsilon 1 plus sigma 2 d, epsilon 2 plus sigma 3; d epsilon 3 then the incremental strain for this is defined as effective stress times d or effective incremental strain. Now this is the definition of our effective incremental strain, so that is one step. Now we are in a position to define what is the effective strain particularly in terms of incremental strain.

Now the next step is to find out in terms of the total strains, now here again we will have to make some assumption and one important assumption here is that let us say we are the strains, the material is being deformed in such a way that epsilon 1, epsilon 2, epsilon 3 which are the principle strains or to be more precise; d epsilon 1, d epsilon 2 and d epsilon 3 remain proportional; meaning one does not increase disproportionately compared to other, whatever strain is being d given; if it is let us say 1 is to 4 is to 1, it remains 1 is to 4 is to 1 throughout.

Now, when you integrate it then the integration will also remain in that order.

(Refer Slide Time: 27:01)



So, it will remain 1 is to 4 is to 1 and if you taking that into account one can write that, one step I missed over here. So, here it has been shown that if your d epsilon bar is your incremental strain then incremental strain can be given as this is for Von Mises; assuming Von Mises criterion, so now this is the equation for incremental effective strain.

Now, like I said if we assume that the ratio between d epsilon 1, d epsilon 2 and d epsilon 3 remain constant. So over the period of integration, it will remain same and their proportions would remain same and therefore, the epsilon 1 epsilon 2 would also maintain the same proportion and hence we can assuming this that this proportion remains same, we can have a effective strain total effective strain defined as 2 by 3 epsilon 1 square plus epsilon 2 square plus epsilon 3 square. So, under certain condition we are now able to get also a relation for effective strain.

So we have now effective stress and effective strain, we will deal with these two a little bit more and we will see the significance and relation with our very old friend uniaxial stress condition or tensile stress strain curve in the next class, but for the time being just go through the equation, you have not defined or derived anything. You just need to see the form of this, just try to play around a little bit to see what are the different values over here. So, we have this, so we will leave it with leave you with this thought and you can think about it, try to solve some problems based on this and will relate this like I said with the uniaxial stress strain plot in the next class.

Thank you.