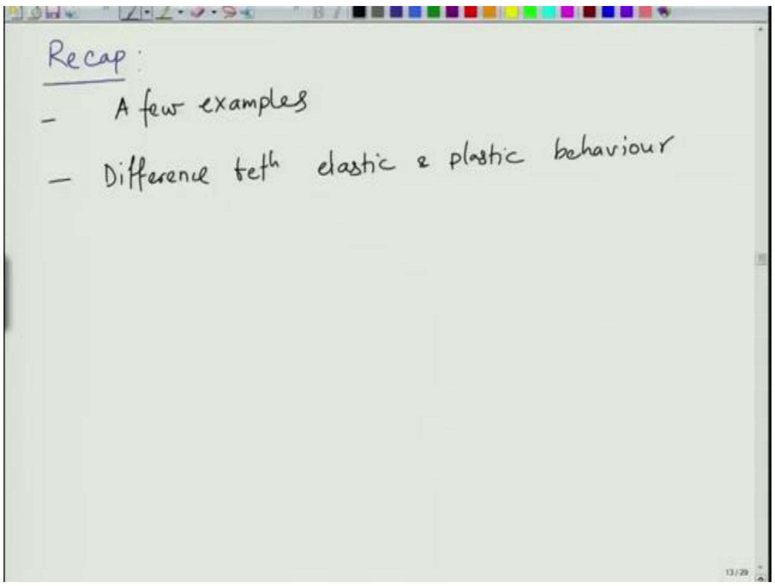
Properties of Materials: Nature and Properties of Materials III
Professor. Ashish Garg
Department of Material Science and Engineering,
Indian Institute of Technology, Kanpur
Lecture 02
Basic Material Properties – Stress and Strain Tensor

So, welcome again to the second lecture of the course Properties of Materials third part of Nature and Properties of Materials series and probably the final part as well. Now, what we will discuss is we will take up some example of again mechanical properties just the basic stuff and then before we move into details.

(Refer Slide Time: 00:40)



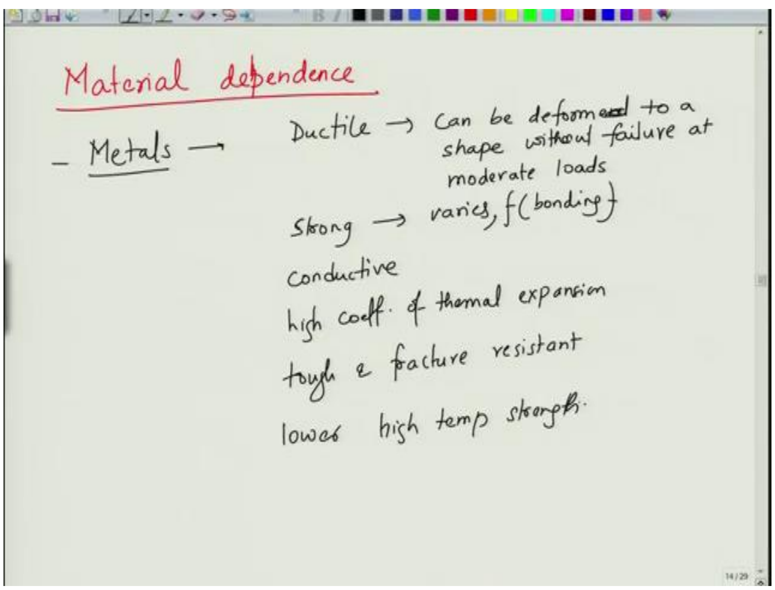
So, we talking about what basically elastic and so if we just recap. So, we in the last lecture, we just gave a few examples, few examples which demonstrate the importance of mechanical properties that why should what which application require a material to be strong, which material require material to be hard and brittle and hard and fracture resistance and so on and so forth.

Then we looked at the difference between elastic and plastic behaviour. So, essentially elastic behaviour is the behaviour of material when you apply a load to a material it sort of it gets and when you apply to a load to a material it may deform either in compression or in tension. But it gets back to its original dimensions when you remove the load.

So, this is called an elastic behaviour with this which is what we generally observe with materials like rubber, it is not very visible in metals or ceramics. Because the deformation limit is extremely small but it is visible in plastics or rubber especially in last (())(02:00) and rubbers. And when you look at plastic behaviour plastic behaviour of material is when you deform it to, so let us say you have a 1 meter of material, when you deform it to 1.5 meters and when you remove the load the 1.5 meter does not become 1 meter it maybe comes back to 1.49 or 1.45 meters, but it does not come back to 1 meter.

So, this extra deformation that you have carried out is nearly half a meter of deformation that is essentially the plastic or permanent deformation, so that material does not get back to its original dimensions and shape upon removal of load. So, this is the fundamental difference between the plastic and elastic behaviour or a macroscopic scale of course there is a lot of science behind it that we will see in the next few lectures.

(Refer Slide Time: 02:51)



So, now let us other than that there is also a difference between behaviour of materials, we will get back to this dependence in details later on. But just to make you conversant with this generally metals could be you know metals are generally ductile, what we mean by ductile is they can be deformed to a shape. So, they can be deformed from one shape to another shape or one dimension to another dimension without failure at moderate loads.

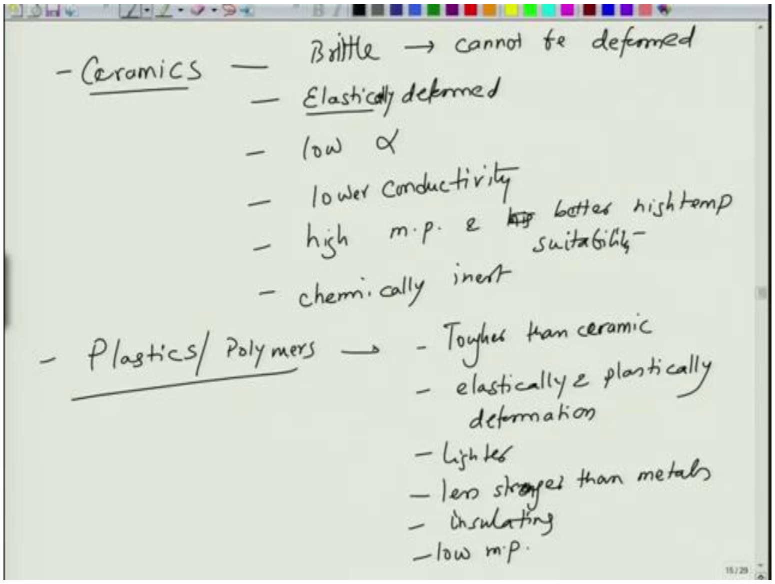
Now, what is moderate, what is high, what is low, that will become clear when we look at the numbers. So, metals are ductile which means they can be deformed to a shape without failure. They are also their strength their let us say they are also strong but the strongness varies. So, for example, if you look at something like led, it is not very strong, led deforms very easily when you squeeze it under normal pressure it deforms.

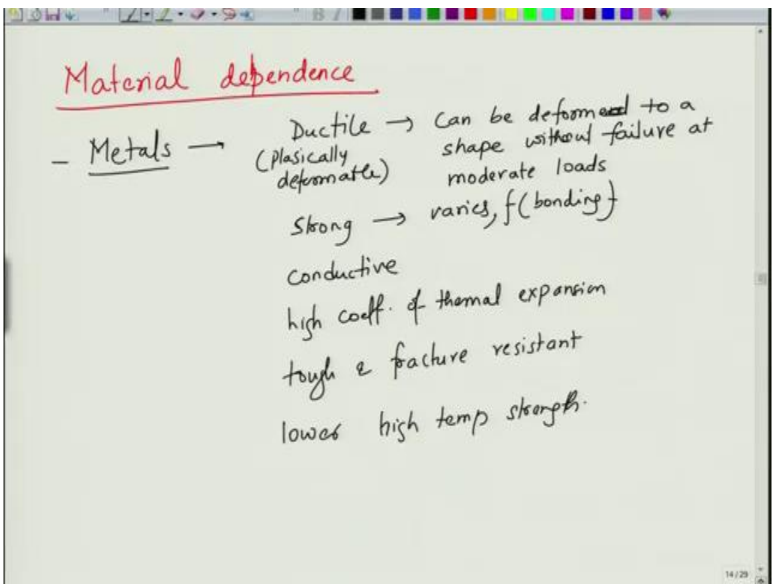
But when you look at steel, when you apply pressure from finger, for example, you are not able to deform it you need to use machines to deform it. So, the strength varies which is a function of bonding, function of bonding. Metals also are generally conductive, they are thermally and electrically conductive and metals are also they have high coefficient of they have high coefficient of thermal expansion by enlarge and they are also tough and fracture resistant which means under impact or sudden loading, they do not suddenly break.

So, if you drop a metal glass on floor, it does not break it. But if you drop a ceramic cup or a glass cup on the floor it breaks, it may break. So, generally metals are ductile, they are they can be strong as steel is very strong, copper is a strong, nickel is a strong, but led is not strong, tin is not very strong.

So, it depends on their bond strength, they generally conductive they have high cushion, they have high cushion thermal expansion, they are also tough and fracture resistant and they have low lower high temperature strength. So, generally you do not use metals for high temperature applications unless it is a high temp high melting point metal itself.

(Refer Slide Time: 05: 33)





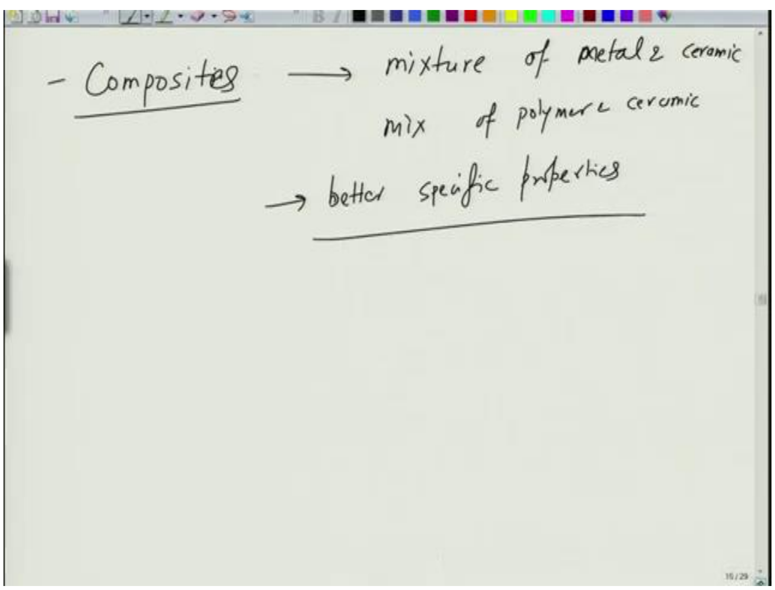
In contrasts if you got ceramic, they are brittle, they brittle means they cannot be deformed. So, ceramics cannot be converted into ceramics cannot sorry I will let me just change the spelling ceramics cannot be converted into another shape by deforming just like metals they are brittle. They are generally they do not go plastic deformation, they generally go undergo elastic deformation. So, when I said here they can be deformed which means metal can be plastically deformed.

But ceramics are generally only elastically deformed they do not undergo plastic deformation. They generally have low coefficient thermal expansion, they have low thermal and electrical conductivity in general than metals and they have high melting point by enlarge as a result they have better high temperature suitability and generally they are also chemically inert, metals on the other hand are not.

We have something called as plastics or polymers, polymers are tougher than ceramics they may or may not be tougher than metals. But they are definitely tougher than ceramics, they can be elastically deformed and plastically deformed. So, there is a possibility of deforming plastics plastically as well as elastically. So, you can convert them into different shapes, they are lighter, but less stronger than metals.

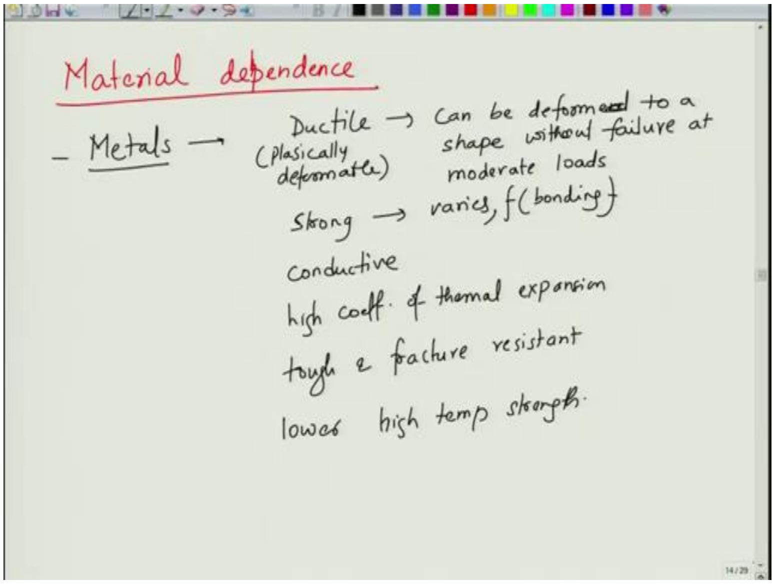
And they are also insulating in most cases and they have low melting point. So, generally ceramic plastics are suitable for low temperature applications that you require moderate strength and moderate melting points or lower melting points you are okay with a light thing. For example, things like buckets etc, we use plastic nothing else.

(Refer Slide Time: 08:15)



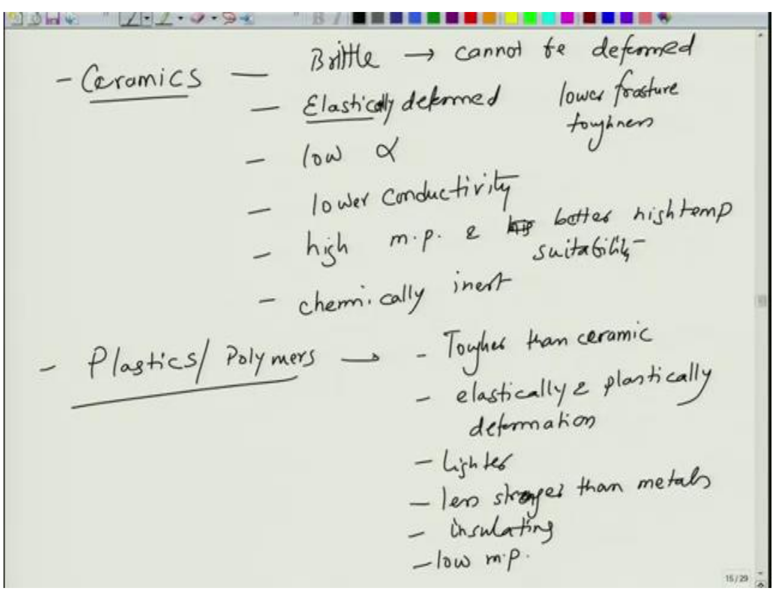
And if you want to mix and match materials what we call as then composites, you want a strong material. But of lightweight then you make mixture of let us say a metal and ceramic or mixture of polymer and ceramic and so on and so forth. So, they are basically they have better specific properties, specific means per unit weight, they have better specific properties, they do not necessarily have higher strength than metals. But or higher fracture toughness and metals, but they do have better properties in terms of stiffness per unit weight and so on and so forth.

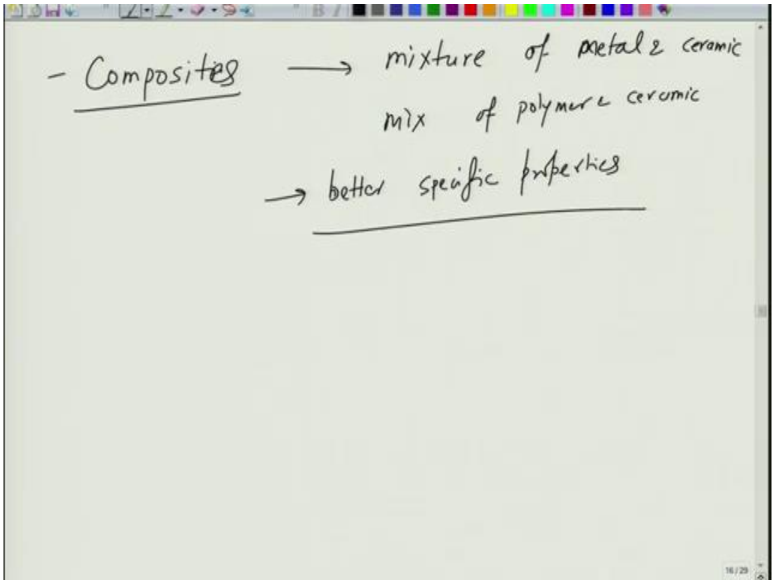
(Refer Slide Time: 09:11)



Now this is a general classification of materials in terms of strength metals are strong, they are ductile that strength can vary from material to material led and tin are not strong but iron and nickel are strong. They are generally very ductile, they are also fracture resistance. So, these are three mechanical properties which are very good for metals.

(Refer Slide Time: 09:26)

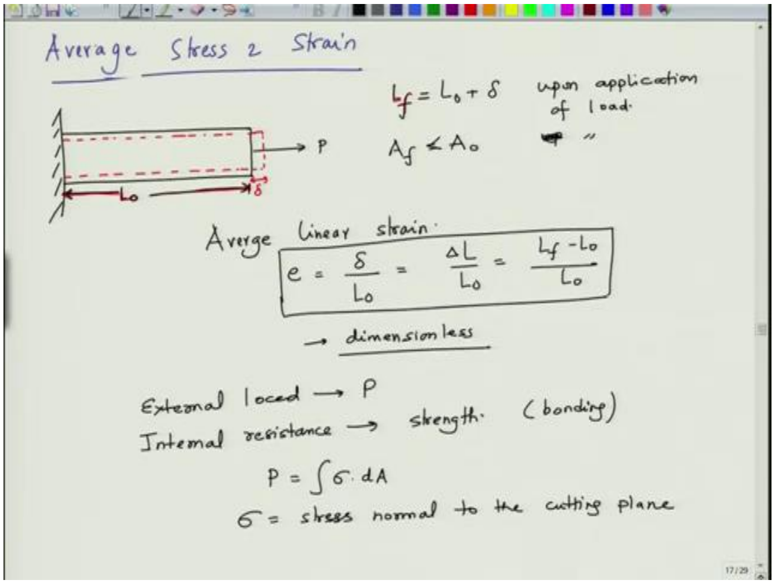




Ceramics on the other hand, they have lower tensile strength, they cannot be deformed and they are brittle and they have lower fracture toughness as well and so lower you can say fracture toughness they cannot withstand impact loading. So, plastic and polymers on the other hand they are tougher than ceramics. But not so much tougher than metals they can be elastic in plastically deformed they are lighter.

They have generally less strength then metals, they are also insulating and have lower melting points, then you can make composite which are mixture of two phases, one hard and one soft or metal and polymer, metal or ceramic, or metal or ceramic and polymer and so on and so forth. They generally have better properties per unit weight than the individual properties then the normal properties.

(Refer Slide Time: 10:22)



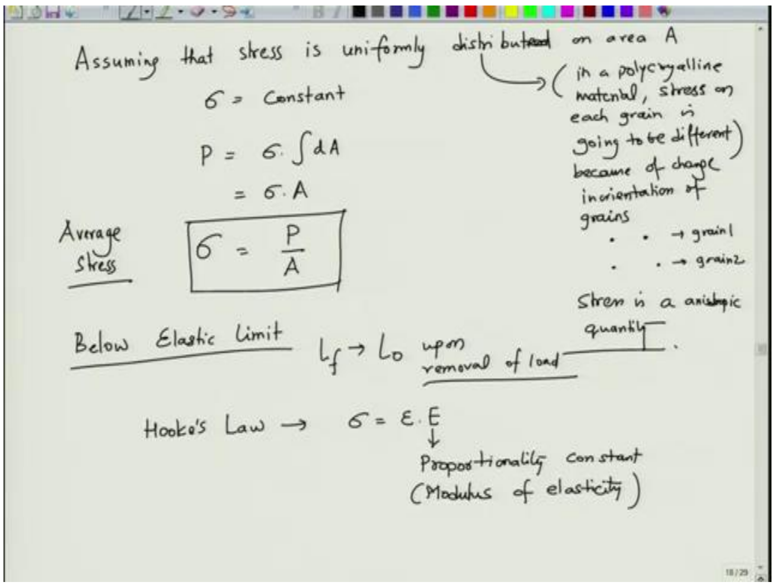
So, now let us look at now mechanical the quantification of mechanical properties. So, let us first look at the concept of what we call as average, average stress and strain. So, when you so again we go back to our same picture we have body let us say a beam attached to a wall we apply a load P and this body takes a different shape upon deformation and the extension which is caused is delta. This is the original length L naught, let me convert it black and the final length that so Lf we can write is equal to L naught plus delta, upon application of load.

So, upon application of load the length changes from L naught to Lf to L naught plus delta the cross sectional area Af, becomes in Af and Af is lower than A naught upon application of load. So, we define a quantity called as average linear strain, so average linear strain is defined as e which is equal to this delta the additional length or increase in length divided by the original length. So, basically you can say this is delta L divided by L naught or equal to Lf minus L naught divided by L naught and this is basically dimension less.

And we are saying this average linear strain assuming that every component every microscopic component in the beam undergoes similar level of deformation. So ,this is average linear strain which is nothing but delta divided by L naught. So, this external load that be apply, so external load is P, when you apply a load to a material that is load is resisted by what we call as internal resistance.

So, this internal resistance is quantified by a property called as strength. When you apply load depending upon the internal strength of the material internal resistance of the material which is depended upon the bounding, it will apply it will have certain resistance. So, as a result this P is nothing but integral of sigma into dA, where dA is the area of a find a small element and sigma is the stress normal to the, in this case, we called as cutting plane.

(Refer Slide Time: 14:15)



Now, assuming that the stress is uniformly distributed on the plane A, so assuming that uniformly distributed on area A, whether it is correct or not we do not, it is mostly not correct because what we are assuming here is that the each infinite decimal element a longitudinal element within the material or the beam experiences the same amount of stress.

For this to happen material must be essentially what we are saying single crystal, if it is a poly crystalline material then different crystallographic orientations will experience different stress. Because as we will see later on stress is anisotropic quantity it is not a isotropic quantity, it depends upon the distance between the atoms and so on and so forth.

So, if the material is poly crystalline different grains will experience different stress because of different distances between the atoms within those planes. As a result this is a approximation that so as a result in reality every element is likely to experience different stress, But to make it simple, we assume that stress is uniformly distributed so essentially basically we can say in a polycrystalline material stress on each let us say grain, I think you know the difference between the grains because grain is a micro is a part of material is going to be different, because of changes in orientation of grains.

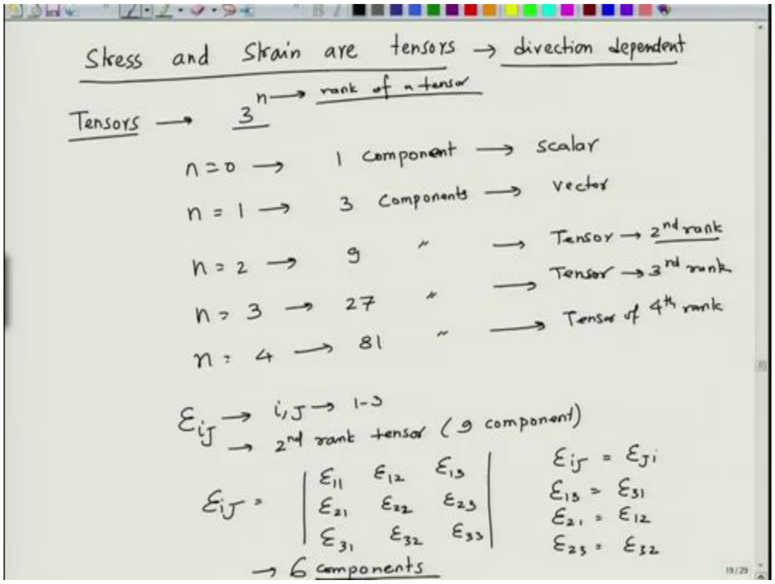
So, some grains are going to have this distance between the atoms, so this is Let us say grain 1, in other grain that distance between this as a result a bone strength will be this. So, this could be grain 2, as a result of stress which is going to be experienced by different grains will be different. But let us say for the sake of assumption that the stress is uniformly distributed on area A which means sigma is constant.

If sigma is constant, then we can write this P as sigma into integral of dA and if A this can be again approximated to A then this P becomes sigma is equal to A or sigma becomes P divided by A, this is your average, average stress. In reality stress, will not be uniform because of an isotropy of stress in materials and stress is an isotropic property an isotropic quantity. But we are assuming that here it is uniform.

So, for average stress in strain below elastic limit let us say below elastic limit means there is no plastic deformation that happens. So, when you increase the strength from increase the length from L naught to Lf and when you remove the load the Lf comes back to L naught, which means Lf comes back to L naught upon removal of load.

Under those conditions Hooke’s law is valid, which can be applied to sigma is equal to epsilon into E and this E is the proportionality constant, which is called as modulus of, so when you write this equation in the scalar form the E is also averaged out quantity that sigma is average epsilon is average as a result the E is also the averaged out quantity.

(Refer Side Time: 18:59)



However in on a microscopic scale both stress and strain they are tensors which means they are direction dependent. So, here we said that the average stress average strain on a plane every element faces the same stress as a result you average out the stress, but in reality is stress and strains are microscopically speaking they are tensorial or vectorial quantities and as a result there is the direction dependents or the isotropic properties.

So, what do we now mean by these stress and strain tensors? So, before we get into that let us look at what a tensors are? So tensors basically you can say tensors are defined by this formula 3 to power n the number of components which are going to, so tensor is basically a you can say a matrix. So, if n is equal to 0 then it has tensor as 1 component, so 1 component will mean that quantity is scalar.

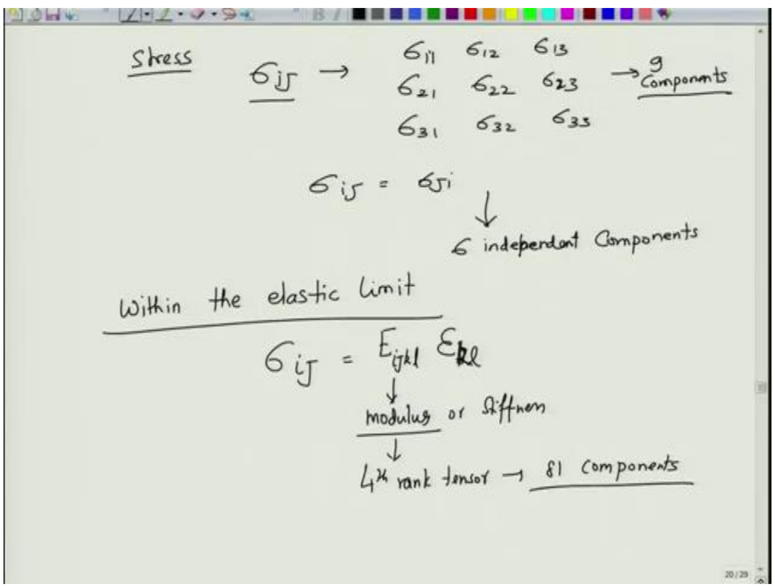
But when n is equal to 1 then it has 3 components, which means the quantity is vector in nature and when n is equal to 2 then it has 9 components and then it becomes what we call as a tensor and this is tensor of 2nd rank this.

So, n here is nothing but the rank of a tensor. So, a 1st rank tensor is vector, 0th rank tensor is a scalar and 2nd ranked tensor is a matrix of 9 components. So, you can keep adding increasing the value of n, so when n goes to 3, it will have 27 components. So it is a tensor of 3rd ranked, n is equal to 4th will mean it has 81 components and it is a tensor of 4th rank and so on and so forth.

So, life is generally lucky we that we do not have so many components in reality because of thermodynamic and material symmetry considerations in most quantities but. So, as far as stress in a strain are concerned, strain is a tensor of strain is directed by the notation epsilon iJ. So where i and J are integers going from 1 to 3, so i and J and vary 1 to 3.

So, as a result, this epsilon iJ is a 2nd rank tensor and it has which means 9 components. So, if you write this epsilon iJ, this will become epsilon 11, epsilon 12, epsilon 13, this is your strain tensor and it has because of symmetry of strain and stress it does not actually has 9 component, it has 6 components because of sigma epsilon iJ being equivalent to epsilon Ji. So, as a result, for example, which means epsilon 13 will become epsilon 31, epsilon 21 will be equivalent to epsilon 12, epsilon 23 will be equivalent to epsilon 32. So, as a result you will reduce this to 6 components because of symmetry, this is about the strain.

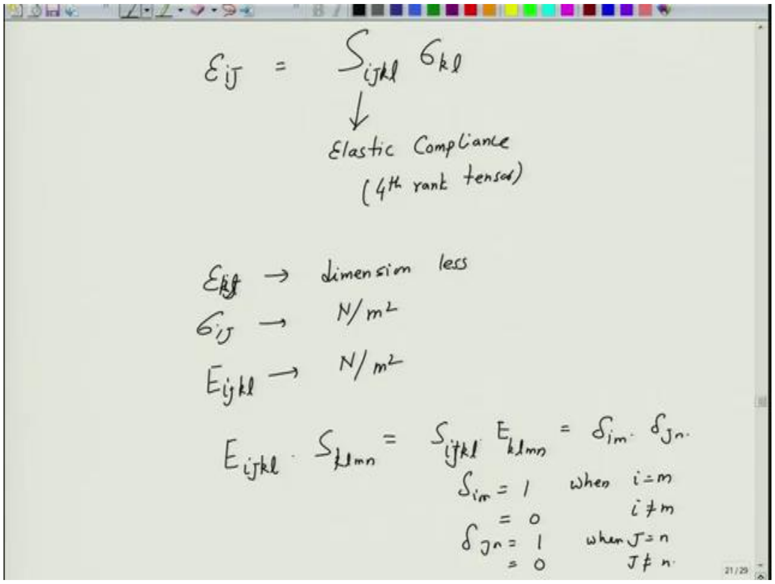
(Refer Slide Time: 23:27)



So, this stress can be written as sigma iJ and again you can write this as sigma 11, sigma 12, sigma 13, sigma 11, 21, 22, sigma 23, 31, 32 and 33 again, sigma iJ is equal to sigma Ji. So, this will essentially have 6 independent components.

So, here it shows 9 components and now within the elastic limit you can write instead I can write sigma iJ is equal epsilon let us say kl and then the proportionality factor is essentially E iJkl. So, this iJ, E iJkl is essentially we can say a modulus or stiffness, which is essentially 4th ranked tensor and this will have 81 components. 81 independent components however because of symmetry and other things the number of components go down so you do no really have to we are not that unfortunate.

(Refer Slide Time: 25:23)



And you can also relate accordingly stress strain that is stress. So, you can write this SiJkl this is called as elastic compliance and this is again a 4th rank tensor. Now, here we can see that strain is dimensionless, stress has a unit which is Newton per meter square. So, as a result the modulus or I can write kl this will again be in Newton per meter square.

So, this is the correlation that we have. So, if you now combine these two quantities the, so we can say that we have written one for the elastic compliance and another for the modulus stiffness. If you combine the two we can write them as EiJkl into Sklmn is equal to SiJkl and Eklmn and this is basically equal to delta im into delta Jn and this delta is called a chronicle delta. So, it is delta im is equal to 1 when so this is because the property of matrix and you can see the two relations should be compellability with each other, so when i is equal to m delta im is equal to 1 and when i is not equal to m, this is equal to 0.

Similarly, delta Jn is equal to 1 when J is equal to 1 and 0 when J is not equal to n. So, for this equation to hold true, because these two equations can be inversed as a result this property must be obeyed as far as vectorial notation are concerned. So, we will stop here, what we have done is we have looked at the what is strain and what is stress, what is average stress, what is the average strain and there is modules conceding that is stress a uniform.

However, in reality stress and strain both are anisotropic properties, they vary different materials in different directions because of properties of materials and we have looked at the equations how they are represented in vectorial form. So, we will do more analysis of these elastic properties in details in the next class. Thank you.