Fundamentals of Materials Processing (Part- II)
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Lecture - 07
Work Hardening and Flow Behaviour

Welcome back, so we were discussing about effective stress and effective strain. So, let me rewrite some of the equations that we discussed yesterday.

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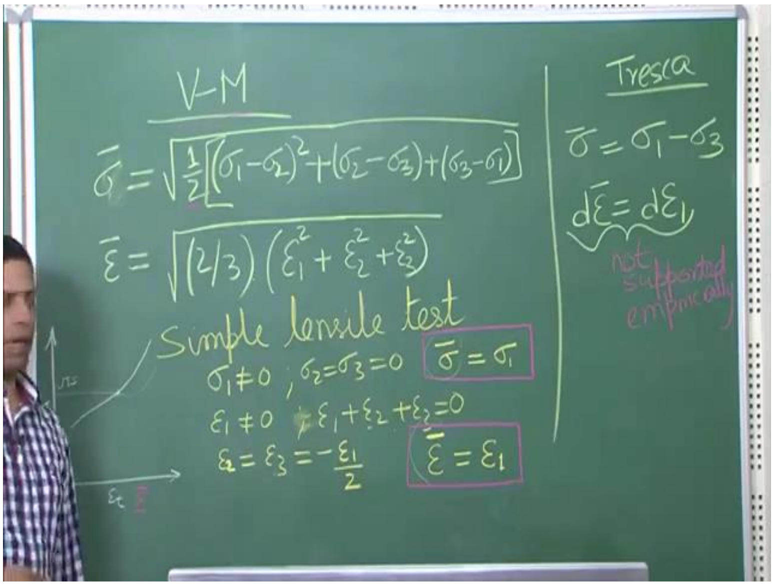

So if we are talking about Von Mises, we talked about effective stress and we came to the relation that sorry we have to (Refer Time: 00:35) stress. So, effective stress is equal to 1 over 2; sigma 1 minus sigma 2 whole square plus sigma 2. Remember we came to this form because this is also the form for the yielding criterion for Von Mises. So, this is the effective stress in Von Mises and effective strain that we came to, we did not derive it, again we just wrote it down that this has been shown assuming that your incremental strains d epsilon 1 and d epsilon 2, d epsilon 3 are proportionate throughout then we get this 2 relation for Von Mises condition.

For Tresca also we said that there is effective stress and that effective stress is nothing, but sigma 1 minus sigma 3 and we said that there is a particular reason why we have 1 over 2 over here, we will get to that again very in a moment very soon, but before that to complete the picture; let me also put down what will be the effective strain if we were to consider only the Tresca criterion and it will be d epsilon 1. So, what it is saying is that in the Tresca condition or in if you are assuming Tresca criterion then the effective strain depends only on one principle strain, but empirically this is not supported empirically therefore, what you will see is that this Tresca condition for or the effective stress and the strain under the Tresca criterion is not used very often and what is used is Von Mises criterion.

Now like coming to the point on why we have using 1 over 2 and 2 over 3. We can understand the form of the equation, it has to be of the same form as our yielding criterion, but why do we have to have 1 by 2 and so on. Now let us consider simple tensile condition or tensile test, so in tensile test your sigma 1 is not equal to 0, sigma 2 is equal to sigma 3 is equal to 0; epsilon 1 is not equal to 0 and we can add the non compressibility rule which is that not plus (Refer Time: 03:46) what I am trying to say is epsilon 1 plus epsilon 2 plus epsilon 3; equal to 0 and since epsilon 2 and epsilon 3 which are on the perpendicular on the transfer sides, they will be of equal magnitude; you can show that epsilon 2 equal to epsilon 3 is equal to minus epsilon 1 by 2.

So, this comes directly from this, so if you have these 2 condition because we have the primary strain in one along that direction in which you are applying stress in the other 2 direction because of the non compressibility. So, remember in tens simple tensile test sigma 2 and sigma 3 which are the stresses on the perpendicular direction are 0, but not the strains on the other 2 direction there is still some strain and that is because there has to be, they cannot be the material cannot get compressed there has to be some amount to strain you cannot or in this case actually if you keep elongating the material without it getting contracted on the side, it would mean that the total volume is increasing which is what we have forbidden, we had taken an assumption and that is the valid assumption volume does not change during plastic deformation, not in the range we are interested in.

This is actually 2 to be précised, so now what you will see is that if you take put this value over here what you will get is epsilon bar. Now put epsilon 2, epsilon 3 equal to minus epsilon y by 1 by 2 over here. So, you will have epsilon 1 square plus epsilon 1 square by 4 plus epsilon 1 square by 4. So, you have 2 epsilon one square by 4 and which is one by 2 epsilon one square and this is epsilon 1 square. So, that is 3 by 2 of epsilon 1 square and that gets cancelled over here. So, you are left with square root of epsilon 1 square therefore, effective strain comes out to be epsilon 1. So, you see in our simple tensile condition, epsilon bar is coming out to be equal to epsilon 1 which is the primary strain although the other 2 strains exist, but in the effective strain is actually coming out equal to the primary strain.

Since sigma 2 and sigma 3 are 0, you put it over here and what you get is that sigma bar is equal to sigma. So, we have 2 very important results over here your effective stress is same as the stress that you get in uniaxial tensile condition. Effective strain is same as the uniaxial strain or the strain in the uniaxial tensile condition which means it has a very very strong implication, if you can understand the meaning you will be really surprised what it is actually saying. So, you have sigma 2 versus epsilon 2; the stress strain curve let us say for a simple tensile test. So, you do the tensile test and somewhere over here you have the UTS remember this is true stress true strain. So, the curve does not come down it keeps on increasing remember, the load will keep increasing. So, after the UTS also there is increase we are we have not talked about the equation that will describe this, but the curve or the actual stress will keep on increasing let us not get into the details of that, but the stress is increasing that is solve is suffice to say.

Now, this true stress, true strain plot that we obtained from a uniaxial tensile test is representative of any kind of deformation that you given the material. All you need to do is get effective stress value from there and effective strain value from that particular deformation and that will be represented by this same graph by the same plot. So, the effective stress and the effective strain plot would be exactly with way the same exactly same as that that you obtain by uniaxial tensile stress tensile test. So, the sigma 2 epsilon 2 for uniaxial tensile is same as the sigma epsilon and sigma bar or the x sigma effective and strain effective for any kind of deformation.

So, that is the importance of putting in these numbers 1 by 2 and 2 by 3 because it is because of this we do not have to put any factor otherwise what would have happened if there were no factor over there you may have ended up with a factor on this sigma bar with respect to sigma true and therefore, although the form of the curve would have been same, it would have been shifted or it would have been multiplied by some factor, but since it is a constant value why not is start with our definition such that these 2 overlap. So, that was the idea behind getting putting on these factors. So, that is like I said a very very important relation that we obtained and this is again do not forget that we are obtaining all this under Von Mises criterion, we are not dealing with Tresca because I said as a like I said that the effective strain relation that is given for Tresca is not supported empirically. So, it is not dealt with or it is not studied or researched to that extent.

Now, let us get to the other another aspect of this which is now we have looked at the stresses, strain, effective stress, effective strain and how our true stress, true stain plot of a tensile curve can also represent the deformation under multiaxial stress strain condition. So, that part is covered now we are in a very good position to be able to predict what will be the stress or what is stress will be required for a given strain, so that is done, but now what would you see is that there is even.

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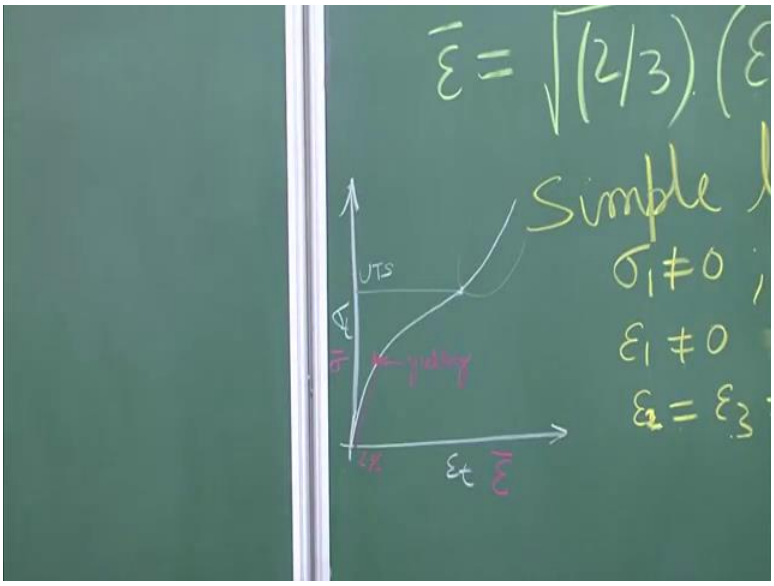

Let us say somewhere over here you get yielding you may not have thought about it. So, much in depth earlier, but now look at it and realize that at this point at this whatever strain if you look at it over here it is 0.2 percent. So, beyond this 0.2 percent or whatever strain, if you look at it subtended over here; once the plastic material is started deforming plastically, you still need to increase the stress. So, that is something very intriguing, will not get into the y of that.

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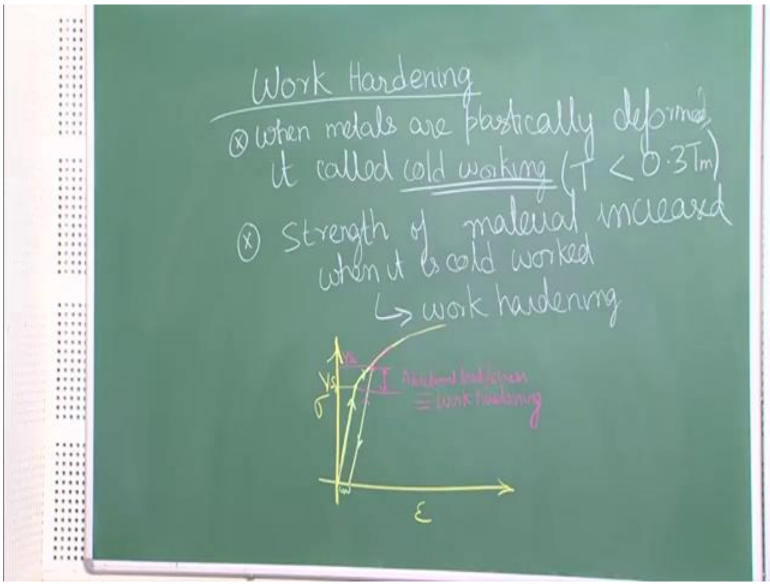

But let us understand or a few basic things about it, it is first of all its called work hardening. So, whenever metals are plastically deformed, it is called cold working provided the temperature is less than 0.3 tm, I have yet described what is tm, but this is melting point and we are more interested in the homologous temperature, we will get to those things little bit later on, but for now just realize that when the temperature is less than 0.3 tm which is the melting point of the material. So, 0.3 times of the melting point of the material and if you are deforming at plastically then it is called cold working.

Now when you are doing cold working, what happens is that strength and this is for to understand it in greater detail you need to understand mechanical behavior of materials, but we are not getting into those details in this particular course because it is altogether a different subject, but we will just give you idea enough, so that you can apply it.

Now, strength of a material and there of some increases when it is work hardening or when sorry when it is cold work. There several types of things that goes on because of which the strength increases. So, that is why I am saying you are not getting into the voice of it, we are just looking at what happens and this is what we call as work hardening. So, the moor the material is deformed higher is its strength that is what work hardening would mean in summary. So, if you have deformed it to 20 percent, it has some higher strength if you deform it. So, to say 40 percent it will have even higher strength; strength meaning if you do the tensile test you will get a yield strength value which is higher, so that is what it means.

Now, let us look at it in terms of a plot again; so let us say again we are doing, now we can do simple tensile test, but of course, we know that it has greater significance, it will also imply the deformation behavior of under multiaxial condition. So, when I say uniaxial tensile test, we cannot think of it as just one simple experiment, it is representing the stress flow behavior or the stress strain behavior under different all different kinds of deformation. Now let us say this is the point which is we call as yield strength, so this is the original yield strength of the unyield material. So, I am using a term unyield it what it means is that the material is not at all deformed you whenever you heat the material at a very high temperature which is called as the solutionizing temperature and you bring it back to the room temperature then that material is in unyield condition and in that condition it does not have any effect of previous cold working. So, even if you may have done any kind of deformation that effect does not carry over if you have done unyielding.

So, let us say we have starting with the material which is unyield so now, it has no history or it does not show any behavior from its past deformation. If you take the material from over here to here which is the elastic region and leave it; it will come back. We know that that; it is because of elastic properties that is the elastic behavior of the material, beyond this which is we call as the yield strength if we the material starts to show plastic behavior, meaning if you take it up to here and then to leave it; it will come back, but it will have some ruminant deformation.

So, this amount of deformation will remain in this, now if you try to deform this material again do not expect the material to go through this cycle again, what it does is actually it goes through. So, here the material now if you start to deform it again or you start to apply stress again, the material follows this curve actually. So, what it is saying is that if you now leave this at this particular point somewhere over here it will come back. So, it will have elastic behavior all the way up to this point and it will get into the plastic mode only beyond this point. So, it means that yield strength of the material I will call it 2 and over here I will call 1, so the yield strength of the material has increased this is what we meant when we said work hardening. This material has being left with so much of plastic deformation. So, now when you try to deform it you will need additional or a higher amount of stress to deform it too plastically to be able to plastically deform it and this is what is called is additional amount of load or a stress that you need to this is what is called as work hardening.

Now, once we know this work hardening behavior there is a related concept which is called flow behavior of flow curve, it is nothing but just try just plotting or just explaining the behavior only in the plastic portion of the material.

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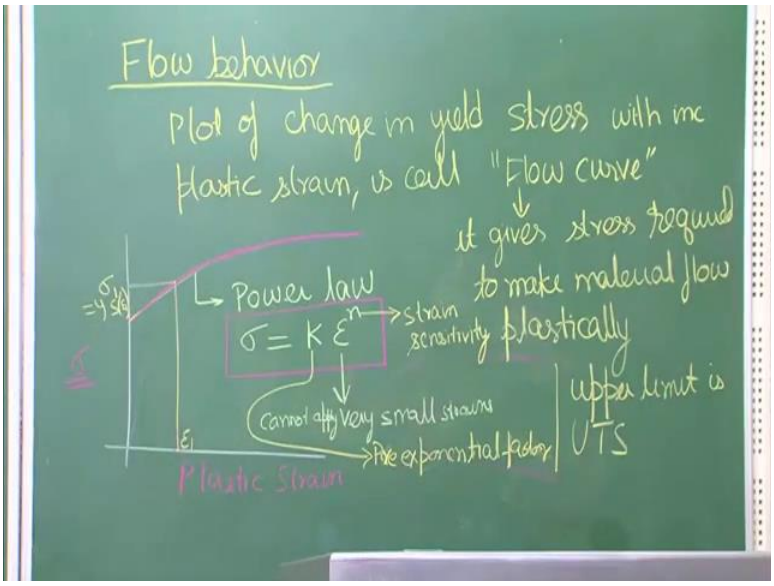

So, what we are talking about is flow behavior, the term flow actually comes from the fact that when you are deforming the material plastically it is it does not come back. So, it flows it does not it has a behavior similar to that of you can say liquid or semi liquid. So, it does not come back it will once you have deformed it, it remains in that shape or it moves it has moved ahead that is what we mean when we say flow behavior. What it is plot basically plot of change in yield stress, so you saw that the yield stress is increasing, the increasing the fact that the yield stress increases is called work hardening.

Now, if you plot that yield stress or you take the (Refer Time: 19:41) of that yield stress with increasing plastic deformation, what we get is called flow curve and this behavior is called the flow behavior, it is called flow behavior because it gives stress required to make the material flow plastically. So, it is not flowing in the sense of liquid exactly, but you can say there are some characteristics similar to that and with which is why it is called flow curve.

So, it gives the stress required to make material flow plastically, now if you remember our again the old stress strain curve, it had elastic region and the plastic region. Now if we get rid of the elastic region which is this one we are remained with only the plastic region and let us plot it over here and we get something like this and here it is plastic strain, no more just true strain. So, we are only talking about plastic strain and this is the stress, so actually it represents nothing, but the yield stress for a given strain. So, let us say you have deform the material up to this point, so let us say you have deform the material up to this point strain 1, so there will be stress required to deform the material beyond this strain and that will be let us say sigma 1 which is nothing, but the yield stress at that point, so sigma 1 is nothing, but yield stress hat strain 1.

Now, what is the form of this equation we that will be very helpful for us then we would be able to predict the stress value, stress required or the stress yield stress of the material after particular deformation. So, you can look at it two ways, if you know the equation then you can say that what will be the yield stress of the material or you can say what is the stress required to deform the material up to that point because you must be able to have stress at least up to that value only then the material will deform to that point and again like I said I have cup said it a cup couple of times earlier. If you are talking about the elastic properties, we have a very exact relation that is given by Hooke’s law, but if we are talking about plastic behavior then we do not have a very exact relation and here also we have one relation which is empirical which has been obtained, you can say based on empirical experiments and which has some amount of you can say which has been verified to some extend by experiments and it is called power law relation between the stress and the strain.

So, what this relation is it is like this, so if you put this is strain over here, you will get this stress over here. So, like I said it is giving you two things; one what is the yield stress after you have deformed it to this extent and what is the stress you must be able to apply before you can get to this deformation.

Now, there is again couple of things that you must be aware of when you are using this relation then you cannot apply it for very small strains. For example, never try to estimate the yield strength of the unyield material from this by using strain equal to 0. 2 percent or which is 0.002, it is not valid in that range. You will have to you can say it is valid beyond something like 0.01 or 0.02.

So, you cannot apply this relation below that strain because as we remember this is valid only when you are in the plastic strain regime, if there is a any amount of elastic regime remaining in that strain then this relation will not hold and therefore, you have to be careful when using this equation. Now this is called pre exponential factor and n is called strain sensitivity. Now we talked about what is the lower range where it is valid it is not valid for very very small strain there is also the upper extreme or the upper limit where it is valid and which is by UTS, you remember we have the uniform strain until UTS and only up to this point we will be able to apply this. So, there is the upper limit is the UTS, so one should be able to apply this in range between this, you can you cannot apply it for very small strains, but and you cannot apply it for very large strains for example, beyond UTS because then the some of the assumptions break down we are no longer in the range of uniform elongation or uniform deformation.

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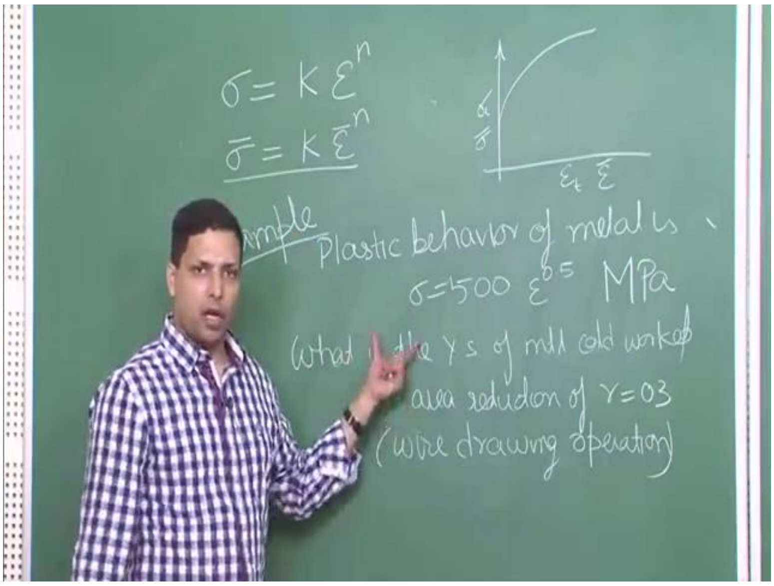

Now, with that let us look at some additional facts or how to deal with this power law relation in a little bit more detail. So, now we have now we have sigma equal to k epsilon to the power n, but we also know that this is describing what our flow behavior from let us says say uniaxial tensile stress strain and we also know that this sigma true epsilon true is also equal to sigma effective and epsilon effective or strain effective which is represented by the same curve. Therefore, this relation can now be also be written as stress effective is equal to k times strain effective times n. So, now we are in a position to also describe this relation for multiaxial deformation. Earlier when we wrote this was meant only for uniaxial stress strain condition, but now given the fact owing the fact that sigma true is same as effective stress and epsilon true is same as effective strain we can write this relation sigma bar equal to k times epsilon bar.

Now, let us let me write down an example will not solve with this time, I will give you time until next lecture I will leave you with this example problem to think about plastic of a metal is given by sigma equal to 500 epsilon, what is the yield strength of material, cold worked to area reduction of r equal to 0.3; meaning 30 percent reduction in area and you can assume something like a wire drawing operation. So, given this relation find out the yield strength of the material, yield strength meaning nothing but apply this finding after finding the strain value.

So, once you are able to find the strain value, you can find what will be the stress, which is the yield stress after such deformation. So, try solving it and we will solve it in the next class. So, once if you are able to solve it then you will be at least you try to solve it, it will give you a good feel of the numbers. So, we will see you in the next class.

Thanks.