Properties of Materials

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Transformation of Axes and Principle Stresses

So, welcome to the new lecture of this course again Properties of materials. So, let us just briefly recap what we were doing in the last lecture.

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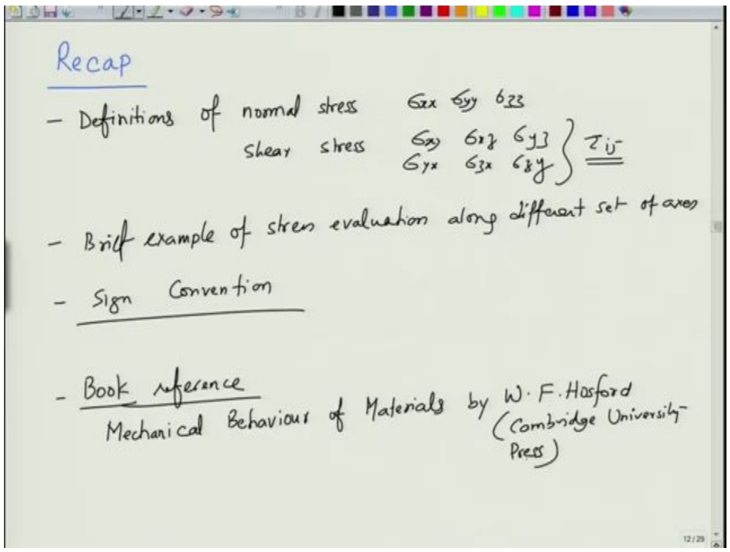

So, in the last lecture, we looked at the definitions of normal stress, which could be sigma x x sigma y y, sigma z z or we can have shear stress, which is sigma x y sigma x x, sigma y z and this could be also y x, z x and z y. And these are generally referred as tau i j instead of sigma and by definition normal stress is basically a stress which is arising from a force that acts perpendicular to an area and shear stress is because of force that acts along let us say in this case j direction and it acts on a plane which is normal to i direction so, this is what is the definition of tau i j. And we also did a brief example of stress evaluation along different set of axes and then we also looked at the sign convention.

By the way, just I forgot to tell you that you can read all this from a book title is, so book reference it is a very good book “Mechanical behaviour of materials” by W.F. Hosford of Cambridge University Press. So, this is a very good book if you can get this, all of this will be available in this particular book.

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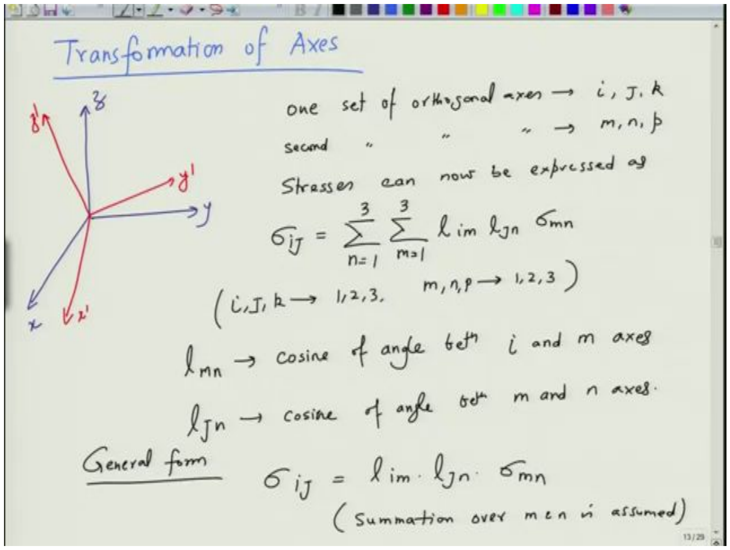

So, at the end we were trying to do the transformation of axes in a general sense. And so basically we said that we start with this coordinate system, which is let us say x, y, and z and we want to transform to x prime, y prime and z prime, which is a new system, which is at certain angle to the original ones. So essentially let us say, we define them as 1 set of the first set of orthogonal axes as i, j, k. And then we have second set of orthogonal axes that can be defined as m, n and p. So, by using this by stresses in a new system in general can be defined as, so stresses can now be expressed as sigma i j is equal to n is equal to 1 to 3, summation of what n is equal to 1 to 3, m is equal to 1 to 3, this will be l i n, l j n and sigma m n.

So the n is here summed over 1 to 3, m is summed over 1 to 3 and of course i j vary from 1 to 3, but at 1 time you can take only 2 of them. So, i, j, k can vary from 1 to 3 and m, n, p can also vary from 1 to 2 to 3. So here what is now this is basically because of matrix transform because the tensors are matrix properties. So essentially, we are using the characteristics of matrix and l m n here is defined as cosine of angle between let us say i and m axes, similarly l j n here would be the cosine of angle between m and n axes, okay. So, in a general form, you can get rid of the summation symbol, which means it is implied or it is assumed that it is there. So, in a general manner you can write this as sigma i j is equal to l i m into l of j n into sigma m. And so, which means here the summation over m and n is assumed, it is by default there.

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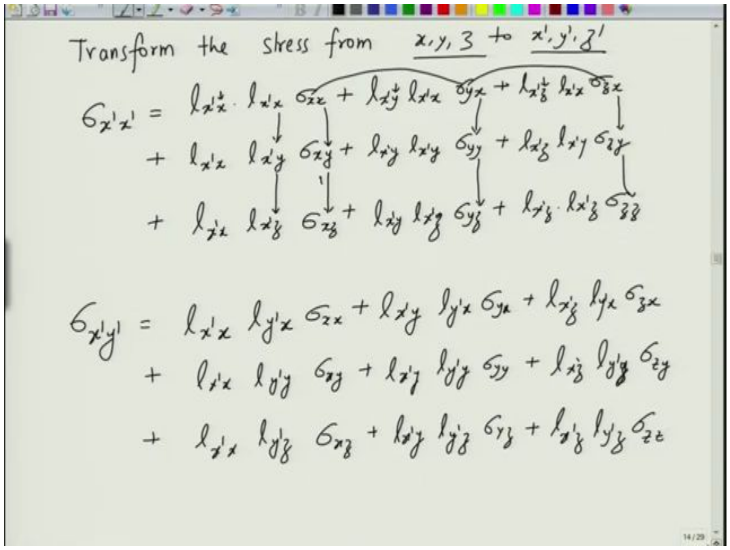

So, let us say you want to transform the stress from let us say x, y, z to x prime, y prime z prime, the new system. So let us say, let us first begin with x prime x prime. So, sigma x prime x prime can be written as l of x prime x into again l of x prime x into sigma x x. And then this becomes now the second term will become l of x prime y l of x prime x into sigma y x. So we can see that we are varying, this was x prime here x so we are varying the x here to x y and this x corresponds to this x. So, this was x earlier it becomes y sorry y x not y z, y x because it was sigma m n. So, we are varying in the first row we are varying m and then the third term becomes l x prime z l x prime x sigma z x.

So, we can see that we have varied only 1 term that is the m term, so m term has changed from x to y to z. Similarly, stress also this was sigma m n, so it has become sigma x x sigma y x and sigma z x. Now, let us look at the other terms, so now we will vary n so, in that first case n was equal to x x prime, now the n will become equal to y. So, the first term remains x prime x so this x prime will be this, this x will now become y, and here sigma will become sigma x y, this will become sigma x prime y, this will become sigma l x prime y. So, now you can see it is varying vertically and this will become sigma y y.

Third term will be l x prime z l x prime y into sigma z y. Coming third row, third row will be x prime x l x prime z so we can see that x has gone from here, and this similarly here it will become sigma x z. Again l x prime y l x prime z sigma y z c l x prime z l x prime z into sigma z z. So, you can see that in this case this digit has changed. In the first case it was the first digit so this changed from here to here, from here to here. Similarly, in this case this particular digit this particular digit and this particular digit. So in the first row we vary m, in the second row wise we vary m and the column wise we vary n so, this is what the general expression for the stress will be.

You can also write let us say sigma x prime y prime. So, what will sigma x prime y prime will be, this will be sigma l x prime x l y prime x sigma x x plus l x prime y l y prime x sigma y x plus l x prime z l y prime x sigma z x plus we can write l x prime x l y prime y sigma x y plus l x prime y l y prime y sigma y y plus l x prime z l y prime y sigma z y plus l x prime x l y prime z sigma x z plus l x prime y l y prime z sigma y z plus l x prime z l y prime z into Sigma z z. Now of course, so these are 2 expressions similarly, you can write other expressions as well.

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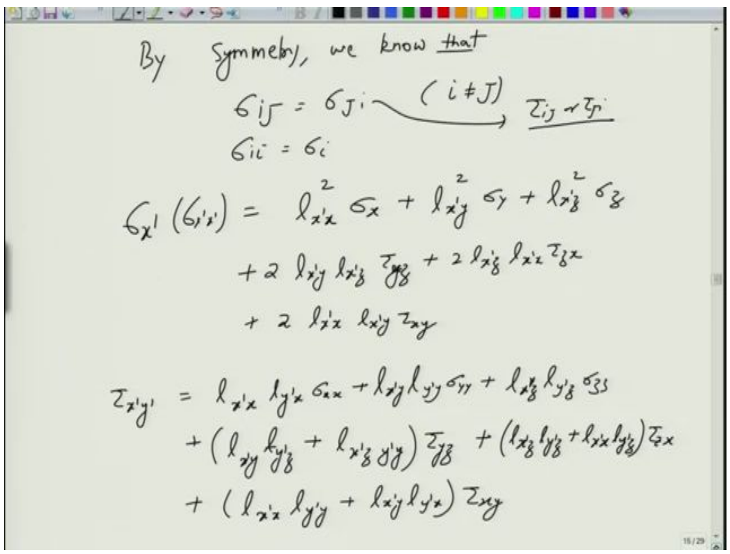

Now, by symmetry we know that sigma i j is equal to sigma j i and this i is not equal to j. So, if we apply this and we write sigma i i as sigma i. So, we can write this as sigma x prime, which is nothing but sigma x prime x prime is equal to l of x prime x square into sigma x plus l of x prime y square into sigma y into l of x prime z square sigma z plus 2 of l x prime y. So, basically we can write this as Tau i j or Tau j i so we will introduce this Tau x y z plus 2 l x prime z l of x prime x z l of x prime x prime x into Tau of z x and plus 2 of l of x prime x into l of x prime y Tau of x y.

And this you can do yourself as a home exercise, it is a little bit tedious, but not complicated. So, it just needs to be done and similarly you can write the expression for this. So l of x prime x l of y prime x sigma x x plus l of x prime y l of y prime y sigma y y plus l of y prime z sigma 3 3. So first 3 principal components and then we write the shear component. So this will be first one, and l of x prime y plus into l of y prime z plus l of x prime z l y prime y into tau y z and l of x prime y plus l of x prime z y prime y. And then second 1 will be l of x prime z y prime z plus l of x prime x, y prime z into Tau z x.

And the third term will be l of x prime x l of y prime y plus l of x prime y l of y prime x into Tau x y. So, these are the stress components that we write by doing the transformations.

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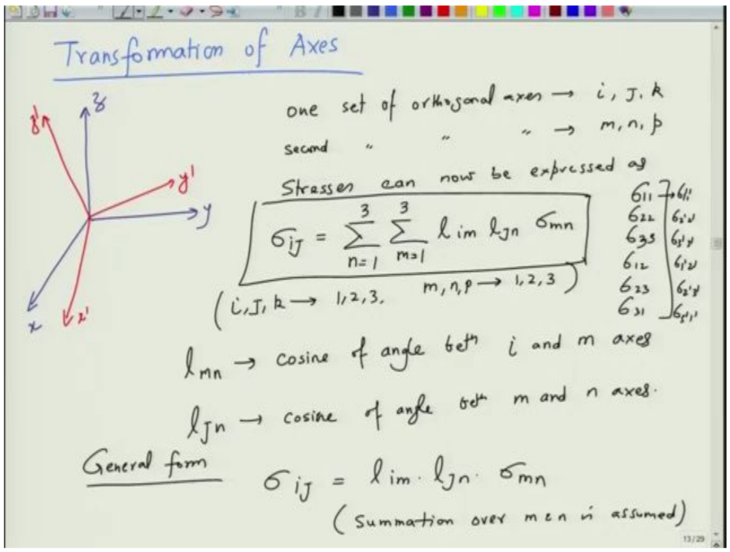

If you go back to previous slide, essentially the transform is this, this is a formula. So, essentially you can calculate any stress it could be i j so essentially if you take you can calculate sigma 1 1, you can calculate sigma 2 2, you can calculate sigma 2 3 and so on and so forth. Sigma 3 3 and 1 2, sigma 2 3 and sigma 3 1. All of these can be calculated in the new system. So, let us say in the new system, it becomes 1 prime 1 prime, in the original system it was so that the transforms could be sigma 1 prime 1 prime it could be 2 prime 2 prime, 3 prime 3 prime, 1 prime 2 prime, 2 prime 3 prime and 3 prime 1 prime.

So, It could be any of them and you can calculate just by using this formula where your original 1 system is i j k, second system is m, n, p and you can transform between the 2 the way you like using this formula which is a bit tedious to write but it is fairly straightforward, you just have to vary, in 1 row you vary let us say m, so in rows you very m and the columns you vary n, and i j remains same as what you write on the left. So, if you suggest as a home exercise, I mean you can write the expressions for sigma 1 prime 1 prime, what will this be in terms of original system.

Similarly, you can try writing expression for sigma 2 prime 3 prime, what will this be in terms of so you can see that, for example, when you write this so let us do a simple.

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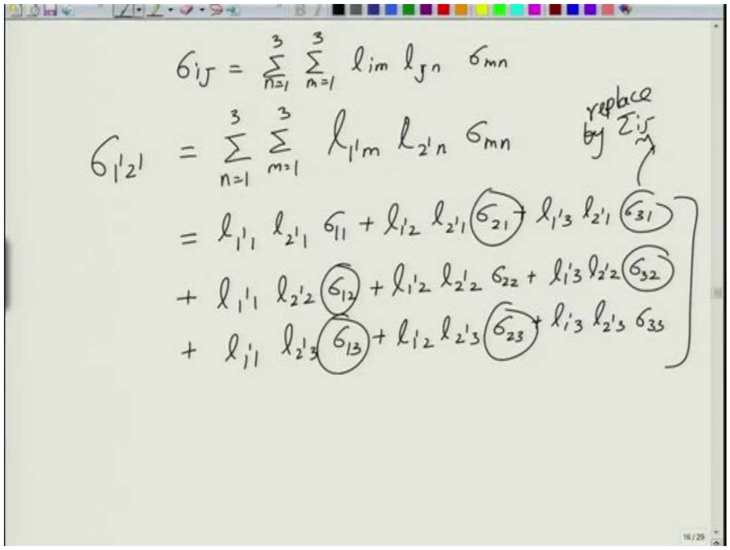

So, we say that the formula is sigma i j is equal to n is equal to 1 to 3 sigma m is equal to 1 to 3 l i m l j n and sigma m n. Let us say I want to calculate what is sigma 1 prime 2 prime, so if I write this as an expression, so let us say the first one, it remains n is 1 to 3. So I fixed certain things here l 1 prime m, the second 1 becomes l 2 prime n then this becomes sigma m n. So I am only now left to vary m and n, first we vary along the row okay. So the first term will become l 1 prime 1 l 2 prime 1 sigma 1 1. Now, we very l of 1 prime 2 l of 2 prime 1 main this 1 this will become 2 1, and this will go 1 prime 3 l 2 prime 1 sigma 3 1 this this fine.

Now, you come to this will remain sigma 1 prime 1 this will be sigma 2 prime 2 this will be sigma 1 2. Now, you again go this way so this will now you are changing l 1 prime 2 l 2 prime 2 and this will become sigma 2 2 and this will now, your this is sigma 1 time 3 l 2 prime 2 and this is sigma 3 2. This will remain 1 time 1, this will become 2 prime 3, this will become sigma 1 3, this will remain 1 prime 2 this will remain 2 prime 3, this will become 2 3 and this will remain l 1 prime 3, this will become 2 prime 3 and this will become sigma 3 3. So, this is what will be the new stress tensor. So, of course, you can replace since these can be replaced by Taus you can replace so this is how you write the stress tensor of various of these stresses.

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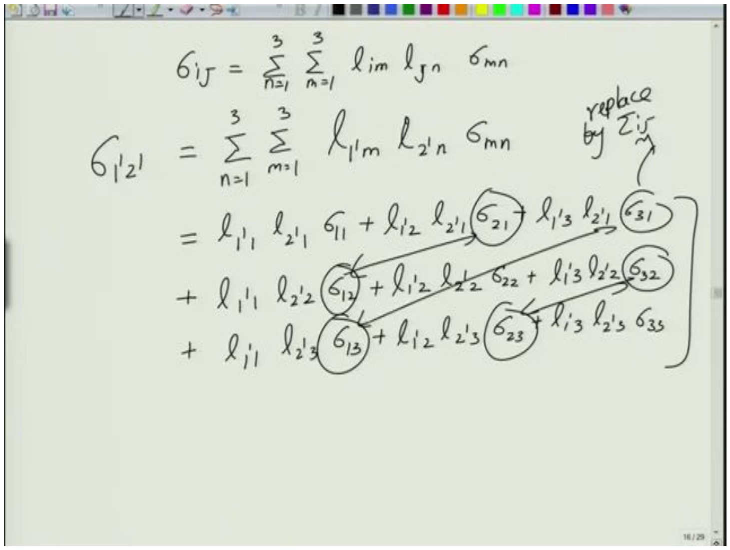

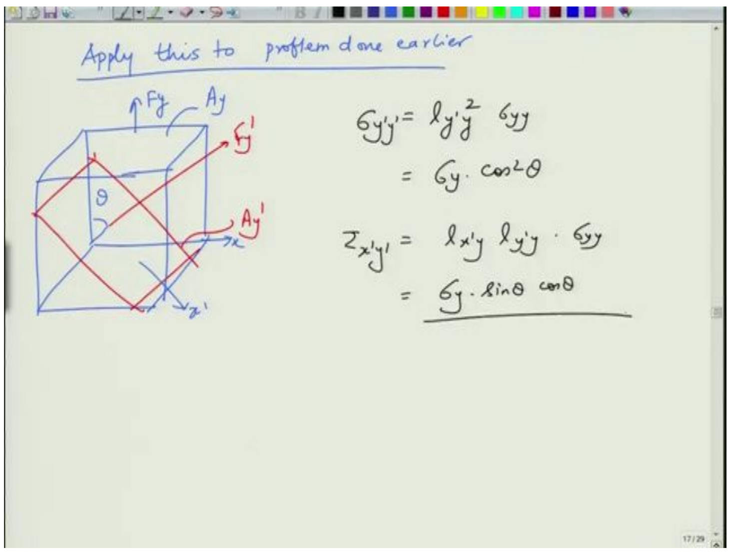

So, if you now apply this to problem done earlier, so now if we apply this to the problem that we did earlier, we draw the same box. So, we made a plane which was like this so this was let us say, A y prime, this was F y prime, this was F y and this was A y, this was x, this was x prime of course, y and y prime are listed there and this angle was basically you can say Theta.

So, now, if I go by the same logic that I have applied here, the sigma y prime will become l y prime y square into sigma y y. So, basically sigma y y prime, and if we do the math, basically l y prime y is the cosine of angle between y and y prime. So, this is basically sigma y into cos square theta. Similarly, if you want to calculate what is Tau of x prime y prime, this becomes Tau of l of x prime y l of y prime y into sigma y y, other terms are 0. So we just calculate this, this will become sigma y l of x prime y and l of y prime y so, this is sine theta, this is cost data. So, this is same result that what we got from our earlier exercise that we did.

So, what essentially we have done in this so far is we have written a general equation through which you can transform the you can estimate the stresses along different axes coordinate system by using this transform, this is nothing but from matrix characteristics. And we tried doing this for let us say two principle and shear stresses, that is sigma x prime and sigma x prime y prime. And then we did a general case for and you can then apply the symmetry properties, which makes it i j being equal to sigma j i and then simplify these above equations. And then we did finally 1 exercise for hypothetical case of sigma 1 prime 2 prime and again, you can simplify this further by making these equal.

So, these will become equal, these will become equal and these will become equal. So, as a result you can simplify this equation and essentially that is what we have done in this lecture. Now finally, what we did was we applied the same Transform to the problem that we did earlier manually and if you apply to transform you come up with the same result. So, it basically validates that what we did earlier was correct okay.

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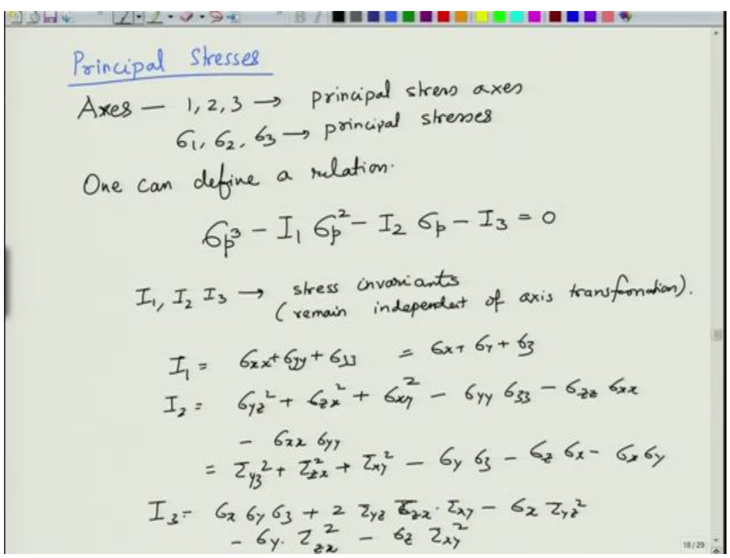

So, now, let us move further and define some more things. So, we define what we call as principal stresses. So, in principal stresses generally, if you define axes as 1, 2 and 3, then these are called as principal stress axes. And the principal stresses will be sigma 1 correspondingly sigma 2, sigma 3, if 1, 2, 3 are principle stress axis the stresses along these are called as principal stresses. So, based on the characteristics of these stresses, one can define an equation that sigma p cube minus I 1 sigma p square minus I 2 sigma p minus I 3 is equal to 0. So, this is a relation which basically defines the principle of stresses. And this p subscript is essentially about the pressure okay.

And here I are I 1, I 2 and I 3 are called as stress invariants, these invariants remain independent of axes transformation, and one can write this I 1 as sigma x x sigma y y plus sigma z z, I 2 can be written as sigma y z square plus sigma z x square plus sigma x y square, you can write them Tau x y Tau and this becomes next one is sigma y y minus of sigma y y sigma z z, then we have sigma z z sigma x x and then we have minus of sigma xx sigma y y. So, you can write them as Tau y z square plus Tau z x square plus Tau x y square minus sigma y sigma z minus sigma z minus sigma x minus sigma y so, this becomes sigma x plus sigma y plus sigma z.

Similarly I can write I 3, which is sigma x sigma y sigma z plus 2 of Tau y z Tau z x into Tau x y minus of sigma x Tau y z square minus of sigma y into Tau z x square minus of sigma z into Tau x y square. So, we will stop here, what we have done is basically we looked at transformation of stresses from one set of axes to another set of axes using a simple formulism, do that exercise at home and it is a bit tedious, but it is simple. And once you get the habit of doing it, you will be alright with how to do it. So will stop here, we will take up this principle of stress formalism further and do a problem to understand what it is, thank you very much.