Properties of Materials (Nature and Properties of Materials: III)
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Lecture 34 - Electrical Conduction in Metals

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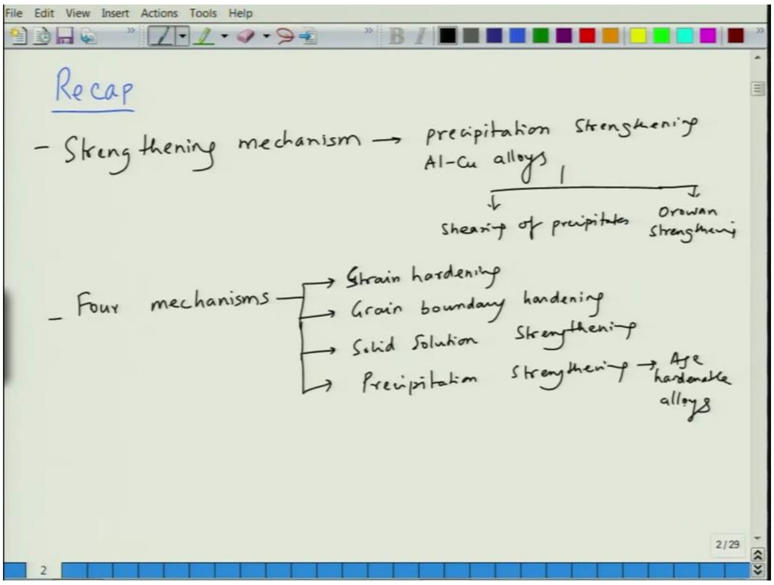

So, welcome to the new lecture of the course, Properties of Materials. So, let us just briefly recap what we did in the last lecture. So, in the last lecture, we talked about strengthening mechanisms and we were mainly talking precipitation strengthening, which is a very common mode of strengthening in aluminium copper alloys and variety other alloys. And depending upon the particles, depending upon the size of the precipitates in their distribution, you may have either shearing of precipitates or the precipitates can help in Orowan strengthening by creating the extensive dislocation motion.

So, this will complete the mechanical behaviour part. So, in total we have discussed 4 strengthening mechanisms. The first one that we discussed was you can say strain hardening, where you deformed the material and it gets harder. Second one is grain boundary hardening, where strength is increased by deforming by reducing the grain size of the material, by the way all of these mechanisms are valid is especially for metals. They may not be valid for ceramics.

And then we looked at solid solution strengthening and finally, we looked at the precipitation strengthening or hardening which is especially true for age hardenable alloys. So, this completes our this course on mechanical properties of materials in general.

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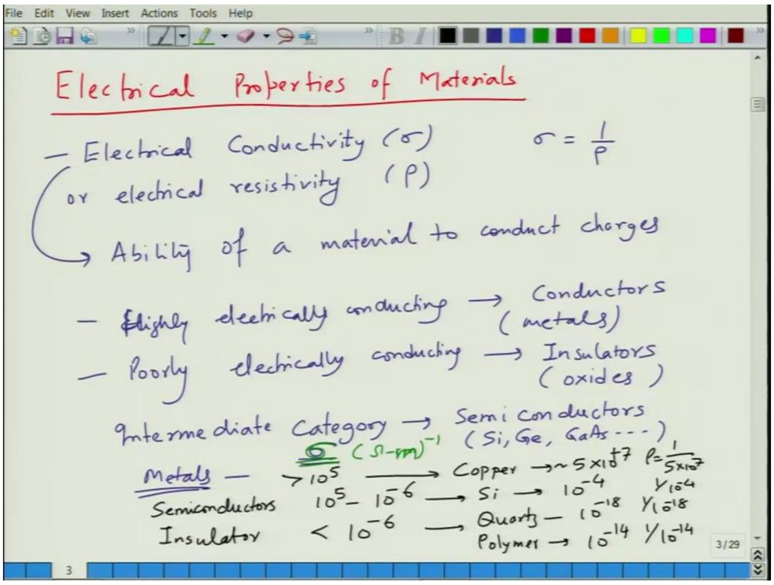

Now let us now move on to the next topic which is on electrical properties of materials. And electrical properties of materials are basically those properties which are determined by, so one of the one of the parameters which actually defines the electrical properties of materials is called as electrical conductivity.

Electrical conductivity which is defined as, which is represented as sigma or you can say it is electrical resistivity, which is represented by rho and this sigma happens to be 1 over rho. So, basically what this does is, it is it defines the ability of a material to conduct electricity. So, electric conductivity is about ability of a material to you can say, conduct charges just like heat conduction, it is electrical conduction. So, there are variety of electrical properties of materials that are there.

So, you have electrical conductivity, you have resistivity, you have dielectric constant and magnetic properties are there. So, we not going to talk all of them. We are just going to restrict ourselves to the conductivity or resistivity here. So, based on the magnitude of this electrical conductivity or resistivity, we define materials as, one which are highly electrically conducting. Such materials are generally called as conductors. And examples are typically metals.

Metals are very good conductors of electricity. And then you have another category, another stream which is poorly electrically conducting which is basically insulator. So, such materials are basically insulators and examples are you know oxides and various other ceramic materials are insulators. And there is intermediate category which is called as semiconductors, which are neither completely very highly metallic and or insulating.

And examples of these could be you know Silicon, Germanium, Gallium arsenide, some of these materials are semiconductors. So, how do you define the values of these to distinguish them? So, for metals the range of sigma, so if I look at the range of sigma in ohm centimetre, in ohm metre inverse let us say, then for metals this is about generally greater than 10 to power 5, whereas for semiconductors, it is between 10 to power 5 to 10 to power minus 6.

And for insulators, it is generally less than 10 to the powder minus 6 ohm metre inverse. And so, if you look at copper for example, which is a very nice electrical conductor. For copper this value is 5 into 10 to power 7 approximately, plus 7 ohm metre inverse. And if you look at the same value for semiconductor like silicon, for silicon this value happens to be of the order of 10 to minus 4.

And if you look at for something like quartz which is insulator, this value is 10 power minus 18. And if you look at something for a typical polymer, for a polymer this value is approximately 10 to power minus 14. So, these are sort of differences in the magnitude in terms of basic electrical property of a material, which is basically electrical conductivity or resistivity. And so, this is the conductivity value. You can see that rho will be so, if you want to now determine rho, rho would be nothing but 1 over 5 into 10 to power 7.

For this it would be 1 divided by 10 to power minus 4. For this it would be 1 divided by 10 to power minus 18 and 1 divided by 10 to power minus 14. So, for metals, electrical conductivity is high or resistivity is low. For insulators, it is it is converse, converse is true which means the electrical conductivity is lower and resistivity is very high and semiconductor conductor fall in between.

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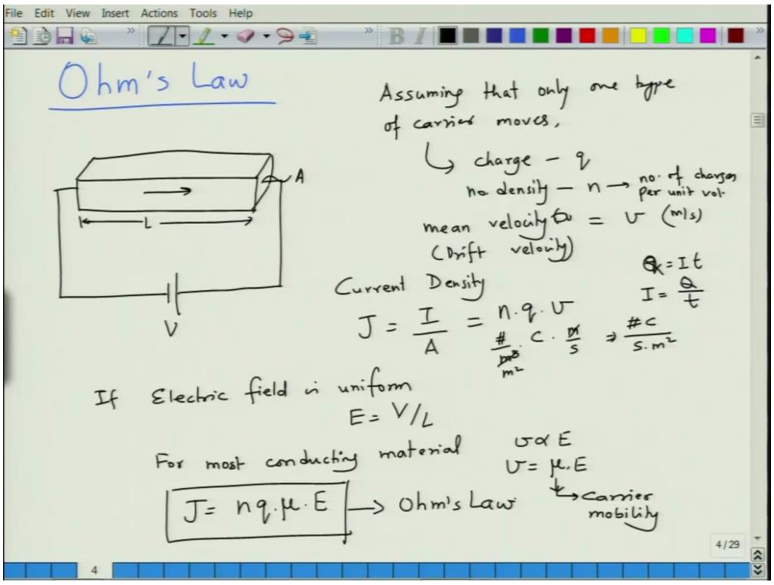

Now to understand the electrical properties, the basic law that we use is called as Ohm's law. And Ohm's law is we have all studied at in our schools so, if you have a bar of length L of cross section area A. So, let us say this is the bar of length L and the cross sectional area is A and if it is attached to a bias, let us say voltage applied is V, the assuming that only one type of carrier moves. We may have multiple types of carriers in materials as we will see, only one type of carrier moves.

Then we can write so, which has let us say for this carrier, it has a charge q, it has a number density which means number of carriers per unit volume, let us say small n. And it has a mean velocity which is one can also say this is called as Drift velocity which is equal to v. And this is along the axis in this direction. So, unidirectional so, if you so from this one can calculate what the current density is.

Current density is defined as J which India current divided by area. So, what is current? So, we know that q is equal to I t. So, I becomes equal to let us say capital Q divided by t. So, number of charges flowing per unit time. So, if we now do the math, so this will be number of charges per unit volume. And this is number of charges per unit volume. And this is the velocity. Velocity is nothing but metre per second.

So, if you do the math here, so total amount of charge basically per unit volume will be n into q. so, the current density in that case would be n into q into v. So, if you look at this, this is number per meter cube. This is coulomb and this is metre per second. So, this will become metre square. So, what we get here is number of charges per unit time per unit area. And that is what we said I was. I was equal to total amount of charge flowing per unit time per unit area will make it current density. So, assuming that electric field is uniform throughout the solid, so if electric field is uniform then, we can write this E as voltage divided by the length.

And for most conducting materials, if we say that the this velocity is proportional to electric field, if this relationship is true, and then we write this v is equal to mu into E where, mu is a constant which is called as carrier mobility. So, I was going to write something else but we will invoke that little later. So, this is called as electrical carrier mobility. So, v is equal to mu E, then I can write this J as n q into mu into E. And this is basically what is called as basic forms of Ohm's law.

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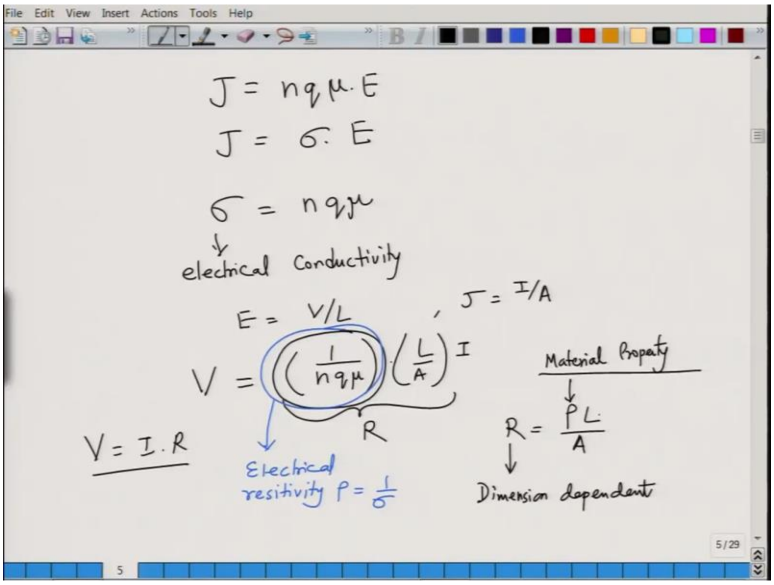

So, you can see that J is equal to n q mu into E. So, we can write this as J is equal to sigma E, where sigma is n q mu, which is basically electrical conductivity. You can also write it little differently. So, if you write this E as v over L, then I can write the value of v which will be equal to 1 over n q mu into L divided by A into I because J is I divided by A.

So, I can manipulate this equation in this form and this we know that another form Ohm's law is, V is equal to I R. So, this term is basically R term and this here, is you can say electrical resistivity rho which is 1 over sigma. So, R you see here is, rho L divided by A. So, you can see that, here if you take R, R is something which depended upon L and A, which is the length and area. Whereas, n is a term which, whereas rho is a term which depended upon number of electrons per unit volume, the charge of carrier, number of carriers if not electrons and the mobility. All of these are constant for a given material.

Or made by so, this is basically you can say, material property where R is dimension dependent. So, it is not the resistance which is materials property. It is actually the resistivity or conductivity which is the intrinsic property of a material which can be changed by changing its processing condition, changing its micro structure, changing grain size and so and so forth as we will see later on.

But for a given material, rho is constant but R can be changed by changing the dimensions. If you increase the length, the resistance will be increased by two times. If you decrease the cross-sectional area by two times, the resistance will decrease by two times. But, the rho will remain constant for the same material. Now the question is we said, carrier? What kind of carrier?

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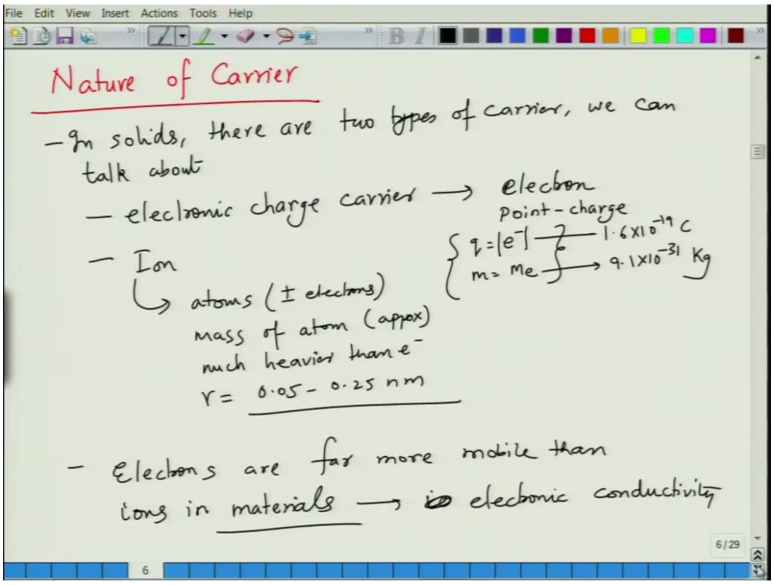

So, let us bring ourselves to nature of carrier. There are two kinds of so, in crystalline solids, in solids there are two kinds of so, in solids there are two types of carriers we can talk about. One is the electronic charge carrier which in case of metals is electron and iron itself, the iron itself. Iron can also move so, this electronic charge carrier is basically if say, it is a point charge with very little mass. So, it carries a charge so, q for this is e.

So, you can say, e and m is equal to m e. These are the fundamental quantities. And this q is 1.6 into 10 to power minus 19 coulomb. And this is 9.1 into 10 to power minus 31 kg. Whereas for ions, ion depends upon ions are nothing but atoms with charges gained or lost. So, basically, these are ionised atoms. So, with plus or minus electrons depending upon it is a cation or anion. It has a mass which is significantly higher than that of electron.

So, basically, it has approximately mass of an atom approximately. And then it is much heavier than electrons and the radius of so, for a point charge, you do not you cannot define a radius. But for ions, the radius varies anywhere from 0.05 to 0.25 nanometre, ball park figure. So, as a result, you can see because of its larger size and heavier mass, electrons are far more mobile as compared to so, electrons so, that is why most materials in case of most materials, we talk about electronic conductivity.

There is something called as ionic conductivity as well which is important in the context of ionic materials, especially in ionically conducting solids. But we will not invoke here. So, for most materials, what we are interested is in electronic conductivity.

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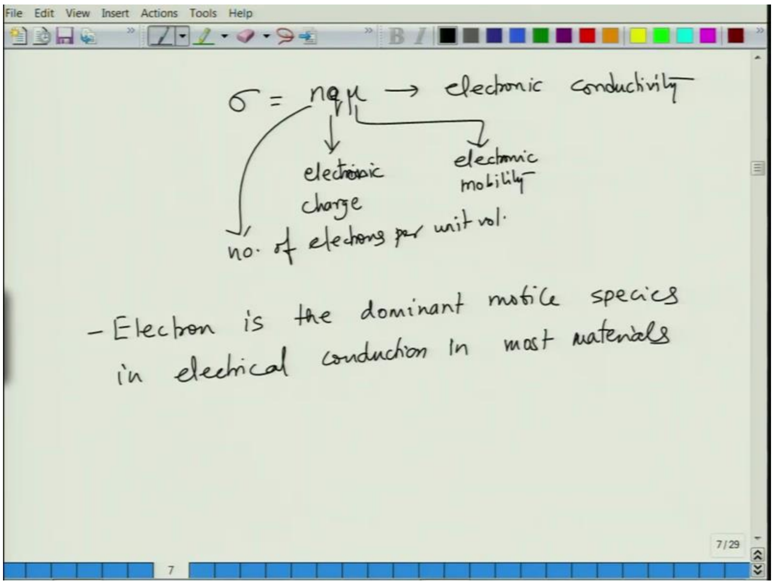

So, the conductivity that we saw earlier, sigma was equal to n e mu. This is basically electronic conductivity. Here, sorry, n q mu, so, q was basically electronic charge, mu you can say is electronic mobility and n would be number of electrons per unit volume. So, these are basically the considerations of these so, basically electron is the dominant mobile species in electrical conduction in most materials, especially in case of metals and semiconductors.

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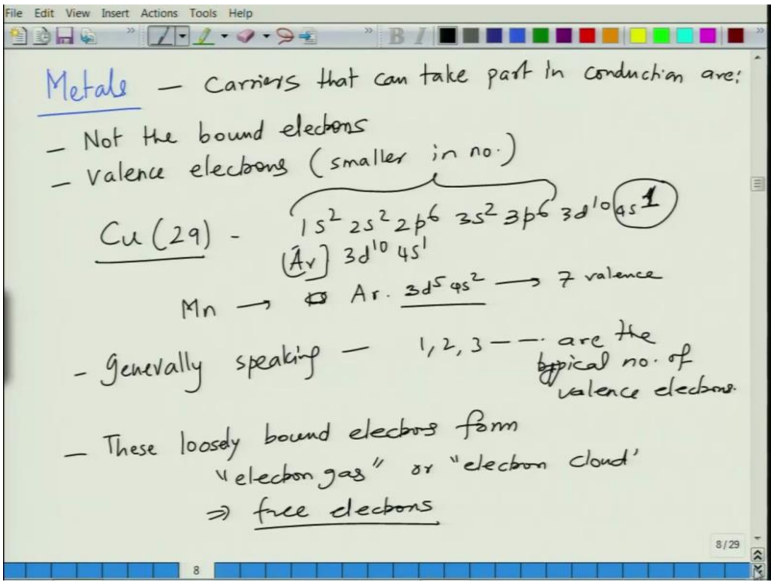

If you look at metals, in case of metals, we generally talk about electrons which can which are mobile or the electrons not the bound electrons. So, basically, carriers that can take part in conduction are not the bound electrons but the what we call as valence electrons, which we call them as free electrons. And valence electrons generally, these are smaller in number. So, all the theoretically, you can have up to 10-12 valence electrons, but generally the valence electron number varies from 1 to 2 to 3 something like that.

So, for example, for copper, copper is 29, so we can write for copper, it is 1 S, 2 S 2, 2 P 6, 3 S 2, 3 P 6 and 3 D 10 and 4 S 2, 4 S 1. So, generally, for copper, this is one electron that is available for conduction. One valence electron. You can write this as Ar 3 D 10 4 S 1 as well because this argon is the argon will complete its shell up to this point. So, that will make it 18. So, 18 plus 11 will make it 29.

If you look at manganese, manganese will give you so, manganese will give you argon into 3 d 5 4 s 2. So, potentially speaking, manganese can have up to 7 valance electrons, but generally the valence states which are available for manganese are 2, 3 and 5. 2, 3, 4, 5, these are the valence states which are more common for manganese. But it can have up to 7 valance electron. So, generally speaking, valence electrons are 1, 2, 3 are the typical number of valence electrons.

And these are the basically outermost electrons which are detached from the atom. As a result, these electrons make so, these loosely bound electrons form what we call as electron gas or electron cloud and these are also called as free electrons. So, we look at the, now having said this, so you have these free electrons which take parts in conduction in materials.

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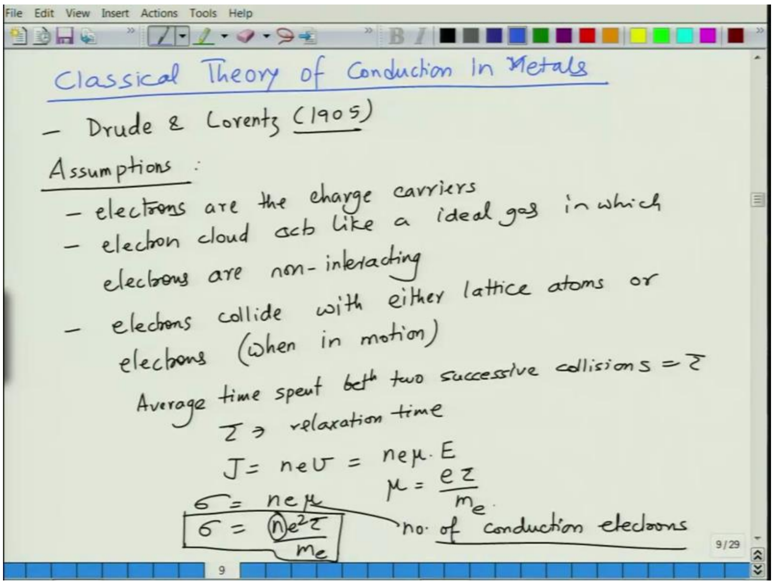

Let us now look at the classical theory of conduction in metals which is based on these free electrons or valence electrons. So, if we look at the classical theory of conduction in metals, it was developed by two people, one is Drude and second is Lorentz in 1905, who assumed that, so what are the assumptions? These were the guys who basically established the basic framework of electronic conductivity of metals. So, first is that electrons are the charge carriers. Secondly, electron cloud acts like a ideal gas in which electrons are non-interacting.

And then these electrons under application of electric pool, they move and when they move, they collide with either lattice atoms or shallow electrons when in motion. So, when they undergo collision with these lattice atoms and electrons, we define a time which is called as average which is defined as tau and this tau is called as relaxation time. And this tau is the one which makes up this term, so, we saw that J was equal to n e v and which was equal to n e mu into E. So, we said that sigma was equal to n e mu but we did not define mu, what it was.

So, mu is equal to mobility which is equal to e tau divided by m. So, you can write now this as, n e square tau divided by m e you can say. This is m e. So, we have replaced q by e and mass of electron is m e. This is what the sigma value is now in the terms of number of electrons and e square tau divided by m. So, when the problem so, this says that, n is so, here you can see that, e is the electronic (ma) electronic charge, tau is the relaxation time, the time is spend between two successive collisions, m is the mass of electron. This n is basically you can say is the number of conduction electron.

So, the question is how many electrons do conduct? Is it all the electrons which are conducting? Do all of them have similar energy, all of them are conducting? Or is it only a certain number of electrons which conduct and give rise to? The answer somewhat very obvious from resistivity calculations. Another problem that occurs is, with this model is, so first of all, we are not able to conclude from this, that how many are the number of electrons which are conducting.

Is it all the conduction electrons which are conducting? Or is it only a certain set of electrons which are conducting? It is not possible to tell the obvious answer from resistivity. However, there are other clues which tell us more clearly with respect to some problems.

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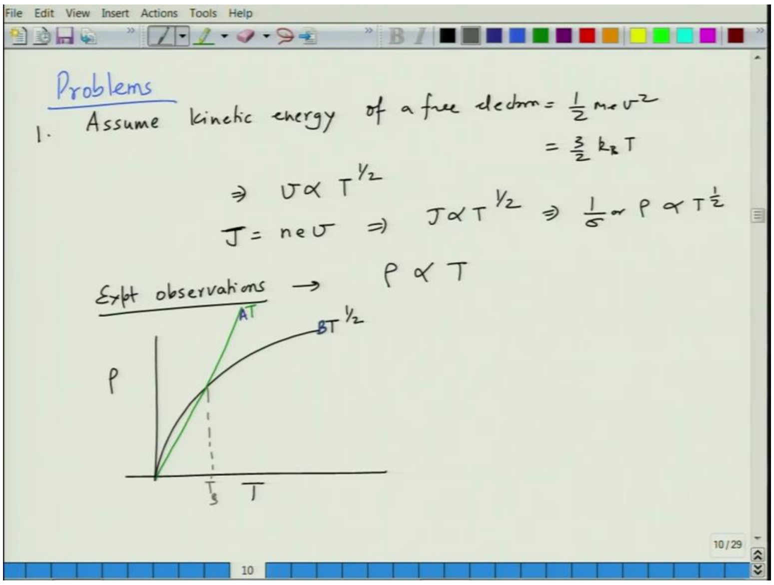

So, the problems which arise are number 1, if you assume that no, you are assuming that this free electron acts like a perfect gas. If it acts like a perfect gas, then we can assume that kinetic energy of a free electron will be half m e v square.

And if it acts like a perfect gas, then this will be equal to 3 by 2 k B T, assuming. So, which means that v should be proportional to T to the power 1 by 2 that is what it follows from here. Which means now, we saw that J was equal to n e v. Which means, this current density should which means that, J should also be proportional to T to the power half. If J is proportional to T to the power half, then even 1 over sigma or rho should also be proportional to T to the power half.

Because J is equal to sigma e, so, which means 1 over sigma that is, rho that should also be proportional to T to the power half. Now, this is what this model says. But experimental observations suggest that rho is a function of T, linear function of T. So, when you plot for instance this rho versus T, the Drude theory says that, my variation should be like this. Rho should vary as T to the power half.

But experimental observations say that, this is a relation is proportional to T so, let us say this is B T to the power half, this is A T something like that. There is however a point at which both of these are similar or they are in the vicinity of each other. So, if you are measuring at this temperature which is let us say, T s then you would not find a difference. So, unless you make a temperature dependent plot of resistivity, you will not be able to find out whether the model is correct or not.

It is only when you make this temperature dependent plot, you say that the model predicts a behaviour of T to power half, but the experimental observation give you a behaviour which is which says that rho is proportional to T. So, this is the first problem that we encounter when Drude's model of, mainly that you know all the electrons are non-interacting, they behave like a perfect gas, they have energy which is half m v square. And all of them have same energy, 3 by 2 k T. So, which means, when temperature goes to 0, the energy also goes to 0. If energy goes to 0, the specific heat also goes to 0.

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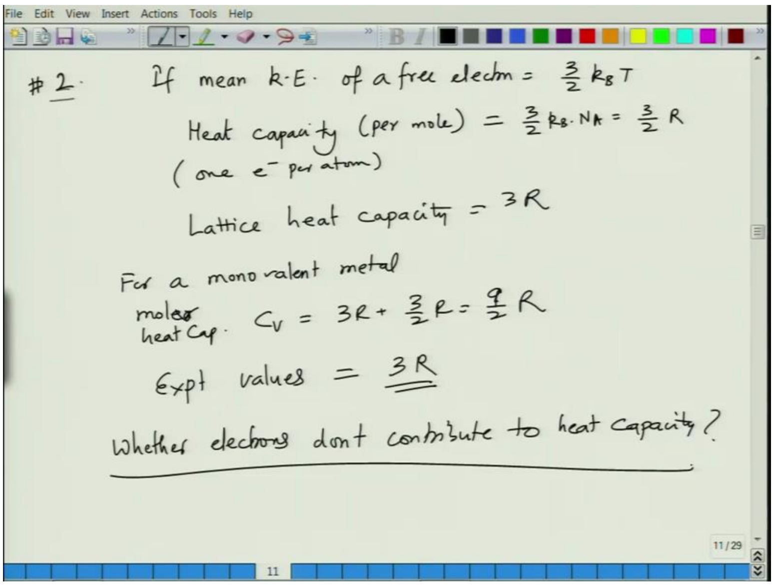

Another problem arises is in the specific heat. So, the problem number 2 is, let us say, if mean kinetic energy of a free electron as we saw was equal to 3 by 2 k B T. That means the heat capacity will be and if we say, per mole, then you just have to multiply this by Avogadro number, this will become 3 by 2 k B into N A, which will be equal to 3 by 2 R.

Assuming that we have one electron per atom. So, one electron per atom is the assumption. So, of course, if you increase the number of electrons, this will increase. So, heat capacity per mole so, each mole, each atom has one electron which means it will have 3 by 2 k B into N A which will be equal to 3 by 2 multiplied by R. Now, lattice heat capacity on the other hand, if you go from the thermal properties, lattice heat capacity turns out to be 3R.

So, which means, for a monovalent metal, the heat capacity, the molar heat capacity will be equal to 3 R plus 3 by 2 R which will be equal to 9 by 2R. Experimental values however, are approximately equal to 3 R. So, does it mean that electrons do not contribute to heat capacity at all? Now, if that is the case, then it is not consistent with the ideal electron gas model. So, the question arise, whether electrons do not contribute to heat capacity? That is the question.

So, these are the 2 disagreements that we see when we apply this ideal gas model of free electrons. Of course, there are other issues like whether all the electrons should be contributing to conduction and things like that. And these answers are solved to some extent by considering what we say is quantum mechanics.

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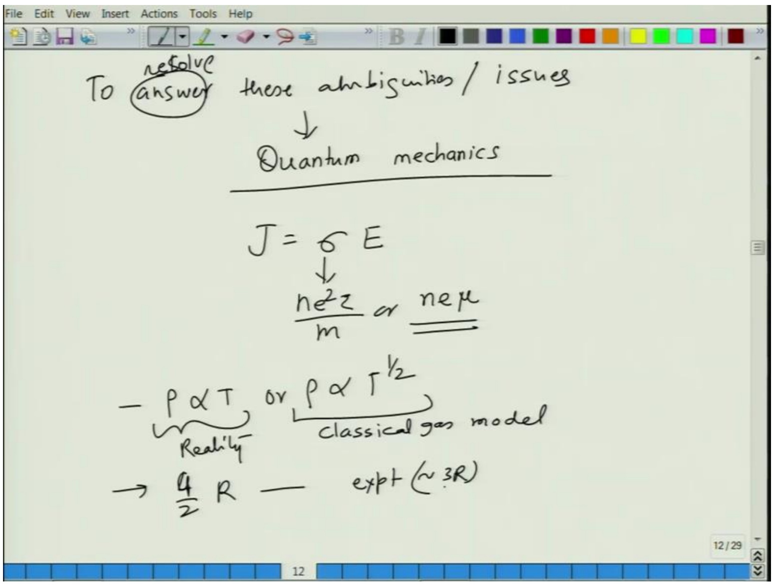

So, to answer these ambiguities, or let us say issues, or to resolve let us say not answer, but to resolve we take help of what we call as Quantum Mechanics. So, basically, what we have done is, we have looked at the classical behaviour of electrons, where we say that where we say that conduction in metals is takes place by the means of conduction of free electrons. And they follow this relation J is equal to sigma E, where sigma is electrical conductivity which is given as n e square tau divided by m or n e mu, where mu is the electronic mobility and tau is the relaxation time.

The but there are questions with regard to consideration of this free electron gas model in terms of 1 rho is proportional to T or rho is proportional to T to the power half. That classical gas models says this. Whereas, reality is this. Similarly, classical gas model says that, my specific heat should be 9 by 2R. But experiment say that it should be equal to approximately 3R. So, there is some problem in terms of number of electrons perhaps contributing to the total cause here.

And this is what is going to be resolved by the use of quantum mechanics that we will take up in the next lecture. Thank you.