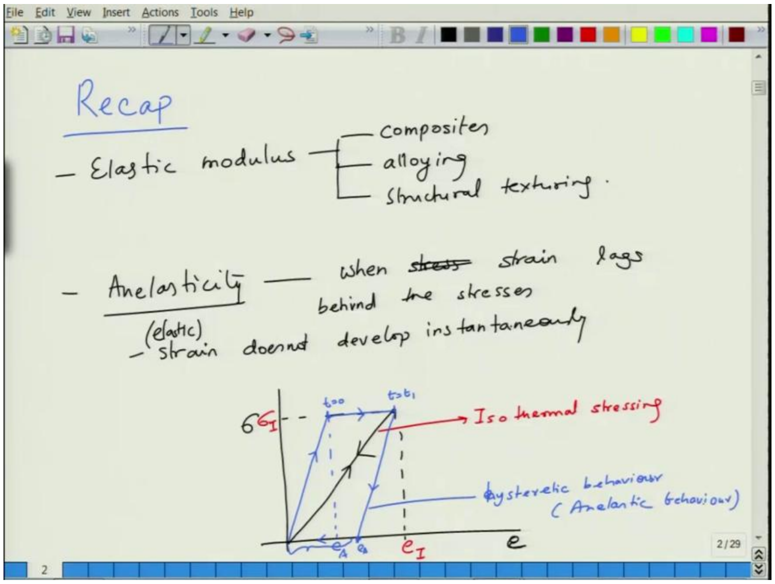
Properties of Materials (Nature and Properties of Materials: III)
Professor Ashish Garg
Department of Material Science & Engineering,
Indian Institute of Technology Kanpur
Lecture 13
Stress strain curve and Anelasticity

So welcome to the course again, this course, Properties of Materials. Let us just briefly recap what we did in the last lecture.

(Refer Slide Time: 00:23)



Recap

- Elastic modulus
  - composites
  - alloying
  - Structural texturing.
- Anelasticity — when ~~stress~~ strain lags behind the stresses
  - (elastic) strain doesn't develop instantaneously

So in the last lecture, we finished our discussion with elastic modulus. Elastic modulus of materials can be tailored by making new materials like composites or you can alloy, alloying, so you can add elements to a given system and then you can change its modulus such as carbon and iron and zinc, copper-zinc alloys and so on and so forth.

And then you can also do the same by structural texturing. So it would not change the fundamental properties, but it can change the directions along which you stress and then you can create the structural texturing leading to different modulus in different directions and so on and so forth. And then we look, moved in to the principals of anelasticity.

So anelasticity we defined as when stress, when strain lags behind the stress that is strain does not develop instantaneously, instantaneously. So this is elastic strain. We are talking about elastic strain here. We are not talking about plastic strains.

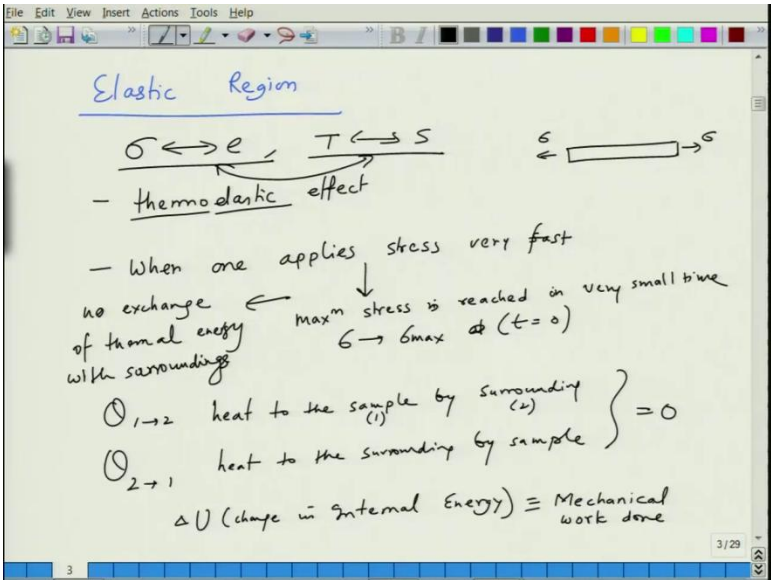
So the elastic strain whatever should develop does not develop instantaneously. So essentially what happens in a ideal situation that you have a, so let us say sigma, e. So ideal situation is like this. When you go on this route, you come back on the same route and the amount of strain, so this is basically we call it isothermal strain or stress. So this is eI, this is sigma I. So this is basically you can say isothermal stressing.

The stressing is done at such a rate so that there is no change in the temperature of the sample. So as a result, the strain follows the stress instantaneously. And when you drop the stress back to 0, the strain also drops back to 0.

Now what happens in real situation, what could happen in real situations is that when you apply stress, let us say, in this fashion. So we reach its maximum stress, the amount of strain that you are able to reach is only this much which is, e let us say, A.

So, basically, and then you leave the sample for some time for the strain to follow this path, then you follow this path. So between this and this, so this is, let us say, t is equal to 0, between t is equal to 1, the additional strain which is eI minus eA develops and then when you drop the load again suddenly at a certain rate, then this strain is again recovered, you can say almost equivalent to eA and then whatever was developed will again drop back to 0 as a function of time if you leave it in this fashion. So this behavior is basically called as hysteretic behavior and basically this is what is called as anelastic behavior. Now what happens microscopically basically, essentially in these processes.

(Refer Slide Time: 04:20)



Elastic Region

$\sigma \leftrightarrow e$, $T \leftrightarrow S$

$\xleftarrow{\sigma}$ $\xrightarrow{\sigma}$

- thermoelastic effect

- When one applies stress very fast

maxm stress is reached in very small time

$\sigma \rightarrow \sigma_{max}$ at $(t=0)$

no exchange of thermal energy with surroundings

$Q_{1\rightarrow 2}$ heat to the sample (1) by surrounding (2)

$Q_{2\rightarrow 1}$ heat to the surrounding by sample

$= 0$

ΔU (change in Internal Energy) $\equiv$ Mechanical work done

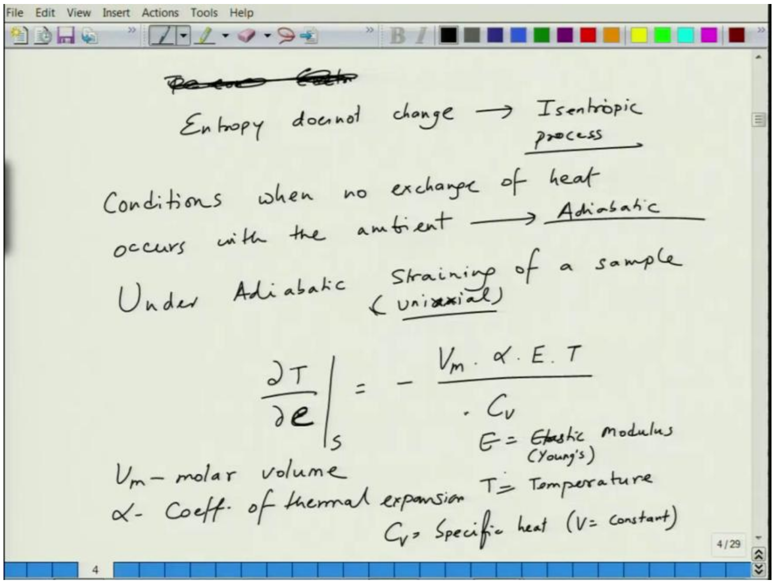
Now just like there is a relation between stress and strain, so in the elastic region, so we are basically considering the elastic region. So just like you relate stress to strain, one can also relate temperature to entropy, because entropy is nothing but a measure of disorder and as you change the temperature the entropy changes.

Now this relationship is known as thermoelastic effect. So these mechanical properties are coupled to thermal properties and coupling of these two is called as thermo, thermo means temperature related, elastic means mechanical related, thermoelastic effect. So these are properties which are dependent upon each other. So we can say these are also dependent upon each other and the coupling between the two is called as thermoelastic effect.

So there are various ways of looking at it. So when you, when one applies stress very fast, when you apply the very, so you have a rod like this and you stress it and you apply the stress very fast, so very fast means maximum stress is reached in very small time. So sigma reaches to sigma max as if it is almost t is equal to 0.

And in this duration, when you apply the stress and reaches maximum, there is no exchange of thermal energy with surroundings. So sample does not exchange any thermal energy with the surroundings. So heat basically which is heat exchanged from the sample. So heat to the sample by surroundings, so Q1, so Q1 to 2, let us say 1 is the sample and 2, so this is 1, this is 2 or Q 2 to the 1 heat to the surrounding by sample both are equal to 0, which means there is no exchange of heat. So in this case, the change in internal energy, let us say, delta U is basically by the mechanical work done, mechanical work done.

(Refer Slide Time: 07:37)



Entropy doesnot change → Isentropic process

Conditions when no exchange of heat occurs with the ambient → Adiabatic

Under Adiabatic Straining of a sample (uniaxial)

$$\left.\frac{\partial T}{\partial e}\right|_S = - \frac{V_m \cdot \alpha \cdot E \cdot T}{\cdot C_v}$$

V_m – molar volume

α - Coeff. of thermal expansion

E = Elastic modulus (Young's)

T = Temperature

C_v = Specific heat (V = constant)

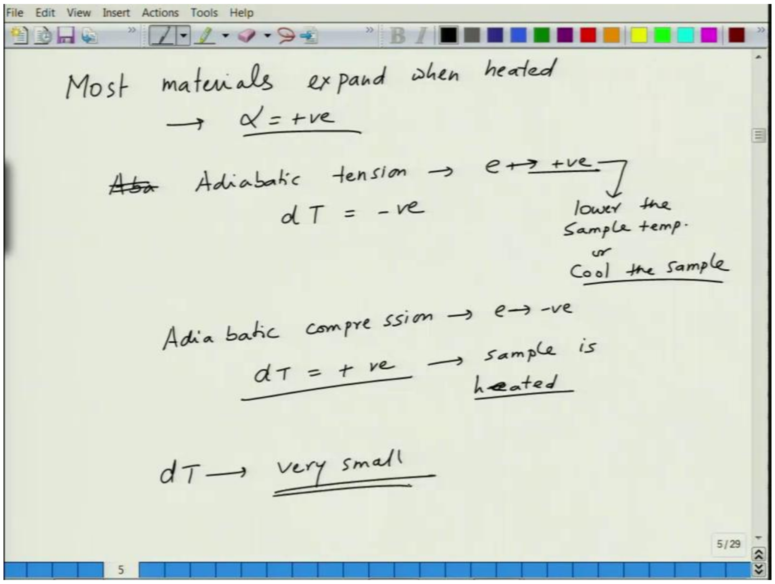
And in such a situation when temperature does not change, entropy does not change, so since t is constant, since t, you can say that the entropy does not change, which means the process is called as this isentropic process, the entropy remains constant.

So, let us say, for, so of the process conditions when no exchange of heat occurs with the ambient, you can define such situation as adiabatic, let us say, condition. So under adiabatic conditions of a sample, let us say, straining is uniaxial, in such a case one can correlate the temperature as a function of, so when I say temperature, so we are not changing the entropy, but of course the temperature is changing.

So del T by del e which is the change in the strain. So change in temperature as a function of strain at constant entropy is written as Vm into alpha into E into T divided by Cv. So we are not going to do the derivation of this, but this can be derived easily. So if you read any book on thermodynamics, you can find the derivation of these. Vm is basically you can say molar volume, alpha is coefficient of thermal expansion and E is the, we can say E is the elastic modulus or Young's modulus you can say, T is temperature and Cv is specific heat at constant volume.

So basically what this tells you, this relation tells you is that there is a dependence of temperature on a strain and these are the terms which take part in this. So, let us say, Vm is constant, E is constant, T is some fixed temperature, Cv is, and it is going to be finite value. Cv is some value. Then depending upon the value of alpha, there will be a change in temperature of the sample to either plus or to minus.

(Refer Slide Time: 10:52)



Most materials expand when heated

→ $\alpha = +ve$

~~Aba~~ Adiabatic tension → $e \rightarrow +ve$ → lower the Sample temp. or Cool the sample

$dT = -ve$

Adiabatic compression → $e \rightarrow -ve$

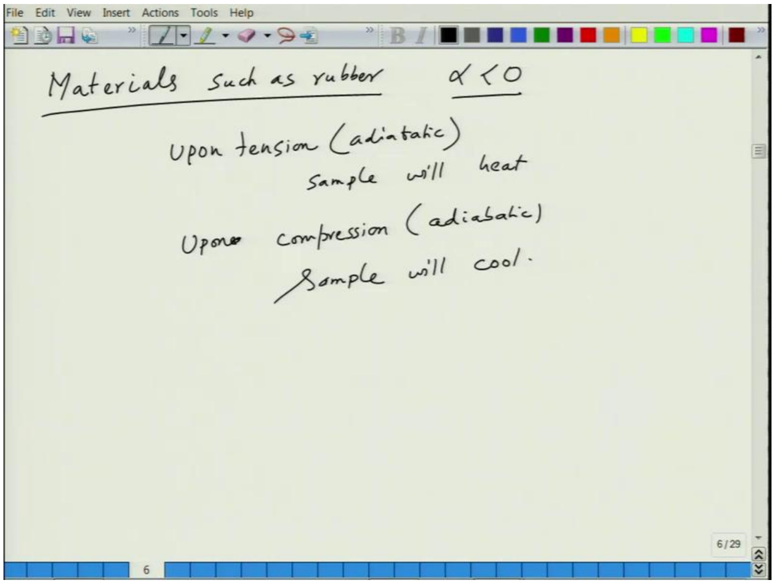
$dT = +ve$ → sample is heated

$dT \rightarrow$ very small

So, most materials expand when heated. So which means basically alpha is positive. If this is the case, then we can say, when you apply adiabatic tension, which means e is positive. Then you can see that dT will be in this case negative, which means, so adiabatic temperature when strain is positive it will lower the sample or sample temperature or in the other words, it will cool the sample.

On the other hand, if you have adiabatic compression, what it means is that e is negative and then dT will be positive, which means sample is heated. So the amount of dT however is very small, but there is a change.

(Refer Slide Time: 12:26)



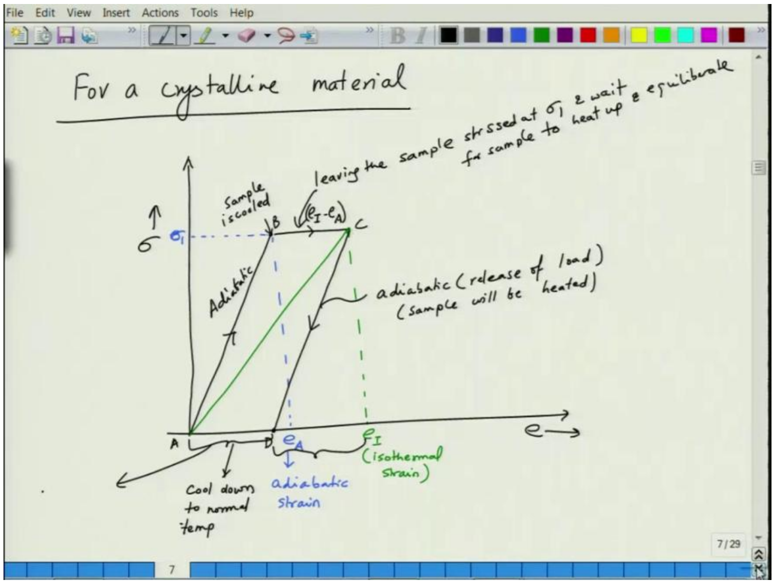
Materials such as rubber $\alpha < 0$

Upon tension (adiabatic)
sample will heat

Upon compression (adiabatic)
Sample will cool.

On the other hand, materials such as rubber, for them alpha is less than 0. So linear coefficient of thermal expansion is 0. So which means, upon tension in adiabatic conditions sample will heat and upon compression adiabatic compression sample will cool. And this is basically because of molecular mechanisms in rubber. We are not going to get into details. So those, but basically, because of the molecular structure of rubber it happens like that.

(Refer Slide Time: 13:32)



So when we are saying, for a crystalline material, so again when we go to a plot like this, so the first step is adiabatic. So this adiabatic tension, so this is adiabatic nature. So up to this point, the sample is cooled. Then it reaches a stress, let us say, sigma 1 and corresponding to this strain is basically you can say eA, this is the adiabatic strain. This strain is smaller than the isothermal strain which would have otherwise developed, if you had isothermal straining up to the same level of stress, then you would have gotten this strain which is eI, which is isothermal strain.

So essentially what will, what you will have to do now is, so this is adiabatic strain, this is isothermal strain, to take to this strain, you will have to allow the sample now to equilibrate. So this is the process of leaving the sample stressed at sigma 1 and wait.

So basically now, when you go through this process, the sample is cooled. Now during this process wait for sample to heat up and equilibrate. So you will get this strain which is eI minus eA. This is the region in which you will achieve this strain.

Now if you again get back to, so now the temperature is higher, it is equilibrated, you reached isothermal strain and when you again get back to the same behavior again, so this is again adiabatic release of load. So under this situation, when you come from A to, when you come from, so let us say, this is A, B, C and D, so when you now, so when you go from A to B, sample is cooled, on B to C, it is like equilibrated, and then again C to D what is going to happen is it is going to be heated. The sample is going to be, will be heated.

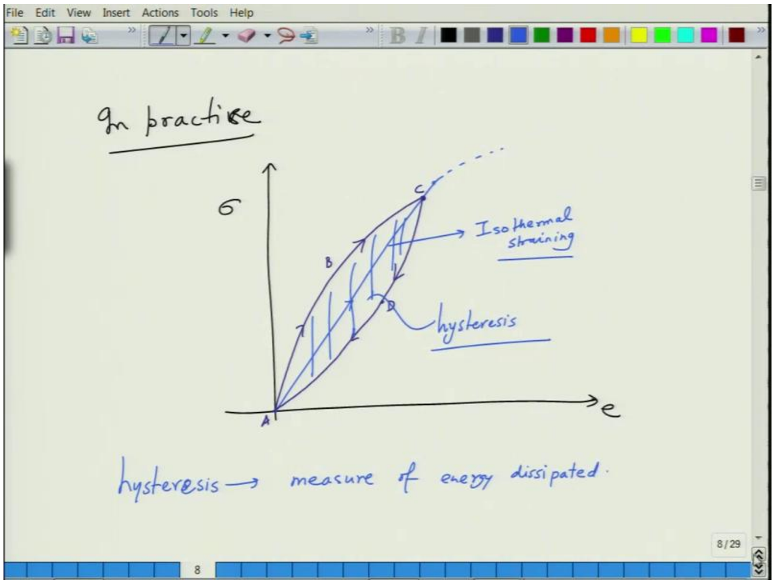
And then again in this region, it will cool down to normal temperature. And again in this region you will have the, so this, up to this point, it will come back from eI to this value and then it will again reach to 0 strain when you leave it to equilibrate. So this is how you will create a hysteresis with the passage of time during interval from D to A, the specimen will transfer its thermal energy to the surroundings or ambient which are cooler, so cool down and then it equilibrates and the strain then decreases by thermal contraction following the route which is D to A.

So essentially A to B it is adiabatic expansion, sample is cooled. You wait from B to C for a sample to trance, sample to get the energy from the ambient to equilibrate and then achieve the isothermal stress, strain level because the strain level which is reached after adiabatic straining is lower than the isothermal strain. And then we reach point C, you again drop the load back to 0. This is adiabatic you can say contraction and this will basically, you, this will lead to heating of sample.

So sample would have been heated by the time you come to point D. And then you leave the sample from D to A, let us say, to come in thermal equilibrium with the surroundings which means it will heat up and you will decrease the strain from D to A by thermal contraction of the sample.

So this is what is going to happen in the hysteresis process and these are basically, so you can say that, these are two adiabatic processes that happen. You have adiabatic expansion during AB and adiabatic contraction during CD. And during BC and DA the sample equilibrate with the surroundings to either go back, go to isothermal strain or to go to 0 strain. So this is what is the anelastic behavior curve that we obtain.

(Refer Slide Time: 19:20)

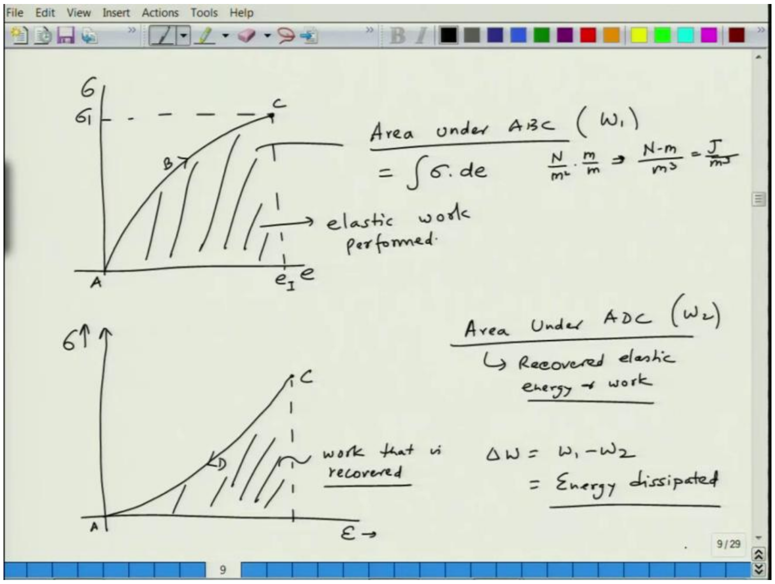


In practice

hysteresis → measure of energy dissipated.

In reality, in practice, what we obtain is not a square shaped curve, but rather we obtain a curve which is more like, so if I plot just the isothermal part first, so what will happen is, if you take the, so you start from A go to, via B to C and then come through D to A. So this is the path that we will follow in reality. And this is the path which is basically you can say is the isothermal and these are the real. So what we get, in whatever condition we do, we get a hysteresis here. So this is the sort of hysteresis which is basically the measure of energy dissipated in the process of elastic deformation.

(Refer Slide Time: 21:00)



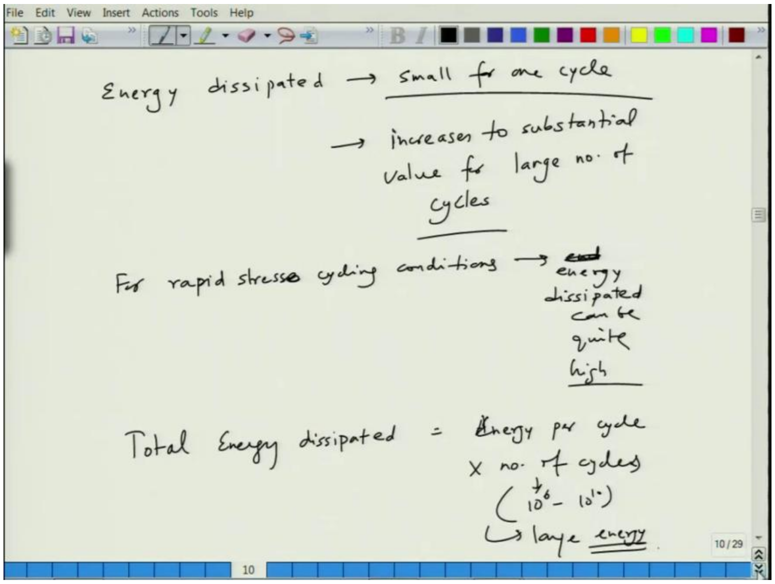
So let us do a bit of calculation, just do a bit of analysis, not exactly a calculation. So when you want to calculate tau, the elastic energy that is stored during the, so this is e, this is sigma, you stretch the sample to this point. So, let us say, we gave from A to B to C. So you reach a stress level of sigma 1 and you achieve a strain level of eI.

So the area under this is basically, so area under ABC will be nothing but square root of sigma into de, which is basically and it has units of stress has Newton per meter square into strain is meter divided by meter, you can say it is Newton meter per meter cube or joule per meter cube. So it is basically per unit volume the energy stored. So this is basically work done. So this is basically elastic work performed.

What happens during the detroit cycle? So, when you come, go from, when you come back from let us say C, so this is C, D and A, so now you are coming back, here you went this direction. So now in this direction the area under the curve is basically, you can say, the work that is recovered. So basically this area under the curve, you can say, ADC. This is the recovered elastic energy or work. This is the energy spent. This is energy that is recovered.

And the difference between these two, so let us say, this is W1, this is W2. So delta W will be, W1 minus W2. And this W1 minus W2 will be the energy dissipated in the hysteresis. So this is the energy dissipated.

(Refer Slide Time: 23:48)



Energy dissipated → small for one cycle

→ increases to substantial value for large no. of cycles

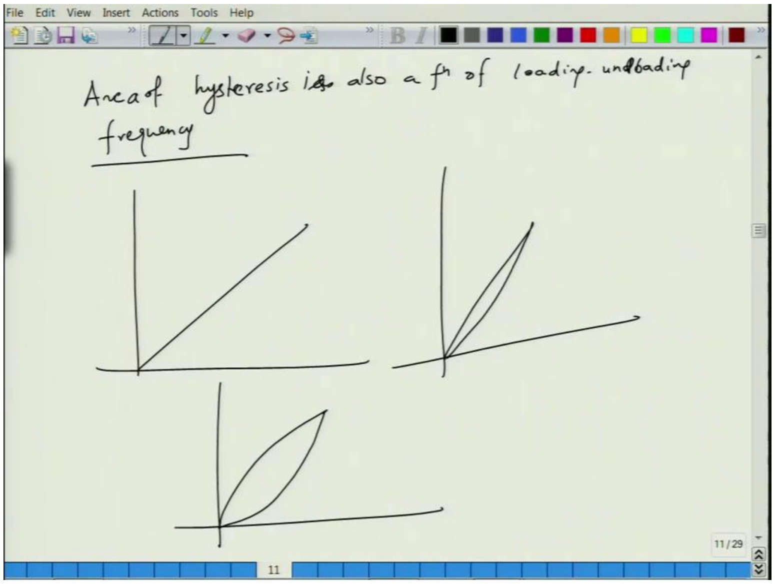
For rapid stresse cycling conditions → ~~end~~ energy dissipated can be quite high

Total Energy dissipated = Energy per cycle × no. of cycles
($10^6 - 10^{10}$)
↳ large energy

Now, although, this energy may sound very small for one cycle, let us say, small for one cycle, but if you have increases, it can increase to substantial value for large number of cycles. For example, if sample is vibrating continuously, let us say, one vibrate, per minute 20 vibrations or so on and so forth will accumulate many cycles within a day, and this will lead to lot of energy that is dissipated.

So for rapid stressing conditions or rapid stress cyclic conditions energy dissipated can be quite high, because total energy dissipated is energy per cycle multiplied by number of cycles. So if number of cycles is 10 to power 6 or 10 to power 10, this will give rise to a large energy. So this will be the situation that might happen and you can do quick calculations to find out the estimate of this energy.

(Refer Slide Time: 26:01)



Area of hysteresis is also a fn of loading-unloading frequency

And thus the area of this hysteresis is also a function of loading unloading frequency. So whether you load very fast, whether you load little fast and whether you load in this fashion. So this is, let us say, at very high frequency or very low frequency, this could be the situation of intermittent frequency and we will do that analysis in the next class, how the hysteresis changes as a function of loading frequencies.

So what we have done in this lecture is, we have looked at basically the hysteretic process that we obtain in case of anelasticity. So isothermal straining of a material would directly take you from 0 to certain strain, which is, let us say, eI isothermal strain. However, would in reality, if you load it too fast or too, if you load it at a rate when strain does not follow the stress, then you will not achieve complete strain and you will have to leave the material for certain time to achieve complete training. And then similarly, when you come back to 0 load, it does not get back to 0 strain, you will have to wait for some time before it reaches a 0 strain.

And the processes which happen are adiabatic processes, especially the loading and unloading process and then you have wait times when you achieve wait for the sample to achieve the isothermal strain or the 0 strain during unloading. So this is what we have done and we also looked at the concept of energy that is dissipated and what is the meaning of this energy that is dissipated.

In the next lecture, we will look at the, this hysteresis as a function of loading unloading frequency and how does it vary. And then we look into things like time dependence and other things. So we will stop here. Thank you very much.