Properties of Materials (Nature and Properties of Materials: III)

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Lecture 07

Tensor Notation of Strain

So welcome again to the new lecture of this course, Properties of Materials. So let us just briefly recap first what we did in the last class.

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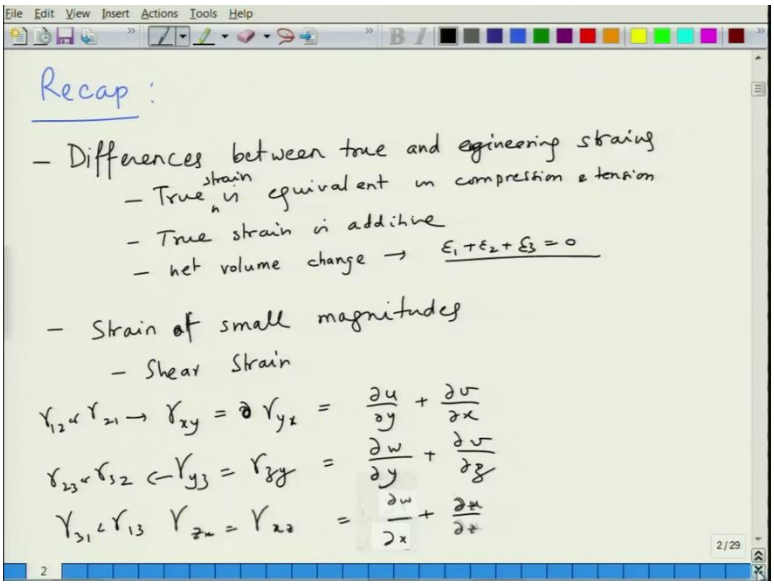

So in the last class we first looked at differences between true and engineering strains. So first thing that we saw that true strain is equivalent in compression and tension, whereas engineering strain, sorry, true strain, whereas engineering is not. And then true strain is additive. So if you deform a material in different passes and you calculate the strain from 1 to 2 and 2 to 3 and 3 to 4 and then you calculate the net strain, thus all those pass strains they add up to net.

And then the net volume change, if there is no volume change, then epsilon 1 plus epsilon 2 plus epsilon 3 will be equal to 0, whereas this condition will not be obeyed by the engineering strain.

Then we looked at the cases of small strain. So we basically looked at the cases of strain of small magnitudes. And there we basically looked at what is shear strain. And shear strain basically is defined as, so we can say that for a 3D body with various displacements, you can define three shear strain gamma xy as which is equal to gamma, sorry, gamma yx, this is equal to del u by del y plus del v by del x.

Similarly, you can write gamma yz, which is equal to gamma zy this is equal to gamma w, del w divided by del y plus del v divided by del z. Similarly, we can write gamma zx, which is equal to gamma xz, which is equal to del w by del x plus del u by del z. You can write these also as gamma 12 or gamma 21. These can be written as gamma 23 or gamma 32. Similarly, here you can write gamma 31 or gamma 13. So these are the relations for shear stresses.

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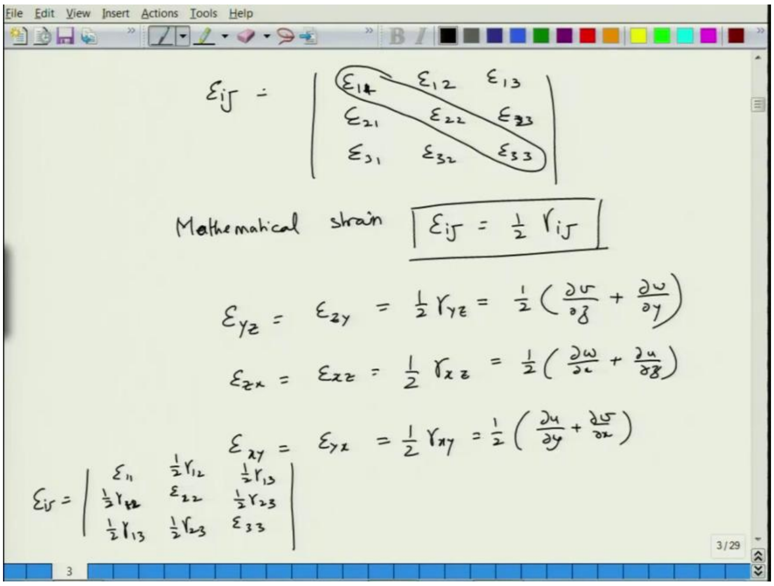

So now if you write the overall tensor, the tensor can be written as epsilon 12, epsilon, sorry, epsilon 11, 12, epsilon 13, epsilon 21, epsilon 22, epsilon 33, epsilon 3, sorry, 23, then 32 and then 33. Now out of these, these three are principal strains and on the off diagonal terms they are shear strains.

So mathematically speaking, however, to the mathematical strain or we can say tensorial strain, epsilon ij turns out to be half of engineering shear strain. So there is a whole derivation for it. We are not going to go into that. But the relation is epsilon ij is equal to half of gamma ij.

So if you just replace this here, so basically the way you define then epsilon yz is equal to epsilon zy is equal to half of gamma yz, this is equal to half of del v by del z plus del w by del y. Similarly, you can write epsilon zx to be equal to epsilon xz this will be half of gamma xz and this will be half of del w by del x plus del u by del z.

Similarly, we can write gamma epsilon xy, which is equal to yx, which is equal to, epsilon yx which is equal to half of gamma xy or you can write gamma yx which is equal to half of del u by del y plus del v by del x.

So if you now replace these terms there, so what you get, basically get is epsilon ij will become equal to epsilon 11 epsilon 22 epsilon 33 these are the diagonal terms. Then we have half of gamma 12, half of gamma 13. Here we have half of gamma 21 or 12 let us say. Then we can write half of gamma 23, half of gamma 13 and half of gamma 23. So this is what the strain tensor is going to look like.

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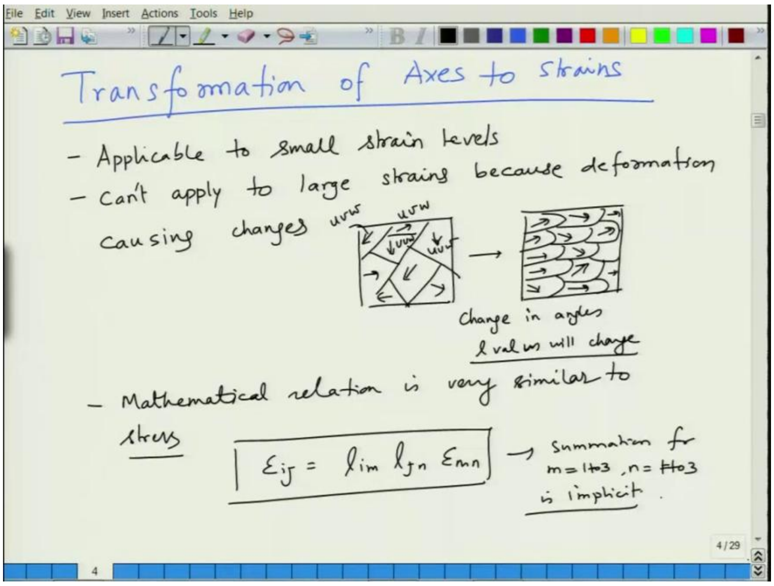

Now what we are going to do is that, we are going to do transformation of, we are going to apply the transformation of axes to strains. We saw the transformation of axes to stress. Now we will look at the transformation axes to strains. So now this is applicable only to smaller strains. We cannot apply to large strains because of deformation causing changes.

So the cosA is the l values they change as the deformation happens on a large scale, because the materials are mostly polycrystalline. So as a result that grains deform, they reorient themselves along certain different directions. So for example, for, in the beginning your grain structure maybe something like that and each grain has a different crystalline orientation and when you deform it, when you extend large deformation, the grains may become like this. So they may become oriented.

So as a result, the stress in a strain tensors will change. Because let us say, if this was, in this case, the uvw was in this direction, this is uvw, this is uvw, this is uvw and so on and so forth like this. It is possible that in the deformed grain structure your uvws will become something like this. To some extent, maybe not to this extent, this is the re-exaggerated picture, but what I am trying to convey is that your stress and strain tensor, your stress and strain tensors will change because the angles will change as a result.

So you will have a change in angles. So as a result, the l values will change and your stresses and strains with respect to reference axes will also be different. So as a result, we cannot apply this to large strains, we can only apply at smaller strains. So this is something that we have to keep in mind.

Now the relation basically is similar. So mathematical relation is very similar to basically you can say stress. So we can say that epsilon ij is equal to lim into ljn into epsilon mn. So the summation over different values of m and n is implicit. So it is built in there. So summation for m is equal to 1 to 3, n is equal to 1 to 3 is implicit.

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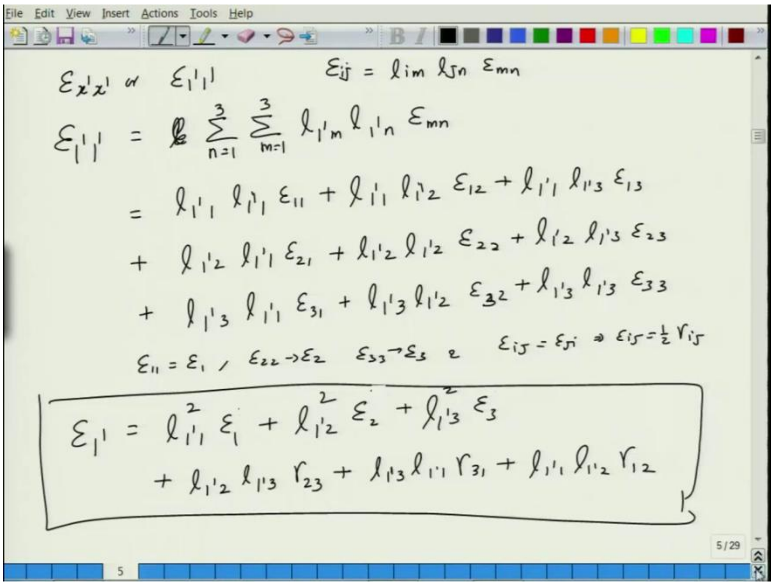

So now let us take first example. Let us take, so you can write epsilon x prime x prime or let us do it differently, let us try to do it in the, or we can write epsilon 1 prime 1 prime. So let us take the case of epsilon 1 prime 1 prime. So if you look at the relation, we said that epsilon ij is equal to lim ljn epsilon mn, okay. So i and j values are fixed, alright. So this becomes basically, so essentially we are going to sum over l of 1 prime m l of 1 prime n to epsilon mn this is what the sum is going to be.

So if we now write it, so this will be l 1 prime 1 l 1 prime 1 epsilon 11, second term so the m remains constant in the row. So let us say it is 1 prime 1, 1 prime 2 and changes from 1 to 2 and this becomes epsilon 12. Similarly, we can write this as l 1 prime 1, l 1 prime 3 epsilon 13.

Next one will be l 1 prime 2, l 1 prime 1 epsilon 21 plus l 1 prime 2, l 1 prime 2 epsilon 22 plus l 1 prime 2, l 1 prime 3 epsilon 23. And this will be epsilon 1 prime 3 epsilon l, l 1, sorry, l 1 prime 3, l 1 prime 1 epsilon 31. This will be l 1 prime 3, l 1 prime 2 epsilon 2, 32. And this will be l 1 prime 3, l 1 prime 3 epsilon 33.

So we can now simplify this. So we can make epsilon 11 is equal to epsilon 1, epsilon 22 as epsilon 2, epsilon 33 as epsilon 3 and we convert these epsilon ij is equal to epsilon ji and epsilon ij is equal to epsilon of gamma of half of gamma ij. So if we make these replacements, what we get here is epsilon 1 prime is equal to l 1 prime 1 square, epsilon 1 square plus l 1 prime 2 square epsilon 2 square plus l 1 prime 3 square epsilon 3 square plus l 1 prime, sorry, l 1 prime 2 into l 1 prime 3 to gamma of 23 plus l 1 prime 3, l 1 prime 1 gamma of 31 plus l 1 prime 1 into l 1 prime 2 gamma 12.

So this will be the expression. So you can see that 2s are missing here because we have replace epsilon gamma. So as a result there is a factor of half. So 2 and 2 they cancel each other in all the terms. So as a result, it becomes equal to 1. So this is the expression that we get for epsilon. Similarly, you can do things for epsilon 22 prime and epsilon 33 prime. All the principal strains can be calculated in the similar manner.

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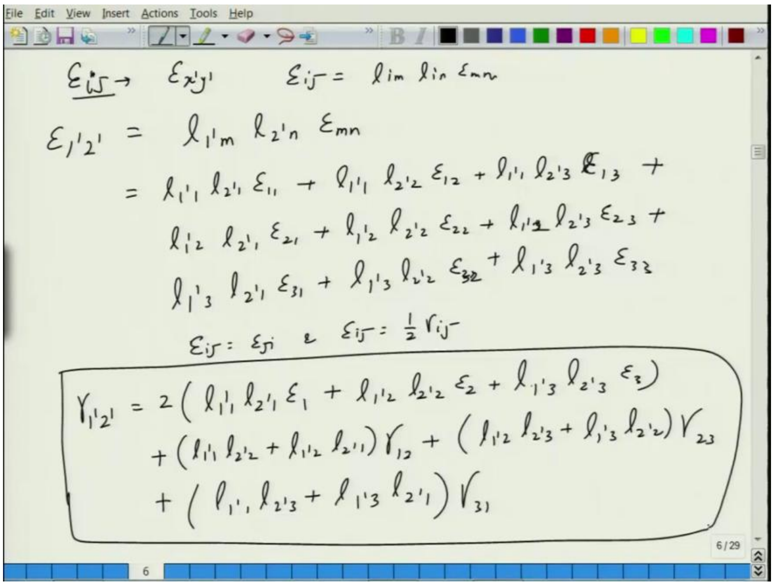

Now let us say, similarly, if I want to do epsilon i, if I want to work out epsilon ij, let us say, so it could be epsilon x prime y prime or let us say we do epsilon 1 prime 2 prime. So if I do epsilon 1 prime 2 prime, again follow the same formula that is epsilon ij will become lim lin epsilon mn. So essentially it becomes l 1 prime m, l 2 prime n epsilon mn. And this is what is going to be summed over.

So when you do this now, it becomes l 1 prime 1, l 2 prime 1 epsilon 1 1 plus l 1 prime 1, l 2 prime 2 epsilon 12 plus l 1 prime 1, l 2 prime 3 epsilon 13 plus l 1 prime 2, l 2 prime 1 epsilon 21 plus l 1 prime 2, l 2 prime 2 epsilon 22 plus l 1 prime 2, l 2 prime 3 epsilon 23 plus l 1 prime 3, l 2 prime 1 epsilon 31 plus l 1 prime 3, l 2 prime 2 epsilon 2, 32 plus l 1 prime 3, l 2 prime 3 epsilon 33. And this will be the full expansion.

So if you now again modify by making epsilon ij is equal to ji and epsilon ij is equal to half of gamma ij, then we can modify this as, so we can write this in terms of gamma 1 prime 2 prime. This will become equal to 2 into l 1 prime 1, l 2 prime 1 epsilon 1 plus l 1 prime 2, l 2 prime 2 epsilon 2 plus l 1 prime 3, l 2 prime 3 as epsilon 3.

And then we will have the terms related to gamma 12 gamma 23 and gamma 31. So this will be l 1 prime 1, l 2 prime 2 plus l 1 prime 2, l 2 prime 1 into gamma 12 plus l 1 prime 2, l 2 prime 3 plus l 1 prime 3 into l 2 prime 2 epsilon, gamma 23 and plus we can make it l 1 prime 1, l 2 prime 3 plus l 1 prime 3, l 2 prime 1 into epsilon 32, sorry, epsilon 13. What have we taken just one second? 1 prime 2, 2 prime 3, 1 prime 3, 2 prime 2 this is epsilon 2, gamma 23 and this will be gamma 31 or 13. So this will be the full expansion for gamma 1 prime 2.

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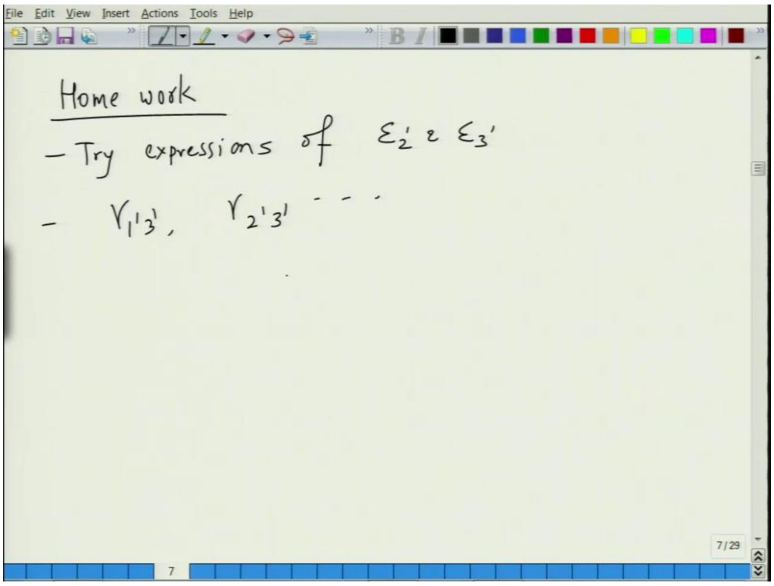

So as a home work try expressions for epsilon 2 and epsilon 3. Similarly, you should try the expressions for let us say 1 prime 3, 2 prime 3 prime, and so on and so forth. So this will help you practice on how to write these transforms. So this is what is about the transformation of strains. So you can do a simple problem on this. So I will not do it in here.

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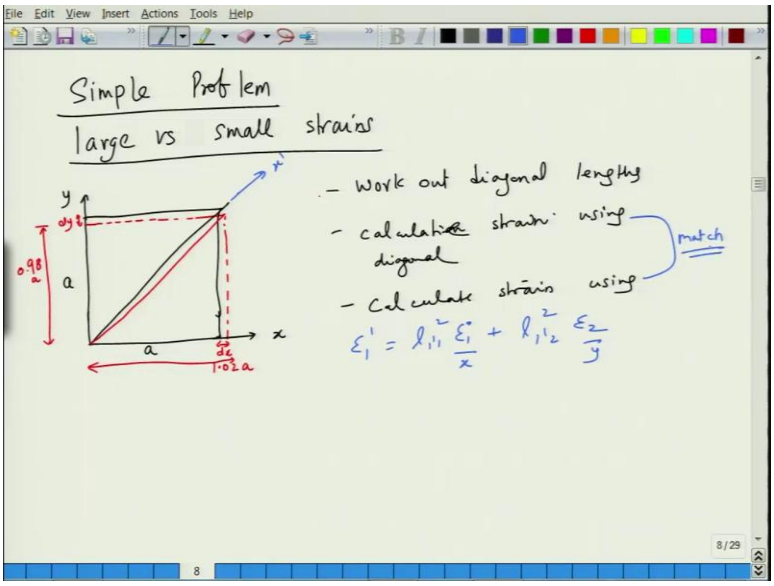

But a simple problem is to verify whether it works at small, so simple problem can deal with large versus small strains. So this could be, let us you take a block of dimension let us say a by a in x and y. So this deforms, let us say, to a very small change. So this could be dx and this could be dy. So let us say this becomes from a to, I do not know, 1.02a and this whole thing, and this thing becomes, sorry, 0.98a.

So you can work out so the thing that you have to do is that work out, so you can just look at the diagonal let us say in this case and diagonal in the next case. So work out diagonal lengths, calculate strain using diagonal, and then calculate strain using, so let us say this is the reference axes. So the axes of, axes will, so this is xy. This will be your x prime.

So assuming that the angle is close to 90 degrees, you can work out the strains as it will be epsilon, so you can say epsilon 1 prime will be equal to l 1 prime 1 epsilon 1, sorry, into l 1 prime 2 square epsilon 2. So this will be the x strain. This will be the y strain. And you can see whether the values match or not. And you will see the values will match. So when you, so there will be a match.

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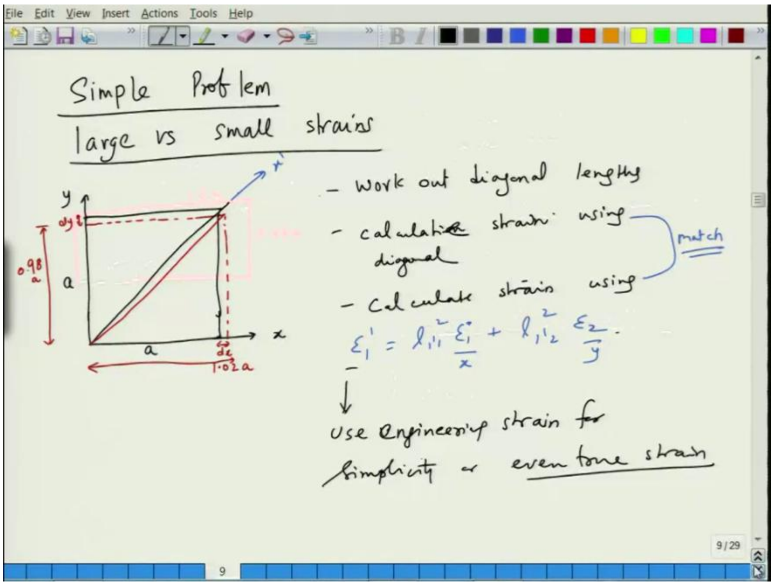

When you do the same thing for a large deformation, so for a large deformation situation is like this. So you have x, y, you have this block. So this was a and a. This block changes to something else. So here we have a change which is let us say this is 1.5a and this is 0.67a, assuming that the volume remains the constant. So this is how you have to choose.

So again, so again look at the diagonal. So calculate, so instead of, what you can do is that, instead of using true strain, you can use engineering strain, use engineering strain for simplicity or if you want to use or even true strain. So, both of them should give you that at least the rough idea.

So calculate the diagonal related strain and again calculate the strain from transformation of axes. So this will become x prime, but here you can see the angles are not, no longer 45 degrees. So as a result, in this case, it is 45 degree, but in this case it is not equal to 45 degrees. So you will have to work out the angles clearly. And you will see the strains will not match. So this will be sort of home work for you.

So what you have to do is that, you take this block, carry out a small amount of deformation where the strain is very small, calculate the strain. So what you have to do is that basically, you look at, you carry out a small deformation with very small strain, calculate the strain using diagonal and then calculate the strain using transformation of axes and you will see both the strains will nearly match.

But if you carry out a large deformation, again you apply the same procedure. You will see that the strains will not match and this will sort of give you an idea of why is it applicable at smaller strain and why is it not applicable.

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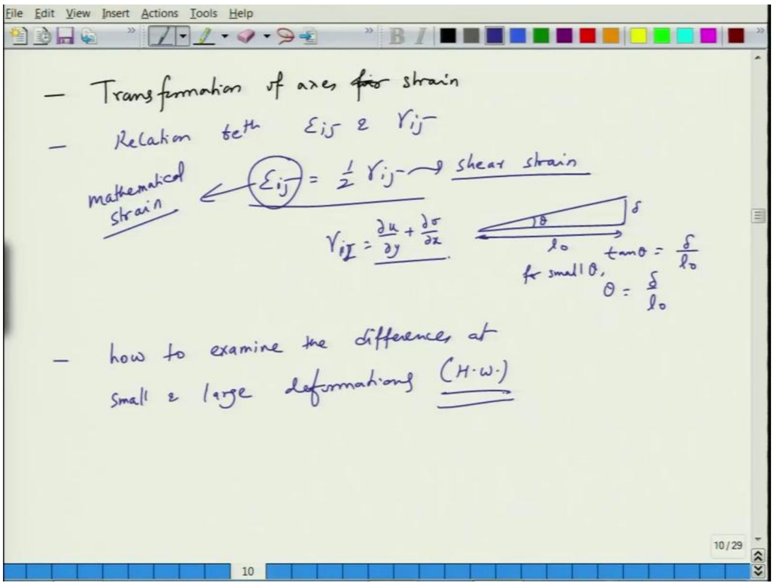

So let us now, we are nearly at the end of the lecture. So let us summarize this lecture. So what we have done is, basically we have looked at the, we have looked at the transformation of axes in, for strain. We also looked at the relation between epsilon ij and gamma ij. So we saw that epsilon ij is equal to half of gamma ij for mathematical reasons.

So this is basically shear strain, which is the engineering shear strain. So essentially you calculate using this tan theta, right. This is theta. So this is delta deformation and this is the original length, let us say, l naught. So tan theta will be equal to delta divided by l naught.

For very small angles, this can be for small theta. I can write this as theta, which is equal to delta divided by l naught. And basically it was the shear strain. So gamma ij was equal to, let us say, for example, 12 was equal to gamma, so del u by del y plus del v by del x. So this was the shear strain, whereas this is the overall trend that you get for. So this combines basically both translations in both the directions.

So this is, for mathematical, so this is your mathematical strain you can say. And it can be proved that this mathematical strain is equal to half of gamma ij. We do not have time to go through the derivation here, but if you look at into any books and mechanics, you can easily find about it.

And then I have given you a problem on how to examine the differences at small and large deformations. So this is something you have to do as a home work. So we will stop here today. In the next lecture we will, we have done the discussion on basically the tensorial stress, the tensor form of stresses, the tensor form of strains, the transformation of axes in both cases.

What we have not done is the most circle thing, but if we get time later on we will do it, but that is something you can read from the books. And so using this information you can get into the mechanics of deformation if you know the tensorial way of working with the stress and the strain. So now what we will start is we will basically move into elastic, theory of elasticity for various materials. Thank you very much.