Properties of Materials (Nature and Properties of Materials: III)
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Lecture 15
Relaxation time and damping capacity

So welcome again to the new lecture of the course Properties of Materials. So let us just briefly do a recap of previous lecture.

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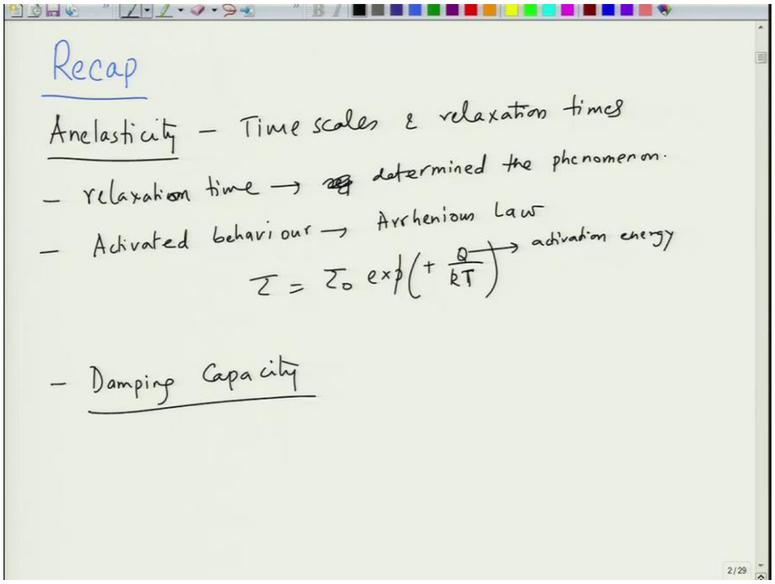

So in the previous lecture, we are talking about the anelasticity and we basically looked at the time scales and relaxation times. So basically depending upon the phenomenon, so your phenomenon could be interstitial diffusion, substitutional diffusion or grain boundary related effects or whatever it is. So you have a relaxation time, which is basically specific, which is basically determined by the phenomenon.

And because it is a diffusional kind of behavior of atoms, as a result, this is activated behavior and this is governed by Arrhenius law. So basically the relaxation time is related to energy called as exponential of Q by KT. So higher the activation, so this is activation energy, so higher the activation energy, more the time you require for diffusion to occur or lower the temperature, more is the time that is required.

And another term at the end that we described was damping capacity, which is basically determined by the ability of materials to dissipate the energy when they are subjected to a cyclic stress and this could happen through various internal processes that may occur inside the material.

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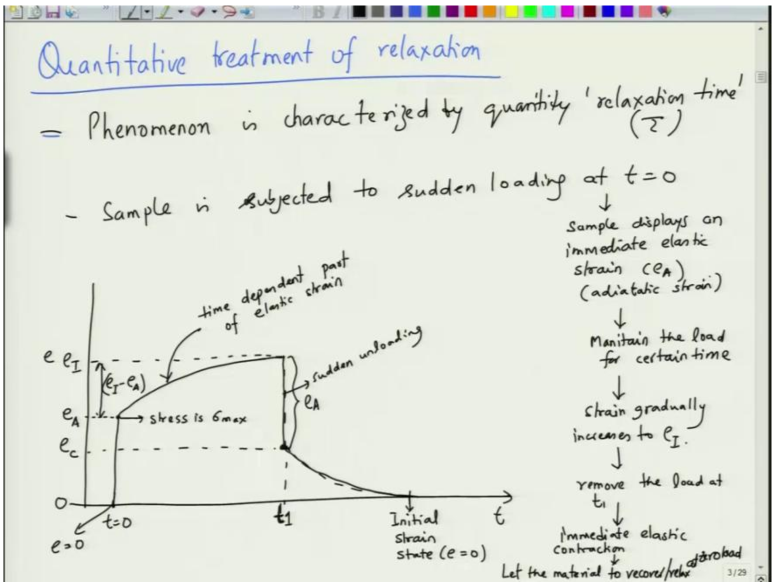

Now let us look at, in this lecture, the mathematical or quantitative treatment of relaxation time. So what we will do is that, we will do the quantitative treatment of relaxation that occurs during anelastic behavior.

So basically we define a quantity which is called as phenomenon is defined by, is characterized by a quantity called as relaxation tau. This is very similar to, see it is not only about this anelastic behavior in these materials, elastic materials, but also it is related to similar phenomenon is also observed in dielectric materials. For example, when you look at dielectric relaxation, so most of the relaxation behavior where atoms move from one position to another giving rise to variety of manifestations like mechanical, mechanical property manifestation or dielectric manifestation, treatment is fairly similar.

So basically, the phenomenon of relaxation is characterized by a quantity called as relaxation time. So you take a sample. So imagine a sample which is subjected to a load, let us say, subjected to sudden loading at time t is equal to 0. So essentially, you have a sample, let us say, so let us say I plot here epsilon as a function of time. So at some time t is equal to 0, you have a sample which is suddenly loaded. So essentially you go to a place where stress is sigma max. So this achieves, as we saw earlier, this achieves a strain, which is, let us say, adiabatic strain.

So basically when you subject the sample to a sudden loading at t is equal to 0, which means you have sample displays an immediate elastic strain, let us say, eA which is basically you can say it is adiabatic strain and the sample is unrelaxed, whereas temperature will change. Then you maintain the load followed by maintain the load for certain time.

So when you maintain the load for certain time, the strain gradually increases, the strain gradually increases and saturates. So this is the final value which the strain achieves which is let us say eI. So the difference between this strain and this eI and final strain and the initial strain is eI minus eA. This is the strain which has been achieved in certain time when you led the material relax. And then, but the load is applied.

Now what you do is that you, so what happens here is strain gradually increases to eI. And let us say this happens until time t1, which is greater than 0. So basically you can say that this is relaxed strain and it allows the sample to equilibrate with the ambient.

And then what you do is that, you remove the load at t1. So when you remove the load at t1, the sample will come back to this strain which was shown earlier. So this is basically you can say sudden unloading at this point. So sudden unloading, which means the sample will immediate elastic, you can say contraction to certain value, let us say, so it comes to certain value, let us say, eC.

And then you, so once that happens, then you let the material to recover slash relax at 0 load. So what will happen that strain will eventually get down to 0. So it will eventually get down to 0 at certain point. So it will go back to the initial state. So this is your initial state that is e is equal to 0. You started from here, where e was equal to 0. So we started at 0 strain, you did the adiabatic loading, let us say, tension, the strain increases to eA suddenly, then for it to grow to isothermal strain, you have to leave it for some time. During that time it goes to isothermal strain. Then again when you suddenly unload by the same amount eA, you will have a drop. So this will be eA. And then it will relax back to 0 strain, if, upon leaving it for certain amount of time.

So basically in this process, you can see that there is this time dependence of, so this is essentially you can say the time dependent part of elastic strain.

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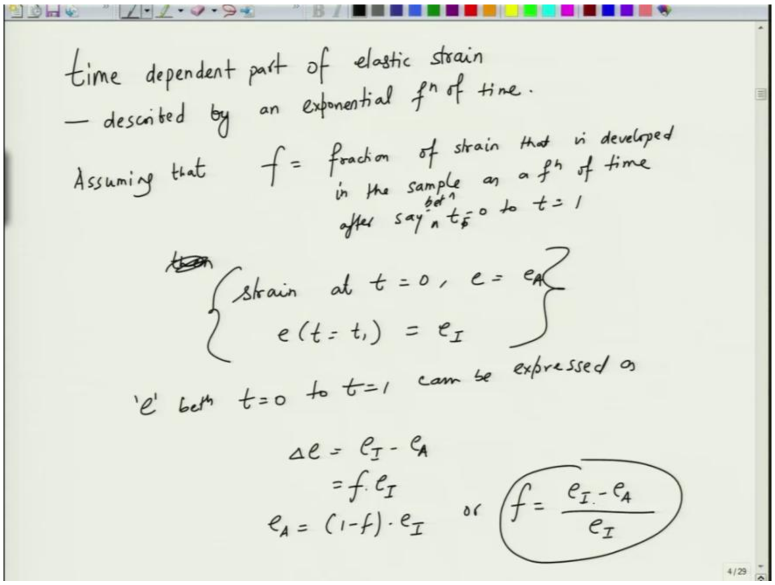

So the time dependent part of elastic strain, this can be described by an exponential function of time. So let us say, assuming that f is nothing but, small f, is a fraction of strain that is developed in the sample as a function of time after say, let us say, sometime between t is equal to 0 to t is equal to t1, between t is equal to 0 to t is equal to t1. Then we can write, so we can say strain at t is equal to 0 is e is equal to eA, which is adiabatic strain and we say that e at t is equal to t1 is equal to eI. So these are sort of boundary conditions. This is at one side, this is on another side.

So a strain which is developed, so this is let us say, these are the conditions that we have. So which means, strain which is developed between, so e between t is equal to 0 to t is equal to t1 can be expressed as delta e which is equal to eI minus eA, which is some fraction of the overall strain, which is isothermal strain. So this is f. So using this token you can write eA is equal to 1 minus f into eI or you can also write small f is equal to eI minus eA divided by eI.

So this is the relation of, this is what is f, definition of f, which is it is a fraction of overall isothermal strain that is developed during the time t is equal to 0 to t is equal to t1.

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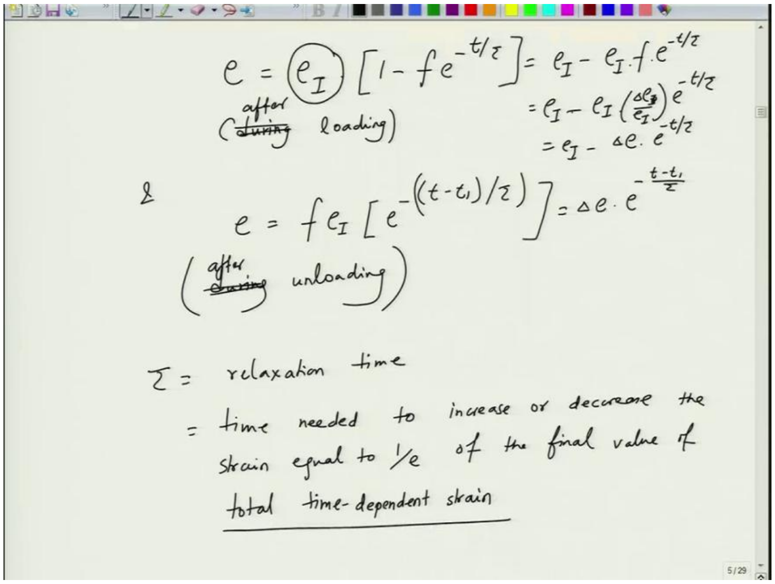

So you can express this strain as, so e can be written as e is equal to eI into 1 minus f e to the power minus t divided by tau. So this is during loading. And during unloading, you can write same thing as e is equal to f of eI into e to the power minus t minus t1 divided by tau. So this is during you can say after unloading, after, and this would be after loading, not during, but after loading, after peak loading, after unloading.

So here this tau is written as relaxation time. And basically it is defined as the time needed to increase or decrease the strain equal to 1 over e of the final value of total time dependent strain. So you can see that here, so this is during the loading process. The strain during the loading process will be eI which is the total this thing. So basically we are saying that it is eI minus eI into f into e to the power minus t by tau.

So we are saying that this is eI minus eI into f was equal to delta eI, delta e divided by, so if you look at the previous relation, delta e divided by eI into e to power minus t by tau. So you can say that this is equal to eI minus delta e into e to power minus t by tau. That is what it is basically.

So strain at any point can be calculated using this relation during the, after the loading process. And after unloading, you can write this as, so you can again modify this expression and appreciate it better. So this will be nothing but delta e into e to the power minus t minus t1 divided by tau. So this is what the strain will be after unloading. So this is how you can calculate the strain.

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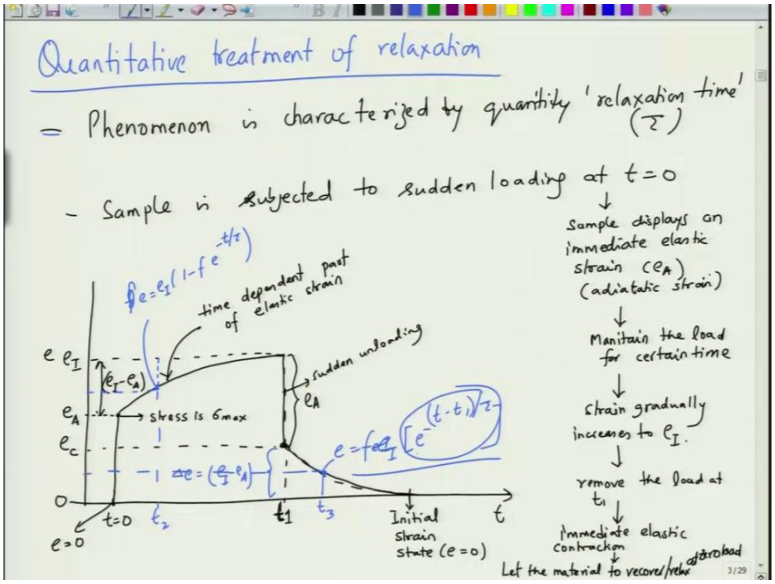

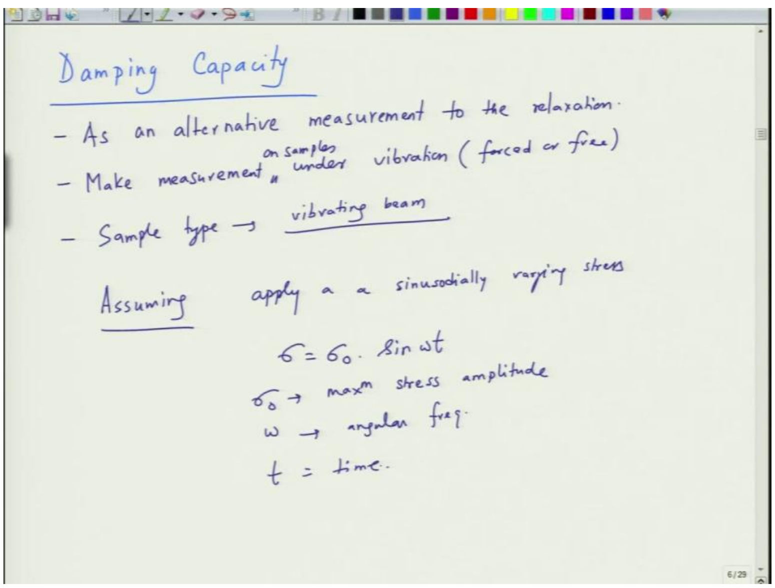

And now, if we look at, what we call as damping capacity. So let me just go back a little bit just to explain this point. So essentially what you are doing is that, if you are going to calculate strain at any given point, let us say at this point, it is given by the relation. So we are saying that this is, sorry, e is equal to, if you look at the relation, eI minus delta e, so eI into 1 minus f e to the power minus t divided by tau. This is what the strain here will be.

And if you want to calculate strain here, so this is nothing but eI minus eA or delta e. So here now it becomes 1 minus eA. So as a result, you can calculate the strain again at some time t. This is again a time t, let us say, t2, t3, whatever you want to calculate and this is how you can write the relation. So this will be f into eI, sorry, f into eI into e to the power minus t minus t1 divided by tau.

So here we have eI minus delta e into e to the power minus tau, t divided by tau and here we have delta e multiplied by, so this is delta e, multiplied by some fraction, some number and this is exponential relation. So that is the only difference between the two. So in one case you have 1 minus f into exponential function and here you have the delta e multiplied by the exponential function.

So, now let us get back to the estimation of damping capacity. So damping capacity is generally useful as an alternative measurement to the relaxation time. Because if you want to do the relaxation time, you will have to do time dependent measurement, time and temperature dependent measurements.

So here we make measurements under vibration, on sample under vibration and the vibrations could be either forced or free. The analysis for both of them is different, but let us just look at the forced vibration.

So in most condition the sample type is either a, so you can say a vibrating beam can we take in as a sample type. So assume that we apply a stress sinu varying stress, so which is written as sigma is equal to sigma naught into sin of omega t. And here sigma naught is the maximum stress amplitude and omega is the angular frequency and t is the time.

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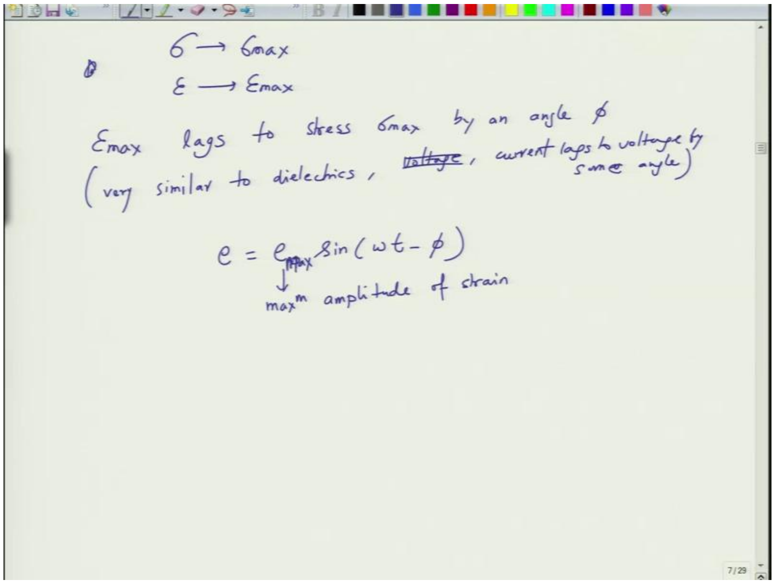

So let us say now that maximum strengths, so let us say the max, so you apply a stress to a sigma max value and correspondingly you will have a strain which is develops to epsilon max. But epsilon max lags to stress sigma max by an angle phi. This is very similar to dielectrics where you apply voltage and current. So current lags the voltage by some angle.

So ideal case it should be lagging by 90 degree, but it lags by an angle which is smaller than 90 degree. But nevertheless, the physics of two of them could be different, but let us say the strain development to maximum value lags with respect to stress maximization by an angle phi. So we can write this strain relation as e is equal to e naught into sin of omega t minus phi. So this is the maximum amplitude of strain. Let us say, we write it as sigma max only instead of sigma 0. And here also we write this as epsilon e max.

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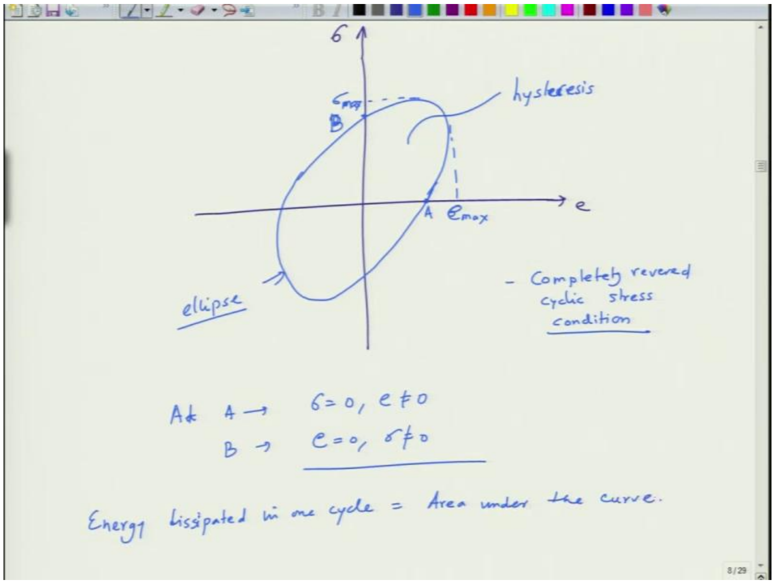

So now these two quantities can be plotted on a let us say a diagram. So on this axis we plot strain, stress. On this axis we plot strain. And these are represented nicely by sort of elliptical kind of, hang on, this kind of, elliptical kind of relation. So this is the ellipse. And the maximum value of stress is given by sigma max, and the maximum value of strain is given by epsilon max and you can see that both of them do not occur simultaneously. And this is basically you can say the hysteresis that we obtain. And the condition is completely reversed cyclic stress condition.

So here we can say that maximum stress and maximum strain points do not coincide. So we can see that at point A, let us say this is point A, this is point, let us say B. So at A, we can say sigma is equal to 0, but e is not equal to 0. At B, we can see that e is equal to 0, but sigma is not equal to 0. And the area within this curve is basically the energy dissipated in one cycle. It is the area under the curve.

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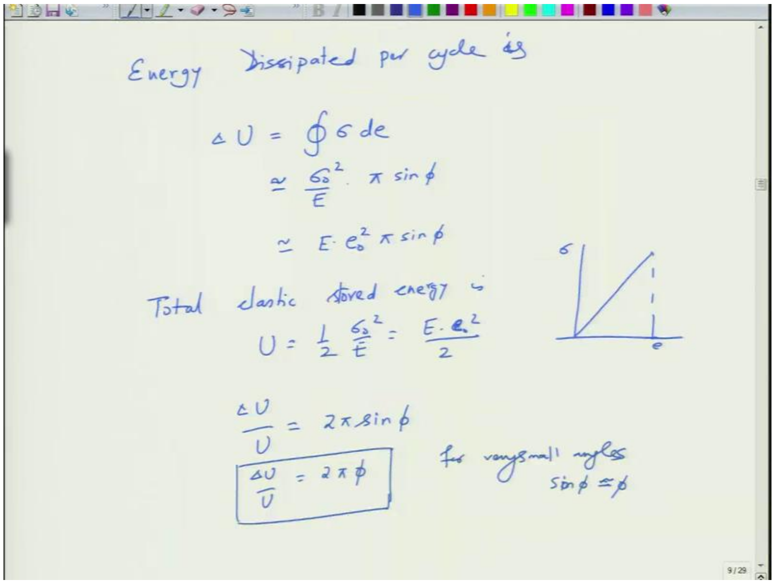

So we can write this energy dissipated as, is can write this delta U as integral of sigma de which turns out to be sigma naught square divided by E into pi sin phi. And this can be approximated as pi sin phi. So the total, and we know the total elastic stored energy is U, which is half of sigma naught square divided by E or you can write E into epsilon naught square divided by e naught divided by 2.

And if you, so U, delta U divided by U becomes, so this is the basically you can see the stress strain curve like this. This is the total, so half of sigma naught square divided by E. This is sigma, this is e. So this is the total elastic stored energy in elastic material.

So delta U by U is equal to 2 pi sin phi or for very small angles, I can write sin phi as phi. So this becomes 2 pi phi. So this is the energy dissipated or you can see the damping sort of capacity of a material. So this is what we have done over a past few lectures. What we have done essentially, we have looked at the anelastic behavior in detail.

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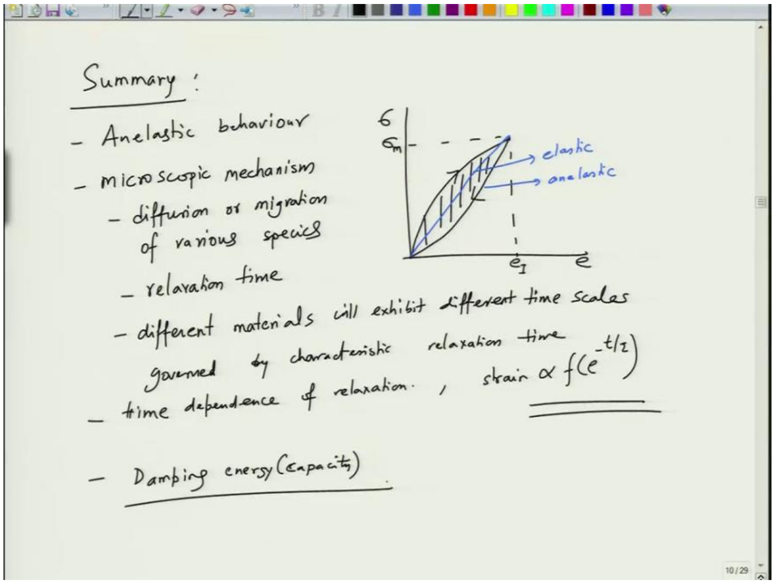

So if we just summarize now. So we have been discussing elasticity for some time. And in the last segment, we discussed anelastic behavior, which is mainly because of inability of strain to develop at the same time as stress. As a result, we obtain a hysteresis in a stress strain curve like this. So this is the path that we obtain. And so instead of having a linear behavior which is like this, this is the linear behavior, so this is elastic behavior and this is anelastic, and this mainly happens because of microscopic mechanisms which are related to diffusion of, diffusion or you can say migration of various species which is get characterized by a quantity called as relaxation time.

So depending upon the magnitude of this relaxation time, every phenomenon will occur at different time scale. As a result, the loading rate and unloading rate which will be manifested in anelastic behavior will be different for different materials, which will have different species. So as a result, it will be different materials will exhibit different time scales governed by characteristic relaxation time.

And then we also looked at the time dependence of relaxation where we saw that strain is a function of exponential relation of minus t divided by tau. So we can see that after loading or after unloading, the strain develops in a exponential function, in exponential manner as a function of time.

And finally, we looked at the relation with respect to damping energy or capacity when material is subjected to a cyclic stress. So this is what we have done over past few lectures. In the next lecture now, we will move on to plastic deformation or the permanent deformation and this will probably continue for rest of the course. Thank you.