Properties of Materials (Nature and Properties of Materials: III)
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Lecture 06
Illustration for True and Engineering Strain

So welcome again to the new lecture of this course, Properties of Materials. So let us just briefly recap the previous lecture.

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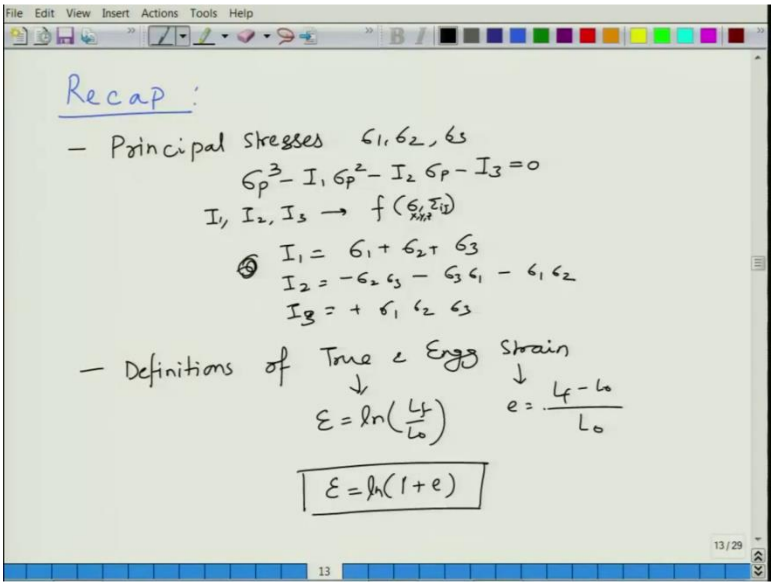

So in the previous lecture we looked that one calculation of principal stresses. Sigma 1, sigma 2 and sigma 3, which are basically from this equation sigma p cube minus I1 sigma p square minus I2 sigma p minus I3 is equal to zero, where I1, I2 and I3 rescues variants, basically they are in variants of stresses independent of orientation of axis and sigma, when you calculate this I1, I2, I3, which are given in terms of normal and shear stresses.

So I1, I2 and I3 are function of shear stresses sigma and tau, and sigma x, y or z and tau i j. You can calculate that three values of sigma p. And these three values of sigma p are nothing but magnitudes of sigma 1 and sigma 2 and sigma 3 and sigma, and I1, I2 and I3 are also related to these values. So I1 is equal to sigma 1 plus sigma 2 plus sigma 3. So you can cross check that. I1 was also equal to sigma x plus sigma y plus sigma z.

So all these three values should match. I2 was equal to minus of sigma 2, sigma 3, minus of sigma 3, sigma 1, minus of sigma 1, sigma 2. And I4, I3 was equal to minus of sigma, sorry, plus sigma 11, sigma 22 and sigma 33 or write or alternatively just write it as sigma 1, sigma

2 and sigma 3. So these I values should match with the I values that you get from the products of summation of products of sigma x, sigma y, sigma z, as well as tau i j’s.

So this is what we did and then we also look at the definitions of true and engineering strain. So true strain is epsilon ln of Lf divided by L naught, where engineering strain is given as Lf minus L naught divided by L naught. And the relation between the two is epsilon is equal to ln of 1 plus e. And we also saw that this that there is a cross, so the values of epsilon and e, they correspond well with each other at very, very small strains, generally below 0.01, but the moment you go to higher strains there is a strong divergence and e values are very different as compared to epsilon values.

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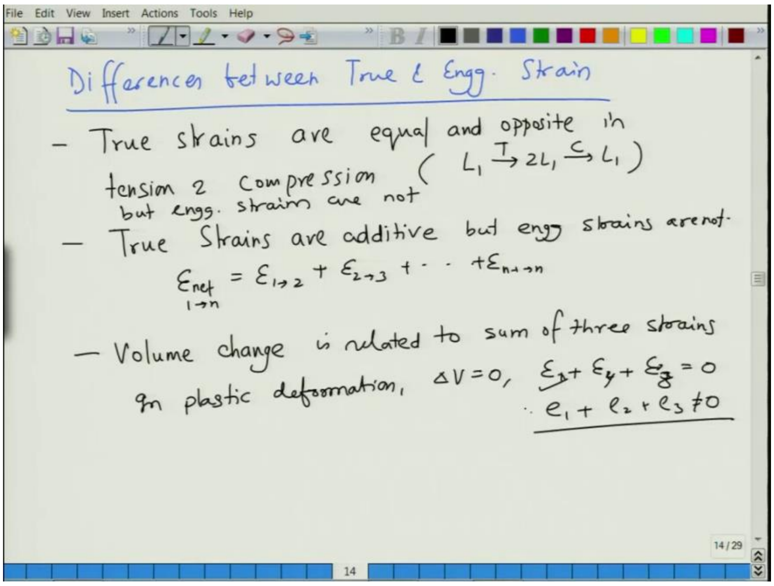

So now let us look at the other differences between the two strains and there are other differences, such as, first one is. So the first difference is true strains are equal and opposite in tension and compression. So let us say if you go from L1 to 2L1, and then go back to L1, they will remain the same. So this is tension, this is compression. They will remain the same. We will see that in the examples. And then another difference is true strains are additive.

So in the first one, true strains are equal and opposite, but engineering strains are not. True strains are additive, but engineering strains are not. So which means epsilon net is equal to epsilon 1 to 2 let us say plus 2 to 3. So you go from one step to second step. First pass, second step to third step. Third pass and so on and so forth. And if you compute the sum, the sum will be equal to overall strain. Let us say if you go from n minus 1 to n, so this would be 1 to n. So this would sum. But engineering strain will not.

And the third is volume change is related to sum of three strains. As we will see in plastic deformation, delta V is equal to 0, which means epsilon 1 plus epsilon 2 plus epsilon 3 should be equal to 0 or epsilon x plus epsilon y plus epsilon z should be equal to 0. But e1 plus e2 plus e3 is not equal to 0. So there is a anomaly between these. So this obeys that principal of net, no net volume change, but this does not obey. So there is a problem there.

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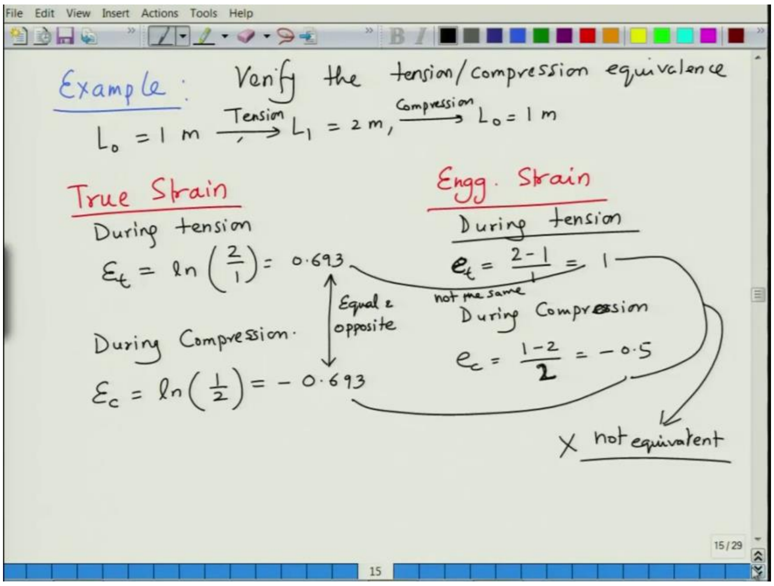

So let us say, let us go to problem number one, let us say. So example number, first example is that let us verify, verify the tension compression problem, tension compression equivalence. So let us say we have L naught of 1 meter, then we have L1 of 2 meter, so L naught goes to L1 of 2 meter, then L1 goes back to L naught of 1 meter. So this is tension and this is compression, alright?

So if I examine the true strain, so during tension, epsilon t will be equal to ln of Lf divided by L naught, which is 2 divided by 1, which is equal to 0.693. And during compression, epsilon c will be equal to ln of 1 divided by 2, Lf is. So in this case, it is going from 2 to 1. So 1 is the final length and 2 is the initial length and this will be minus of 0.693. So you can that these are equal and opposite.

Now let us see the example of engineering strain. So during tension, I can see epsilon t, sorry, et is equal to Lf minus L naught which is 2 minus 1 dividend by 1 this is equal to 1, alright. During compression, ec is equal to 1 minus 2 divided by 1 which is equal to 1 minus 2 divided by 2 which is equal to minus 0.5. So you can see that not only these are different, so these are not the same, because these are very large strains. These are not equivalent and there is a problem. That is why true strain is a better measure of strain than the engineering strain.

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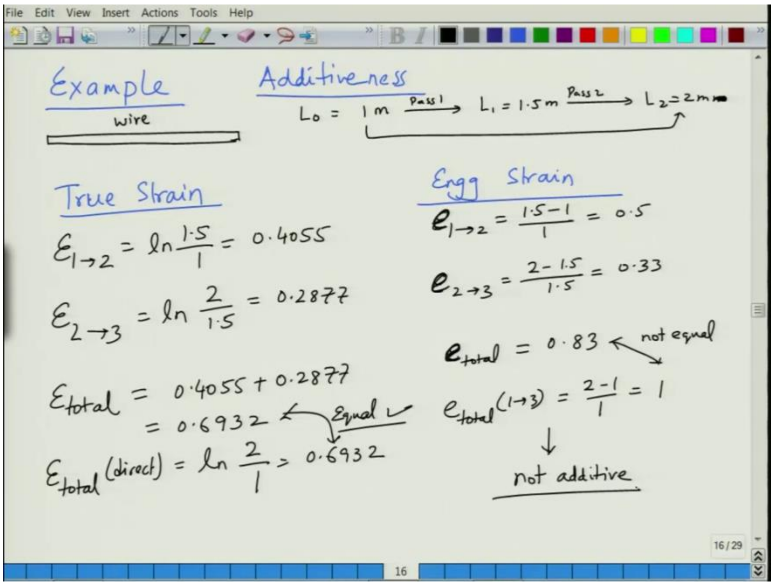

Second example, let us take off, let us take that off additiveness. So let us say we have a piece of wire. This wire has initial length of 1 meter. We stretch it to L1 is equal to 1.5 meter. So this is pass 1. It goes through pass 2 to L2 is equal to 2 meters. So we can see that there is a net change from 1 to 2 meter in two passes. So let us calculate the strain at every step and the overall strain and compare the two.

So let us first look at the case of true strain. So when we calculate epsilon 1 to 2, it is ln of 1.5 divided by 1, which is 0.4055. And then we calculate epsilon 2 to 3, this is ln of 2 divided by 1.5 this is 0.2877. And as a result, epsilon total will be equal to 0.4055 plus 0.2877 this should be equal to 0.6932. And if I work out the epsilon total directly, then I go from 1 to 2 direct, so this is ln of 2 divided by 1 which is 0.6932 and these two are equal, which means true strains are additive.

Now let us look at the example of engineering strain. Engineering strain is epsilon 1 to 2 this is equal to, we are going from 1.5, 1 to 1.5, so it is 1.5 minus 1 divided by 1. So this is 0.5. So the second pass it is 2 minus 1.5 divided by 1.5. So this is equal to 0.33. So if I total them, sorry, this is e, not epsilon. My apologies. So e total will be equal to, did I write e earlier, so e total is equal to 0.83. And what is e total direct that is e 1 to 3, then it is 2 minus 1 divided by 1 which is 1. And we can see that these two are not equal, which means it is not additive. So this is the problem with the true engineering strain.

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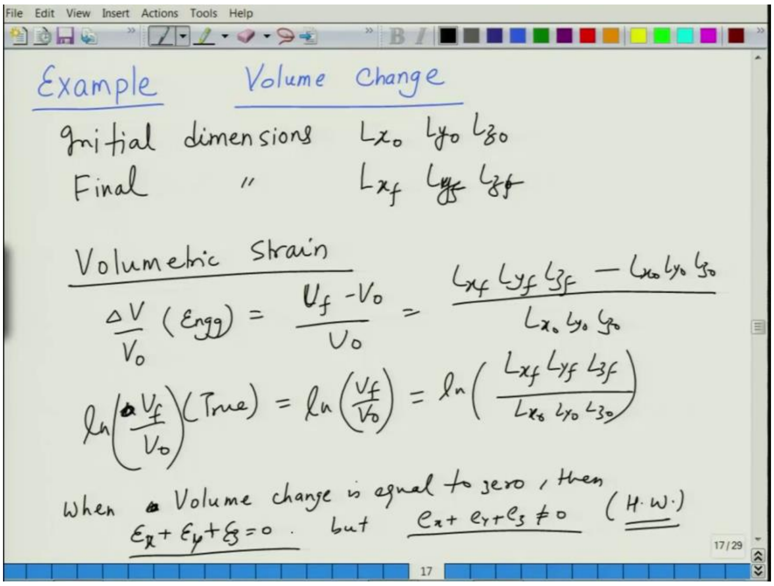

Now let us look at the third another issue of volume change. So I will not do it completely. Let us say we have initial dimensions of Lx naught, Ly naught, Lz naught and the final dimensions are Lxf, Lyf, Lzf. We need to calculate what is volumetric strain? Volumetric strain is delta V divided by V naught. So if you calculate for engineering one, then it is Vf minus V naught divided by V naught.

So this is Lxf Lyf Lzf minus Lx naught Ly naught Lz naught divided by Lx naught Ly naught Lz naught. And if you calculate delta V by V naught for true, this is ln of Vf divided, sorry, this is Vf divided by V naught. So this will be ln of Lf, Lxf. Now I will leave it here. I will ask you to prove that when volume change is equal to zero, then one can see that epsilon 1 plus, epsilon x plus epsilon y plus epsilon z is equal to 0, but the same is not true for. So I will leave it to you to prove that. So this is home work.

So what we have seen so far is basically we have looked at the differences between engineering and true strain. We have looked at the true strains are equal and opposite that we have proved using this. They are equivalent in tension and compression. True stresses are additive and engineering strains are not additive and volume change riddle that you have to do yourself that in the plastic deformation when volume change, and that volume change is equal to 0. It is represented by epsilon x plus epsilon y plus epsilon z is equal to 0, but ex plus ey plus ez is not equal to 0.

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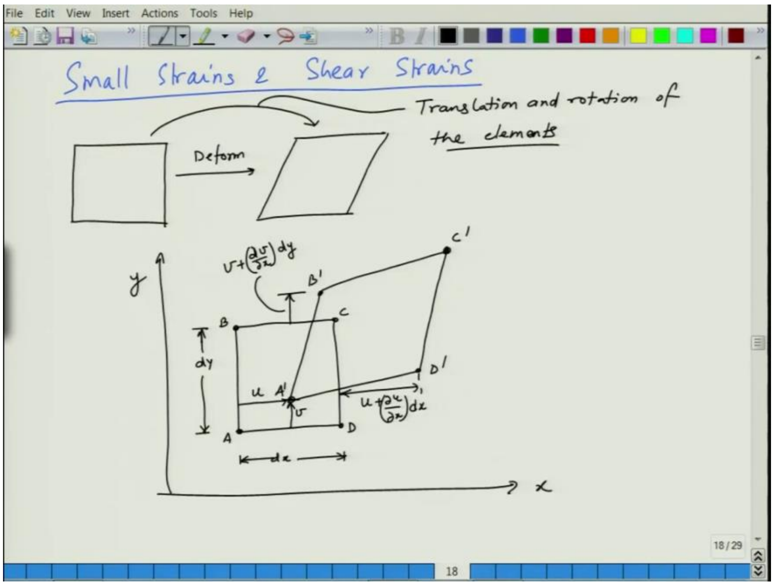

So now, this is what the first part of this lecture is. Now let us look at the concept of what we call as small strains and shear strains. So generally when a body deforms, so generally when a body deforms, let us say, this body when you deform it, this body may convert to a, let us say, a shape like this. But this the process through which this happens involves both translation and rotation of the elements or body parts you can say.

Now the strains have to be calculated in such a manner so that you are able to do them independently off rotation and translation. So you should look at the overall effect rather than getting into the integrities of translation and rotation. So a strain must overcome on a combined, the individual translation and rotation effects. So let us say we have a situation like this, in which let us say this is x and y axis.

We start with a body in this fashion. So we have these four points. C, D. These four points get move to, let us say, so let us say, this is A prime, B prime, C prime and D prime. Let us say, this length is initially dx, small lengths, this is dy. So let us say this point has moved by translation u in this direction and v in y direction. And the corresponding movement of this particular point from BC line is v plus del vy del x into dy.

Similarly, the corresponding movement here is u plus del u by del x into dx for a very small translation, and this is dx. So basically there is the extent of movement that we are having in x and y directions for… So you can see that this B point does not move just by, in this direction, just by v, it also moves by another small amount which is del v. So you can see there is asymmetry. A point moves by different magnitudes as compared to a B point and so on and so forth. So let us calculate what the strains are.

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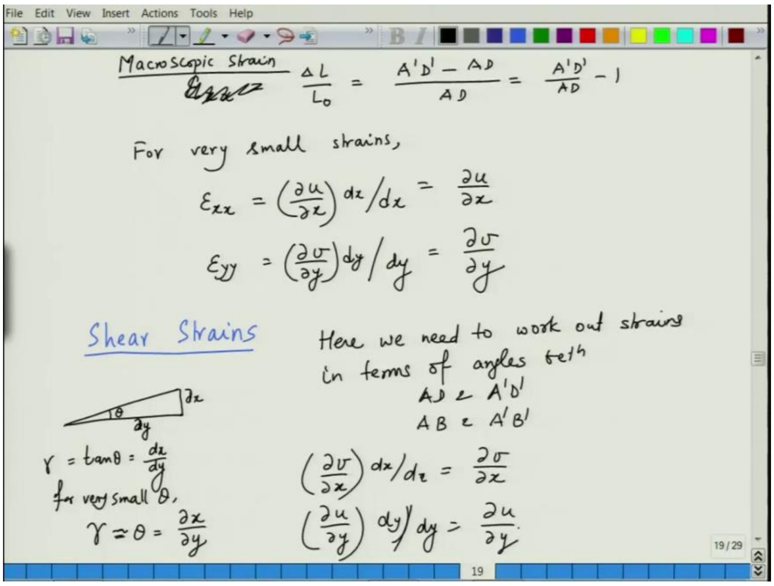

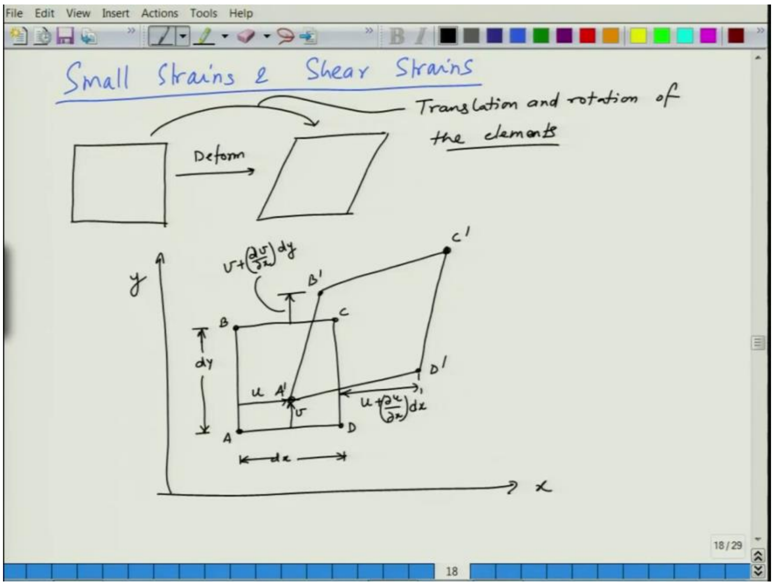

So epsilon xx, let us say we want to calculate, the macroscopic strain will be delta l divided by l naught and we can write this as A prime D prime minus AD divided by AD which is equal to A prime D prime by AD minus 1. For very small values of l strains, sorry, for very small strains, one can write epsilon xx as, so this is let us say, let us just write this as a macroscopic strain.

So epsilon xx for a very small strain will be equal to del u by del x into dx divided by dx. So essentially it is del u by del x. Epsilon yy will be equal to del v by del y into dy divided by dy this will be equal to del v by del, sorry, y. Now let us now see what the shear strains are. Shear strains are defined by, at the small level, so let us say if this is the angle theta, let us say if this is the part dx, if this is the part dy, then tan theta is equal to dx divided by dy and for very small, small theta so this is gamma, shear strains represented by gamma.

So we can say gamma is equal to theta which is equal to dx by dy, let us say del x, del y. So using the same analogy for shear stresses, in terms of, so basically here, we need to work out strains in terms of angles between AD and AD prime, A prime, D prime, sorry, AD and AD prime and AB and let us say AB prime, so this is AD and this is AD prime. This is AB and this is AB prime. So if we do that, sorry, A prime, B prime it is not. Let me just correct it.

So it is AD and A prime D prime and AB and A prime B prime, I am sorry. So basically between this and that, between this and this. So if we now work out these strains, so we can write the first one as del v by del x into dx divided by dx, this will be equal to del v by del x. And the second one will be del u by del y into dy divided by dy this will be equal to del u by del y.

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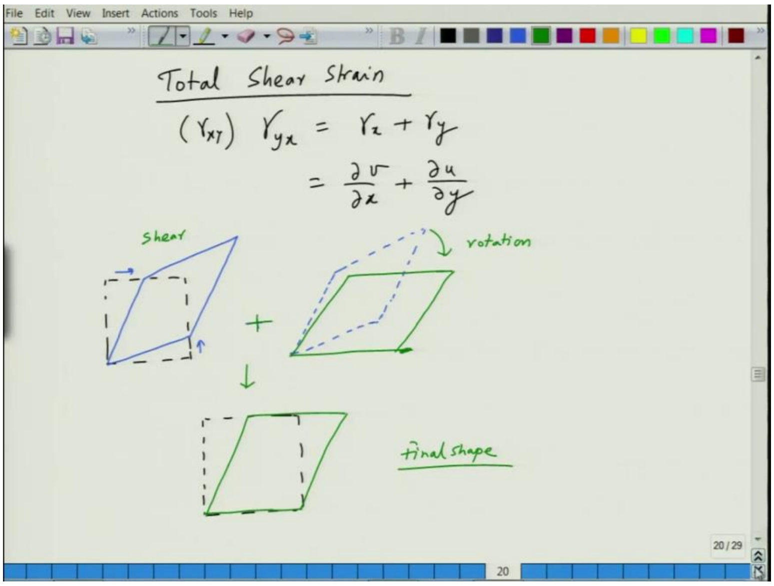

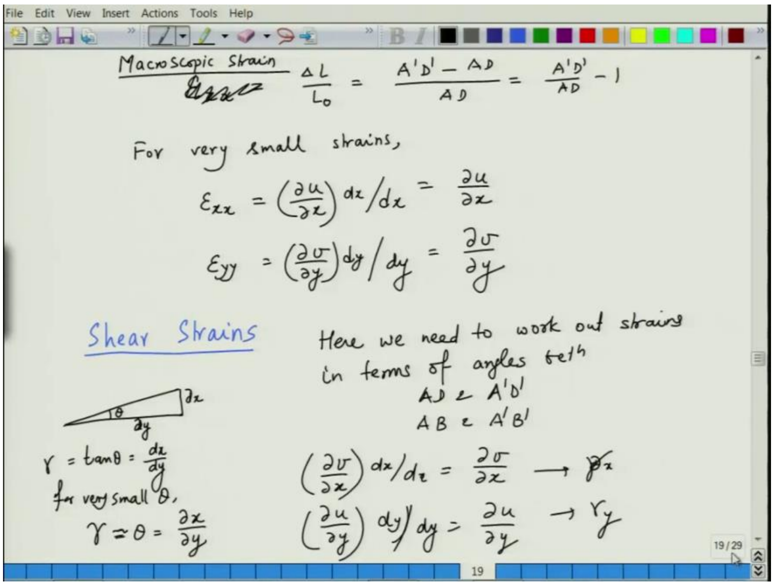

So the total shear strain then in that case, which is gamma of y and x is equal to gamma of x. So this is gamma x, this is gamma y. So this will be equal to gamma x plus gamma y and this will be equal to del v by del x plus del u by del y, and this is also equivalent to let us say gamma xy. So basically what is happened is here, the process of rotation that is happened is, you start from let us say this shape, and then the first thing that you do is that, you have a shear and this allows.

So essentially it is come from here to here and then you create what we call as a, so you have this shape now. And this shape is now converted to which is the final transformation. So if you combine these two essentially, what you get is like this. So you start with this shape and you end up with the green one. So this is what we say is the shear and this will be rotation.

So this is shearing in this direction and this is rotating in this direction. So we are combining the process of shear and rotation to the overall deformation that we see in the final shape. So that is why these two shear strains have to be added, one in the x and other in the y. They give you the net shear strain for a two dimensional body.

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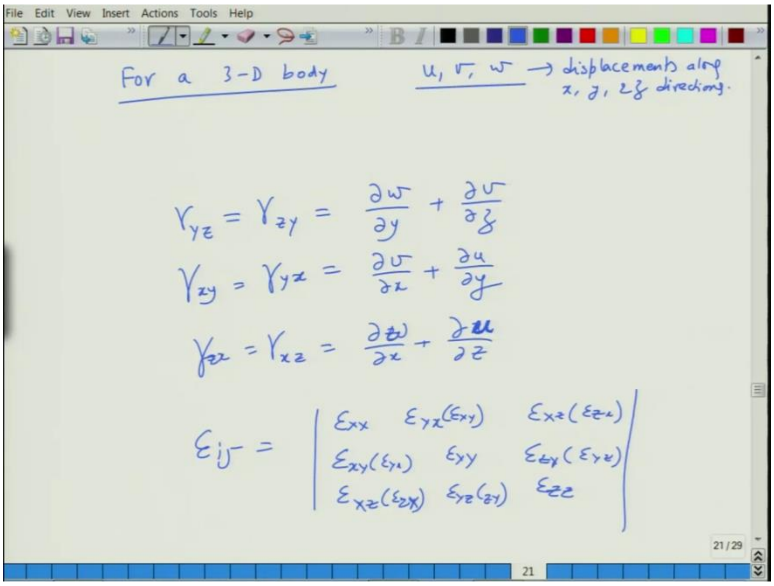

So let us say, for a three dimensional body now, so you have u, you have v, you have w, okay, u for x, v for y and w for z. So displacement is w and z direction, u is the displacement in x direction, and v is the displacement in. So these are displacements in along x, y and z directions. So if you now write them, then epsilon, epsilon yy we know is equal to del v by del y. So if I write gamma yz, so we do not need to actually write this. We just need to worry about gamma yz. Gamma yz is equal to gamma zy.

This is equal to del w divided by del y plus del v divided by del z. Similarly, I can write gamma xy which is equal to gamma yx, this is equal to gamma del v divided by del x plus del u divided by del y. And if you want to write gamma zx, sorry, what have I written, it should be gamma not delta. Gamma zx which is equal to gamma xz, this will be equal to del z by del x plus del x by del z, sorry, del w by del x and del u by del z. So these will be the three shear strains, six shear strains which will be basically for the three dimensional body.

So then you can write a shear stress tensor, shear strain tensor in the same way, epsilon xx, epsilon yx or xy, epsilon xz or epsilon zx and then you can write epsilon xy epsilon yx epsilon yy, here you will have epsilon zz, this will be epsilon zy or epsilon yz, this will be epsilon yz or epsilon zy, this will be epsilon yz or, sorry, zx, xz or zx and this will be zy. So this will be their strain tensor that we can write.

So we will stop here. We will come back to this and especially the transformational strain to different axis in the next class. And so what we have done is basically we have looked at the concept of shear strain, which is does not in solids deformation and we also look at, looked at the differences between the true and engineering strain in this lecture. So thank you very much.