

Properties of Materials (Nature and Properties of Materials: III)

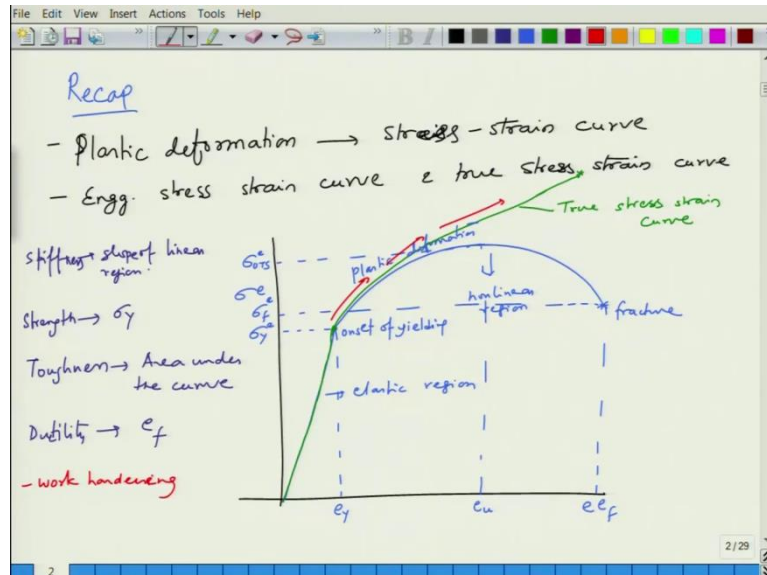
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Lecture 17 - True and Engineering Stress-strain curves

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So, welcome again, to the course, Properties of Materials. So, we will just briefly recap contents of last lecture. So, in the last lecture, we discussed, started our discussion on plastic deformation and we initiated with stress-strain curve. And the most common of these is engineering stress-strain curve, but at the same time you can also have a true stress-strain curve. So, if you plot them together, so, the engineering stress-strain curve will appear as something like this. So, there is onset of yielding.

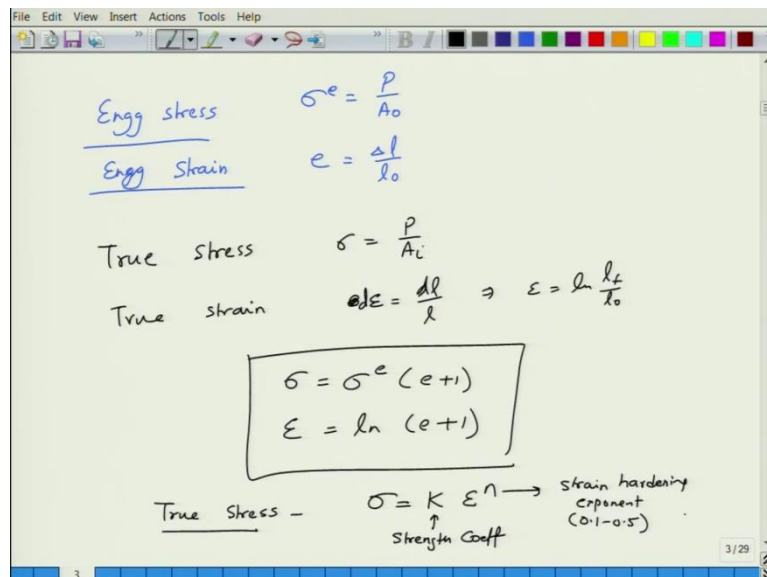
So, this is elastic region. So, this will be σE and this will be E , okay, the engineering stress engineering strain. This is nonlinear region and so, in this region you have plastic deformation and at the end we have what we call as fracture. And the various characteristics that we can determine from this σY and then the maximum stress required for deformation is σUTS , ultimate tensile strength and then we have σF , which is $E F$, it is a the stress to fracture. And this is $E F$, $E Y$, $E U$.

There could be another curve, which is fairly similar up to this point and then it sort of takes off like this. So, this is true stress-strain curve. And true stress-strain curve coincides the yielding roughly at the same point, but the stress in the true stress is always higher than the stress in the true engineering stress and strain and we will see the reasons a little later. But

quantities that you can extract from this stress-strain curve are mainly stiffness, which is the slope of linear region, which is basically the resistance to elastic deformation.

Then we can derive strength, which is basically strength, stress that is required to cause the deformation or resistance to plastic deformation, which is derived by sigma Y the yield strength. Then we have toughness, which is area under the curve that is energy absorbed before fracturing so that is measure of toughness. And then ductility is basically strain to fracture, how much can you strain the material before it fractures? And then we also noticed that the material gets work hardened, which means, as you continue with the deformation, the stress required to deform the material further increases and this is called as work hardening or strain hardening.

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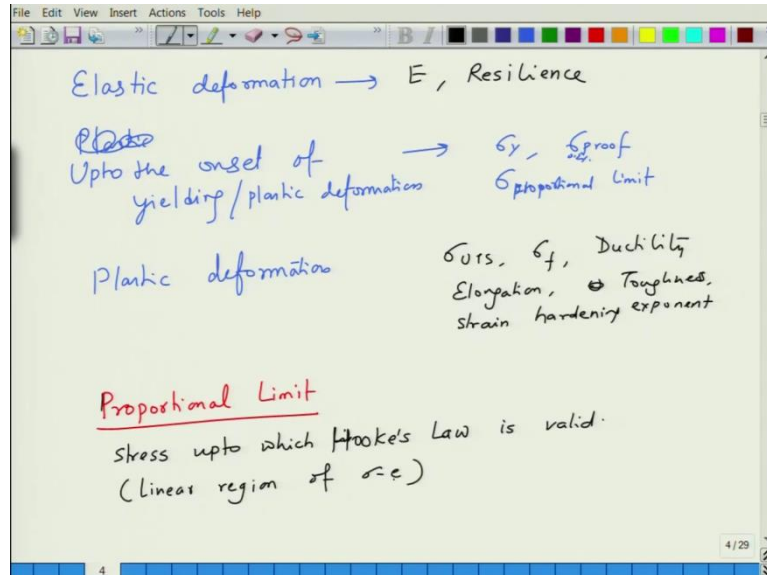


And we also looked two other quantities. So, and we need to distinguish between true stress and engineering strain. So, engineering stress as we defined was Sigma E is equal to P divided by A naught, where slope divided by the original area, and engineering strain is delta L divided by L naught, change in length divided by original length. Whereas, true stress is defined as load divided by instantaneous area and true strain is defined as dl over l.

So, we want to (calc) D epsilon and if you want to calculate overall strain then you have to integrate this from initial to final length. So, essentially, we can say this will become, epsilon will become ln, lf over l naught. So, this will be the, and the difference between the two is summarized in these two equations. So, sigma true stress is equal to sigma e into e plus 1 and epsilon is equal to ln of e plus 1, these are the two relations which summarize that. And then

we also looked at the true stress follows a relation $\sigma = K \epsilon^n$, where K is strength coefficient and n is strain hardening exponent typically from 0.1 to 0.5. So, this is what we did in the last lecture.

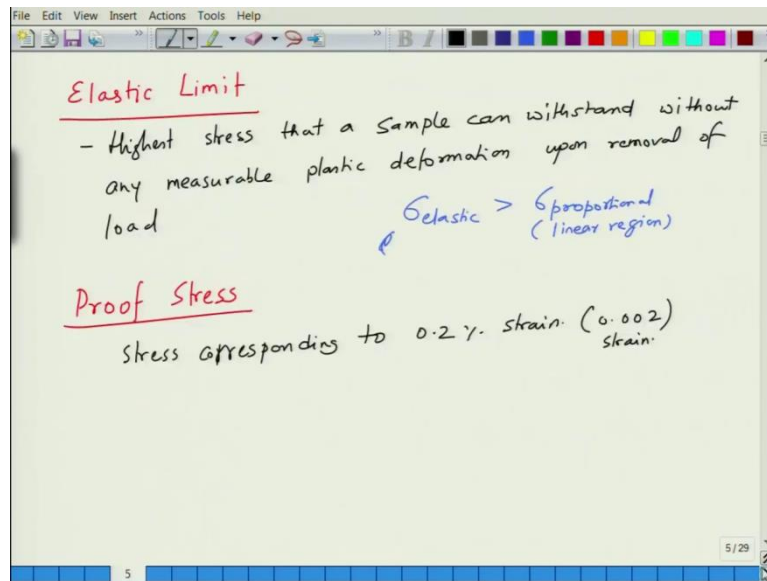
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And now, let us move onto the contents of this lecture that, so, we saw that from elastic deformation, we can calculate properties such as Young's modulus and resilience. So, we will explain in a while what resilience is from plastic deformation. So, up to the onset of yielding or plastic deformation, we can determine things like yield stress is something called as proof stress. So, this is basically 0.2 percent proof and then sigma proportional limit. And then plastic deformation region gives us information of sigma UTS, sigma F, ductility this is what we have seen, but we can also find out things like elongation or toughness and strain hardening exponent.

So, there are some terms, which we have not seen earlier. For example, what is proportional limit? Proportional limit is basically we can say that stress up to which Hooke's law is valid. So basically, in the strict sense it is a linear region only. So, that is linear region of stress-strain curve.

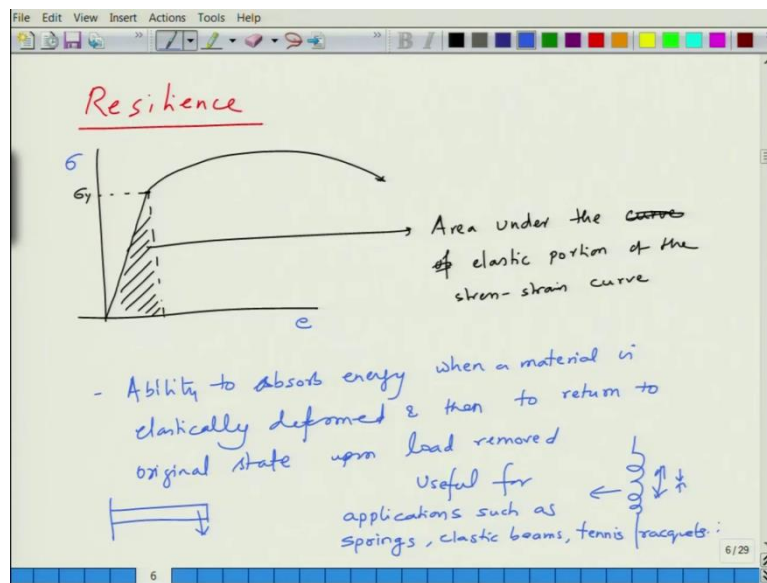
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Then second thing is elastic limit. Elastic limit is highest stress that a sample can withstand without any measurable plastic strain. So basically, this has to do with the accuracy of measurement. So, since every measurement equipment has certain limitation, as long as you are below that detection limit, that is the stress which is called as elastic limit upon removal of load.

Now, the next thing is proof stress, generally, you will see that $\sigma_{elastic}$ will be higher than $\sigma_{proportional}$, because, remember this is strictly for linear region whereas, this can slightly exceed the linear region because of measurement constraints. So, this is always little higher. Now, proof stress is stress corresponding to 0.2 percent strain, okay, or 0.002 strain. So, this is what proof stress will be. So, these are the differences between proportional limit, elastic limit and proof stress.

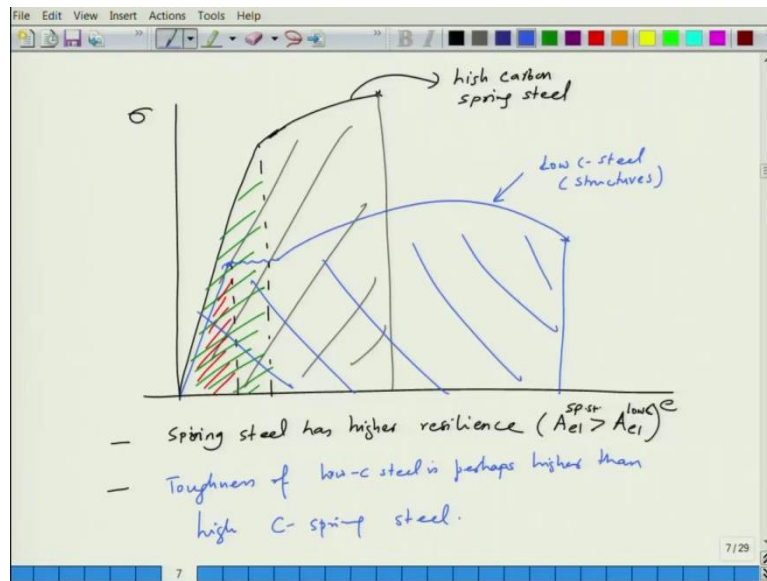
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Now let us look at what is resilience. Resilience is, as we said we plot stress versus strain. So, this is the point at which the yielding will start σ_y , so area under the curve of elastic region. So, this is basically you can say area under the elastic portion of the stress-strain curve. And what does it depict? It depicts ability to absorb energy when material is elastically deformed and then to return to original state upon load removal.

So, basically this is useful for things like springs. So, you have springs can stretch it in the elastic region and then unload it. So, things like springs or if you have elastic beam, which is supposed to deflect up and down, so elastic beams. Even for things like tennis rackets and so on and so forth, wherever things need to deform elastically and then come back to their original condition without getting plastically deformed. So, higher energies, higher resilience is always good.

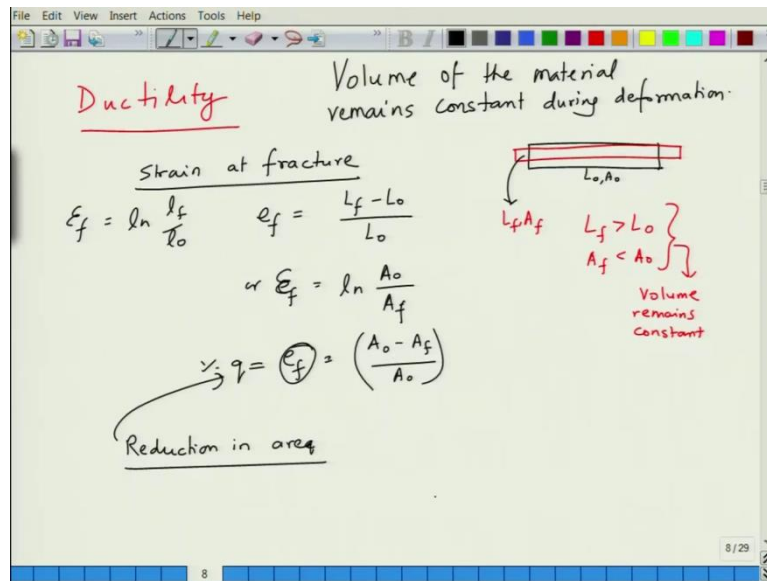
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So let me give you just an example. So let us say we plot two materials here. One is a material like this and another is a material which is something like this. So let me just say, so we have one sample, let us say as high carbon spring steel and another sample is let us say, some sort of low carbon steel, let us say for structures. So, if you look at this plot, then we can see that area under the elastic region for this and the area under the elastic region for this, these two are different.

So, basically, we can see that spring steel has higher resilience because area in the elastic region is higher for spring steel than for the low carbon steel. But if you look at the toughness, toughness for this is, this is the area under the curve and this is the under the curve for this. So, basically, here it seems that toughness of low carbon steel is perhaps higher than high carbon spring steel. So, this is where you can see for applications such as springs of elastic beams or tennis rackets. So, wherever things have to deflect a little bit elastically then come back to the original position, they are, they have to be, they have to have high resilience. But the things which require high toughness that is before breaking, they have higher ability to absorb energy that is why they are, they require high toughness. Okay. So, this is the fundamental difference between the two characteristics.

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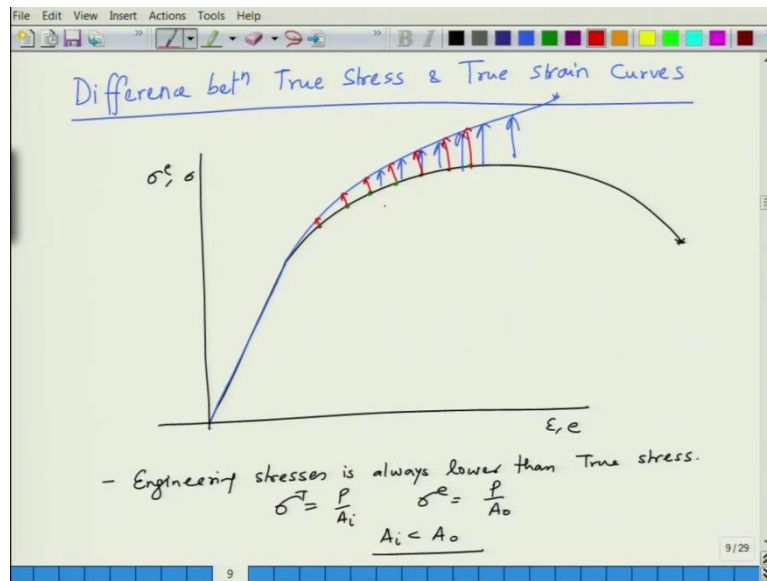


Now, let us look at what is ductility. So, ductility is basically defined as, it can be defined by strain at fracture, which can be defined which can be written as ϵ_f could be equal to L_f minus L_0 divided by L_0 or it could also be written as, so, when you have a sample like this, not only it is so, when you deform it, it becomes like this. So, basically, if the initial length is L_0 and area is A_0 , cross-sectional area, the final one is L_f and A_f . You can see that L_f is greater than L_0 , but A_f is smaller than A_0 because the volume remains constant.

So, this constancy of volume is an important thing in plastic deformation that as volume of the material remains constant during deformation. So, alternatively, we can also express the elastic strain as \ln of A_0 divided by A_f , we can also write ϵ_f here, ϵ_f could be \ln of L_f divided by L_0 and this can be also written as ϵ_f can be written as \ln of A_0 divided by A_f or you can write this as ϵ_f can be written as A_0 minus A_f divided by A_0 . So, these are the different ways of writing strains for (diff), and basically higher the strain is more the ductility is.

We can also quantify this as reduction in area. So, this reduction in area can be called as Q , generally, it is not written as ϵ_f but more as Q , which is percentage reduction in area is the measure of percentage ductility that is A_0 minus A_f divided by A_0 . So, this is basically increase in length or decrease in the area are two measures of ductility of a material that are considered.

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Another difference between stress, so now the question is what is the difference between the true stress and true strain. So, having known the quantities that you can derive from a stress-strain curve now, let us get back to the question of what is the difference between true stress and true strain curve.

Now when you plot them together, this is what we see. So, we have σ_e and σ , ϵ_e and ϵ , okay. So, the true engineering stress will look like something like this and the true stress will look something like this. One thing first we notice is that the engineering stress is always lower than the true stress, okay. So, basically, first thing that we notice is that engineering and this is because σ is P divided by A_i , whereas σ_e is, so σ_T let us say σ_T is P divided by A_i and σ_e is P divided by A_0 .

And since A_i is always lower than A_0 , the true stress is always higher than engineering stress, that is one difference. Second difference is, every point on after yielding between elastic region and up to ultimate tensile stress, every point on engineering stress plot, corresponding point on the true strain, true stress-strain plot will lie slightly to the left of the point on engineering stress-strain curve. And that is because of difference between the two values.

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Engg		True	
$e (\Delta L/L_0)$	$\sigma_e (\text{MPa}) (P/A_0)$	$\epsilon = \ln(1+e)$	$\sigma_T = \sigma_e(1+e)$
0.032	550.83	0.0315	568.48
0.044	559.49	0.043	584.05
0.052	563.43	0.051	592.77
0.060	567.07	0.058	601.14
0.072	570.54	0.069	611.85
0.080	572.36	0.077	618.17
0.092	573.74	0.088	626.54
0.115	576.66	0.109	643.19

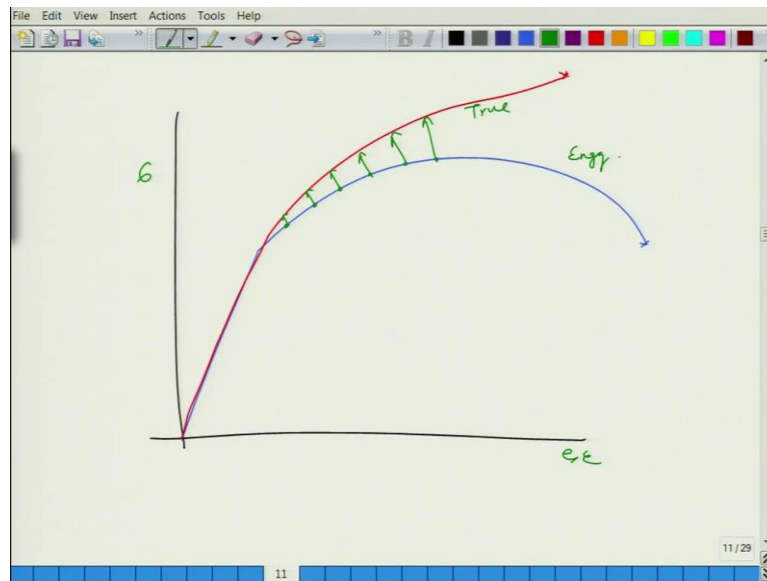
Applicable betⁿ σ_y & σ_{UTS} .

So let us compare that. When we make a table, for example, for engineering and true values, so let us say, let us make a table. So let us first calculate what is e, which is delta L by L naught. And then let us say we first, we calculate sigma e in MP a, which is nothing but P divided by A naught. Let us write the value for this, it is 0.032, 0.044, 0.052, 0.06, 0.06, 0.072 so 60 okay, then 0.080, 0.092, 0.115. If we calculate the stresses at this point, so this is 550.83, 559.49, 563.43, 567.07, 570.54, 572.36, 573.74 and 576.66.

So, these are the strains and corresponding stresses for engineering stress-strain curve. Let us look at the same values for true. So for true, epsilon can be worked out as ln of 1 plus e and sigma T is equal to Sigma e into 1 plus e. So, if we just do the calculations, we will find out this will be 0.0315, this will be 0.043, this will be 0.051, this will be 0.058, this will be 0.069, this will be (point), sorry 0.069, this will be 0.077, this will be 0.088 and this will be 0.109.

So, you can see that the strains, true strains are a little lower, roughly similar in the initial stages, but they diverge from engineering strains as we move on, and this we saw earlier also. Now, if we look at the true stress them true stress is 568.48, okay, 584.05, 592.77, 601.14, 611.85, 618.17, 626.54, 643.19. So, you can see that corresponding true stress values are also significantly higher as compared to engineering stress value. And this is applicable between sigma Y and sigma UTS, and we will see why in a little while.

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So, if you go to this curve, so every point here if you now just rub it, let us just make maybe different plot. So, this is engineering stress-strain curve, this is true stress-strain curve. So, if you take points here, here, here they will be slightly to the left and the leftward tilt increases as you move towards... So, this is true, this is engineering, okay.

I hope it is clear the difference between the true stress and true strain curve. So, we will stop here in this lecture. We will discuss some more differences in the next class before we learn more about the plastic deformation. Thank you.