Properties of Materials (Nature and Properties of Materials: III)
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Lecture 09
Theory of Elasticity

So welcome again to the new lecture on Properties of Materials, and let us just briefly recap what we did in the last lecture.

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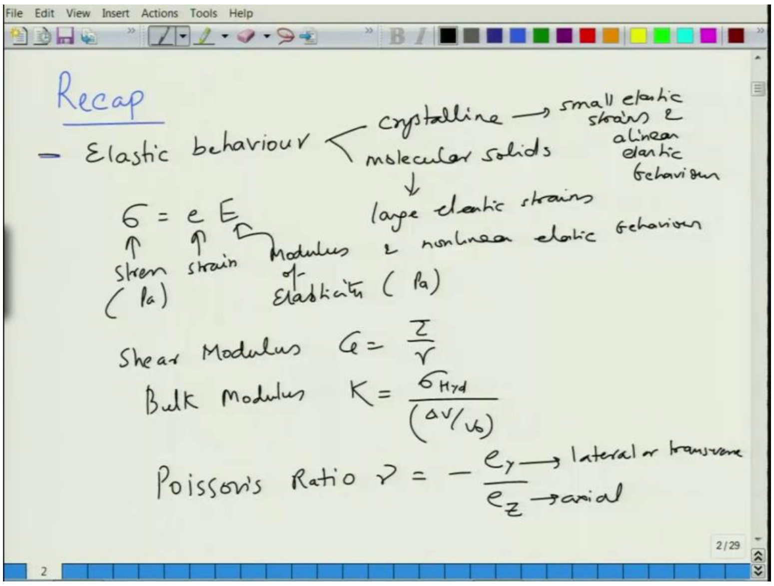

So in the last lecture, we started discussion on elastic behavior of materials. So here we basically looked at what is the elastic behavior of crystalline solids, just a qualitative plot, and then of molecular solids, like polymers. So generally these exhibit small elastic strains and linear elastic behavior. Whereas, molecular solids show a considerably large elastic strains and can show non-linear elastic behavior, especially in materials like rubber.

And here, the elastic behavior which is in linear region, it is represented by this relation sigma is equal to eE, so this is the stress. Stress is proportional to strain and the proportionality constant is modulus. So this is called as modulus of elasticity or Young's modulus and it has a same unit as stress. So if stress is in pascals, this is also in pascals. So let us not worry about mega or giga. They just, if it is in pascals, this is also in pascals.

There were three, few other quantities such as Young's, so we, this is Young's modulus, then we have, what we call as shear modulus and a log of elastic tensile strain or compressive strain, the shear which is G is equal to tau divided by gamma.

And then we have bulk modulus, which is basically related to hydrostatic stress, which is defined as K and this is equal to sigma hydrostatic divided by delta v divided by v naught. So hydrostatic stress divided by fractional volume change.

And then one, another quantity that we define was poisson's ratio, nu, which is ratio of transverse strain to axial strain or lateral strain to axial strain. So axial is at the bottom and lateral or transverse is in the numerator.

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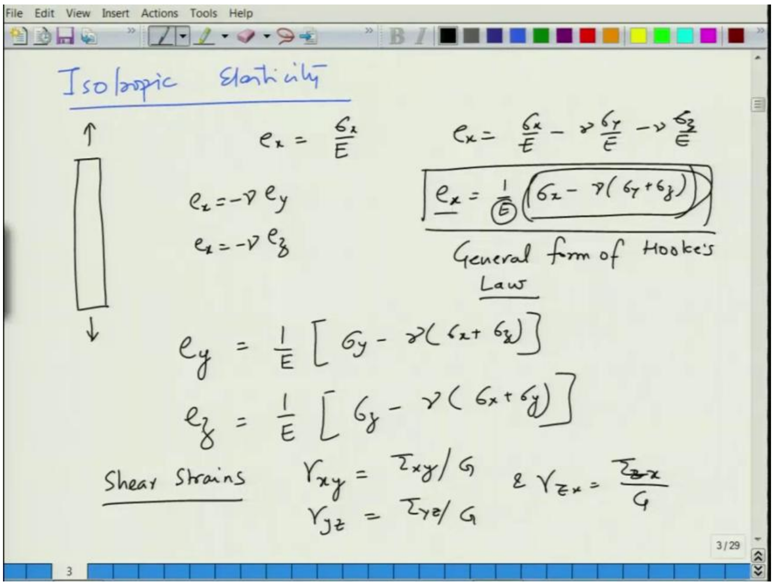

And then we were looking at the isotropic case of elasticity. So basically, so here, let us say you have a bar which is elongated by tensile stress and this tensile stress gives rise to tensile strain. So, but this, it will also lead to contractions. So as a result, you will have tensile strain as well as you will have axial tensile strain as well as lateral contract, lateral strain which is contracting in nature. So as a result, we will have epsilon x which is sigma x divided by E. Epsilon, then we will have epsilon y and epsilon z as well. So epsilon x, because of epsilon y will be minus of nu ey and epsilon x because of epsilon z will be minus of nu ez.

Now because of these strains, if you combine all the three components, then we have epsilon ex as sigma x divided by E minus of nu sigma y divided by E minus of nu sigma z divided by E. And this gives rise to relation ex is equal to 1 over E into sigma x minus nu to sigma y plus sigma z. So, this is what is basically you can say a general form of Hooke's Law. This is where we were in the last class. So this is called as general form of Hooke's Law.

So this is strain, this is modulus and this is the stress, overall stress that the material faces and giving rise to a net deformation. Corresponding shear strains are, one can also get corresponding shear strains, let us do them a little later. So basically we have an expression for ex. If we want an expression for ey which would be similar, ey will be sigma y minus nu into sigma x plus sigma z. And ez will be equal to 1 over E sigma z minus nu into sigma x plus sigma y.

So these are corresponding equations for strains along x, y and z direction in the form of stresses, overall stress divided by modulus. Corresponding shear strains can be written as, one can write shear strains, let us say, gamma xy, this is tau xy divided by G. One can write gamma yz, this is tau yz divided by G. And then we can write gamma zx which is tau zx divided by G. So these are 3 strains, shear strains that we have. So the first 3 are tensile strains and the other 3 are shear strains.

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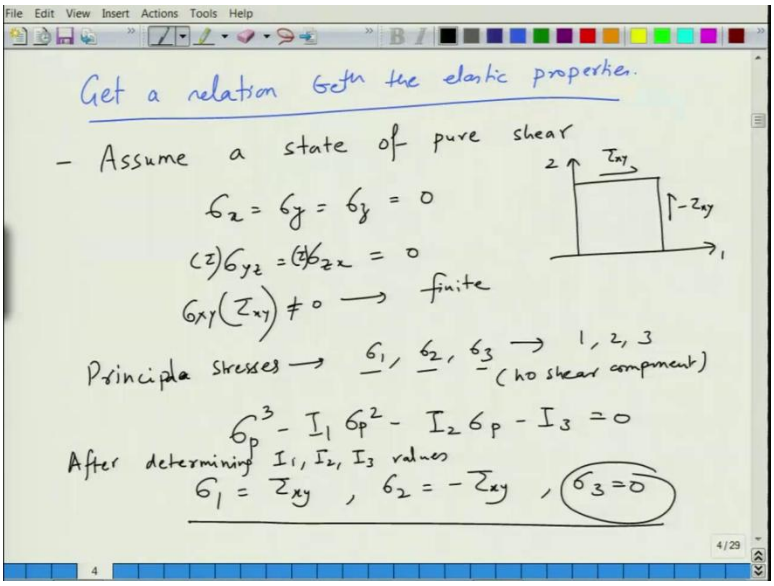

Now let us say, now what we want to do is that, we want to get a relation between the elastic properties. So, for example, how do you relate, let us say, the shear modulus and the bulk modulus. So for this, what we do is that we first consider the state of pure shear. So assume, state of pure shear by this what we mean is that sigma x is equal to sigma y is equal to sigma z is equal to 0. Sigma yz is equal to sigma zx or you can say tau, yz of tau zx is equal to 0 and only tau xy is not equal to zero. So this is the only finite component of stress tensor.

So if you now apply this to the principal stresses, so we defined principal stresses earlier, which are sigma 1, sigma 2, sigma 3 corresponding to certain axes 1, 2, 3. I forgot to mention earlier that this principal stress concept basically relies on the choice of an axes 1, 2, 3 in such a manner so that you are, you only have sigma 1, sigma 2, sigma 3 without any shear component. So there is no shear component. I think, I forgot to mention this particular part when we talked about principal stress.

So basically sigma 1, sigma 2, sigma 3 are again normal stresses chosen on a axes system 1, 2, 3 in such a manner so that there is no shear component and only the normal component remains. So these are determined from this formula sigma p cube minus I1, sigma p square minus I2, sigma p minus I3 is equal to 0 as we have seen earlier. And these Is are nothing but stress invariants.

So you can calculate stress invariants by looking at the formulas for them in the previous lecture and if you put in these values of sigma x, sigma y, sigma z, sigma yz, sigma zx and tau xy or sigma xy in them, then we will find that sigma 1 will be equal to tau xy, sigma 2 will be equal to minus of tau xy and sigma 3 will be equal to 0.

So basically after determining I1, I2 and I3 values, by plugging in the stress values as we mentioned earlier, we will find this is the stress state. So among three principal stresses 1 is 0 and other 2 are equal and opposite. So this is the basically, so the when the material is in pure shear, then this is what the stress state is going to be. The principal stresses are going to be tau xy and tau.

If you want to mention this, so this is let us say 1, this is let us say 2 and this is what the stresses are going to be like. So in one case it is going to be tau xy and here is going to be minus tau xy. So these are the stresses, which are going to act on the material.

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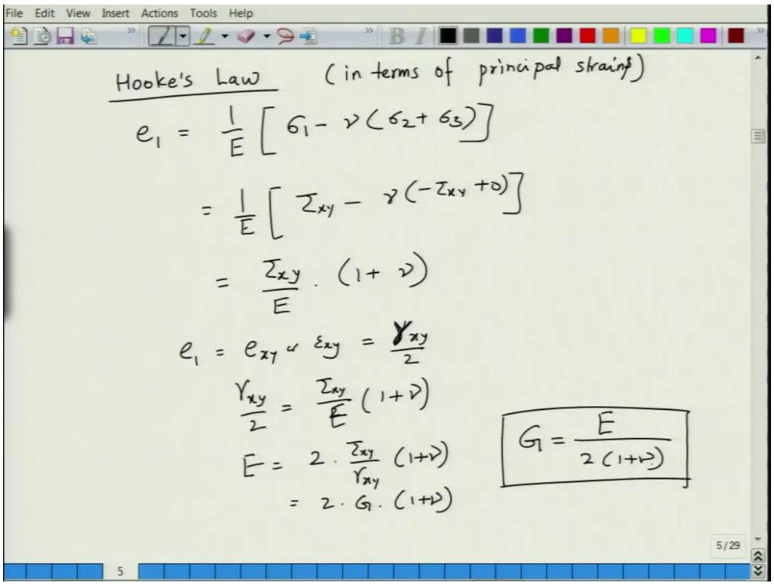

Now when you apply these stresses to the Hooke's Law, so the Hooke's Law that we wrote, so now Hooke's Law, if we rewrite that, this is equal to, so we can write in terms of principal strains. So, e1 is equal to 1 over E into sigma 1 minus nu into sigma 2 plus sigma 3. So basically in terms of corresponding principal strains.

So if you now put in the value here 1 over E, this is equal to tau xy minus of nu minus tau xy plus 0. So this will be tau xy divided by E into 1 plus nu. This is what your e1 value is going to be. And e1 is nothing but, e1 is nothing but exy or you can say epsilon xy, which is related to gamma xy divided by 2. As we saw earlier that mathematical strain is equal to gamma xy divided by 2.

So if that is the case, then if we write the expression for e1, then this becomes gamma xy divided by 2 this is equal to tau xy divided by 2, divided by E into 1 plus nu. So we can see here E is equal to 2 into tau xy divided by gamma xy into 1 plus nu. And this is equal to 2 into G into 1 plus nu. So what we get a relation here the shear modulus is nothing but Young's modulus divided by 2 into 1 plus nu.

So this is a relation that we get between the two elastic properties that is G, E and, three elastic properties, G, E and nu. So if you know E and nu, you would note G. If you know any two of them, you would know the third property.

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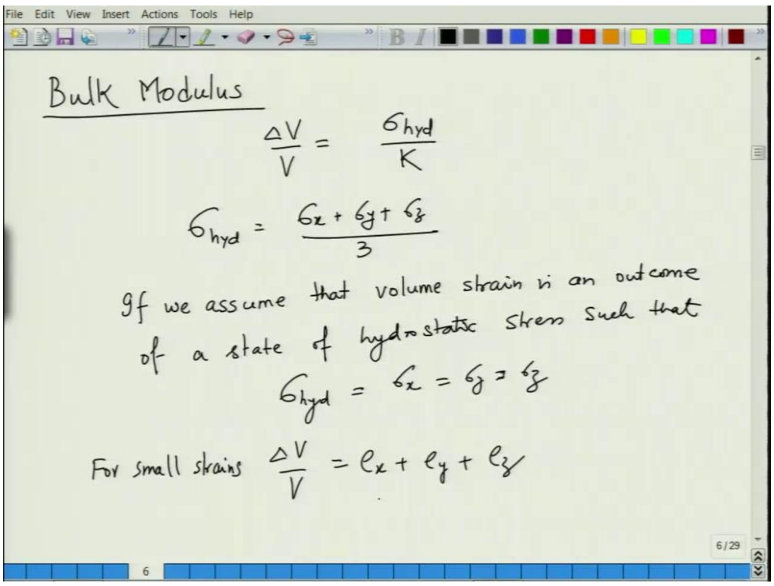

Similarly, you can derive a relation, relationship between bulk modulus. So for bulk modulus we write this as, fractional volume change as, sigma hydrostatic divided by K. This was the value of bulk modulus that we saw earlier.

Now sigma hydrostatic stress is generally given as, in a general form, is sigma x plus sigma y plus sigma z divided by 3. However, if we assume that, that volume strain is an outcome of state, of a particular state of hydrostatic stress such that sigma hydrostatic is equal to sigma x is equal to sigma y. And this will, this also satisfy the above condition, but this is a particular case of hydrostatic stress state.

So this is a particular case. In this case if sigma hydrostatic is assumed to be equal to sigma x plus sigma y plus sigma z, then we can write delta v by V for small strains, this can be approximated as ex plus ey plus ez for very small strains, because for very small strains the true strains will, we are using e and epsilon repeatedly, but remember for very small strains, when we say earlier, here, when we say e1 is equal to epsilon, ex or epsilon xy, what it means is that very small strains.

So essentially we are looking at very small strains where e is equal to epsilon. So there is a correspondence between the two. So similarly, this is equal to ex plus ey plus ez for very small strains.

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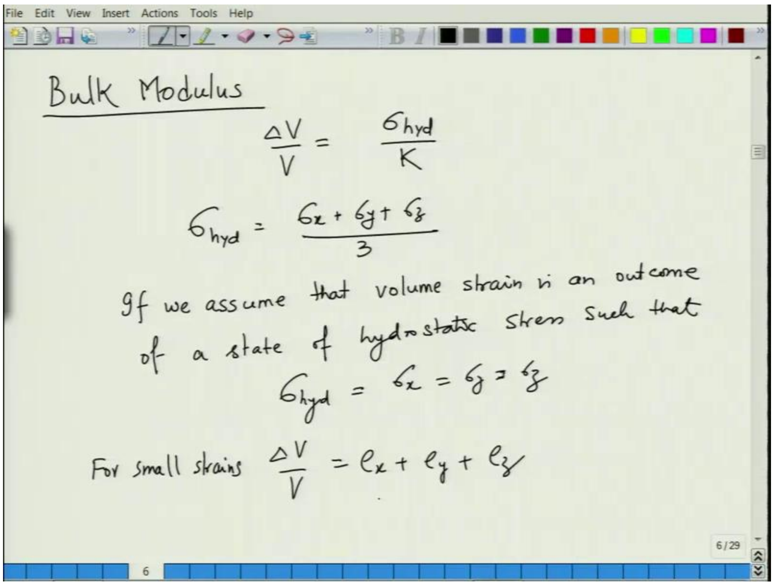

So here now we can determine what is ex, ex can be determined as 1 over E plus sigma x minus nu sigma y plus sigma z. So now let us replace all these values here. So this will be sigma hydrostatic. So basically it will be sigma hydrostatic minus nu into sigma plus sigma hydrostatic. So this will be sigma hydrostatic divided by E into 1 minus 2 nu.

So basically delta and, we are saying, now this is equal to, essentially we are saying that, so delta v by V is equal to 3 times this which is 3 into sigma, so basically we are saying that delta v by V is equal to 3 times ex for a special case, for a special state when sigma x is equal to sigma y is equal to sigma z, which means ex is equal to ey is equal to ez. That means delta v by V is equal to 3ex, which means it is equal to 3 into sigma hydrostatic into 1 minus 2 nu divided by E and this is equal to 1 by K into sigma hydrostatic.

So, these two cancel each other. What we have a relation between K and E which is E divided by 3 into 1 minus, K is equal to E divided by 3 into 1 minus nu. If you combine this relation, so this is a relation which relates the bulk modulus, the elastic modulus and the poisson's ratio. The previous case, you related poisson's ratio, Young's modulus and the shear modulus.

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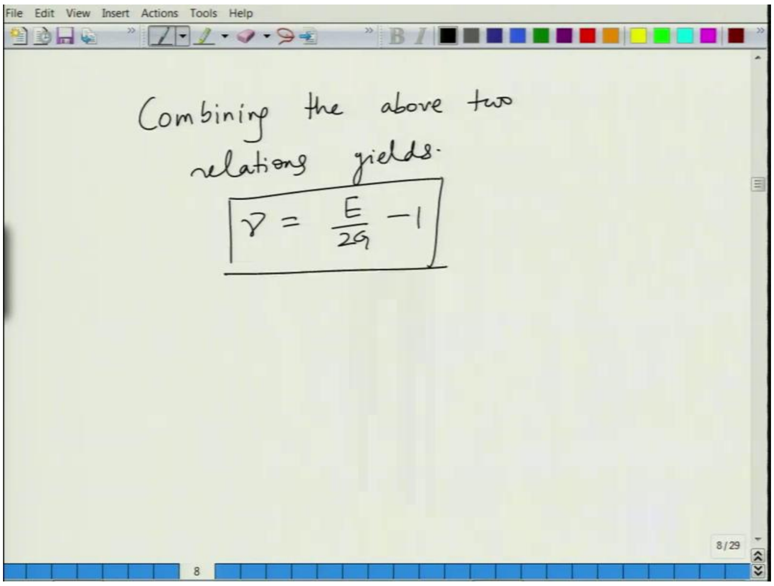

So naturally you can see here. If you combine these two equations, you can obtain, so combining relations yields, nu is equal to E divided by 2G minus 1. So this is the third relation that you get between these quantities. So this is what basically we have done.

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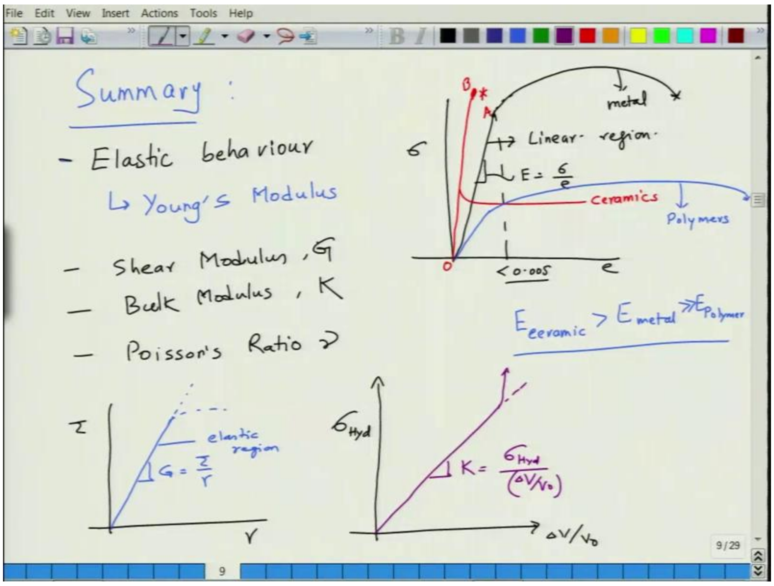

So in this, so this is a sort of a small, short primer to basically elastic properties. So essentially let me now come to sort of a summary of this particular part. Summary at this point, it is not the overall summary, but summary at this point basically. What we did was we have looked at what elastic behavior is.

Essentially when you plot stress as a function of strain, then there is a liner region before non-linearity starts and this linear region is essentially, so this is we can say a linear region and this is obtain for most of the solids, especially crystalline solids, metals, ceramics, et cetera. And this linear region will give you E which is sigma divided by e.

So there is a difference, fundamental difference between the metals and ceramics that you will obtain. So for metals, you will have a behaviour generally like this before material fails. Whereas, so this will be for metals. So they will show a pronounced region with the strains less than 0.005, very small strains. Whereas, for ceramics, you will obtain basically something like this and they will fail at the, so this is let us say A, this is B, this is O, so this is for ceramics.

So naturally you can see the slope is higher and as a result their modulus is higher and we will see microscopic regions a little bit later. And for polymers, generally, you will see a behavior like this. Very long, and this is where somewhere they will fail. So this is polymers.

And we see generally that E of ceramics, in general, is higher than E of metal and which is much higher than E of polymer. So this slope is very, very low in case of polymeric materials. So this is what we will see later on in the, and then what we did, so we, from this we learnt about quantity called as Young's modulus, which is valid basically for tension or compression kind of thing.

Now there are other values we looked at. We looked at shear modulus and we looked at bulk modulus. Shear modulus is G, bulk modulus is K. And then we looked at what we called as poisson's ratio nu. All these properties as we saw they are interrelated and so you can determine your shear, this is how you can determine your Young's modulus. But if you wanted to determine shear modulus as a function of shear strain, then, of course, you have a similar kind of plot for, so this will be the G which is equal to tau divided by gamma within.

And if you wanted to plot the same thing for bulk modulus, you can say this is sigma hydrostatic, this is delta v divided by v naught which is the fractional volume change. Then we have and this can continue further, the slope of this K is equal to sigma hydrostatic divided by delta v divided by v naught.

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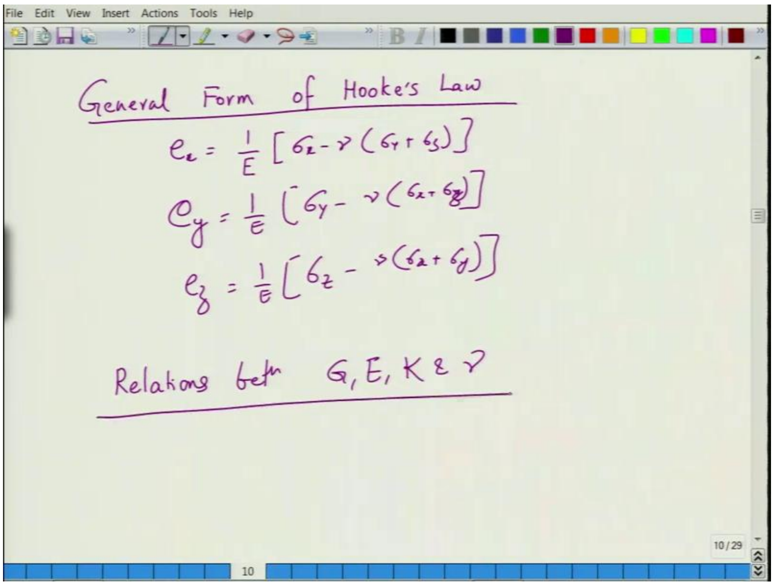

And then we looked at the general form of Hooke's Law, which says that ex is equal to 1 over E sigma x minus nu into sigma y plus sigma z. Similarly, you can write ey, this is equal to sigma y minus nu sigma x plus sigma z. Then we can write ez, which is 1 over E sigma z minus nu into sigma x plus sigma y. So this is what we did for learning about the Hooke's Law. And then we worked out the relations between G, E, K and nu.

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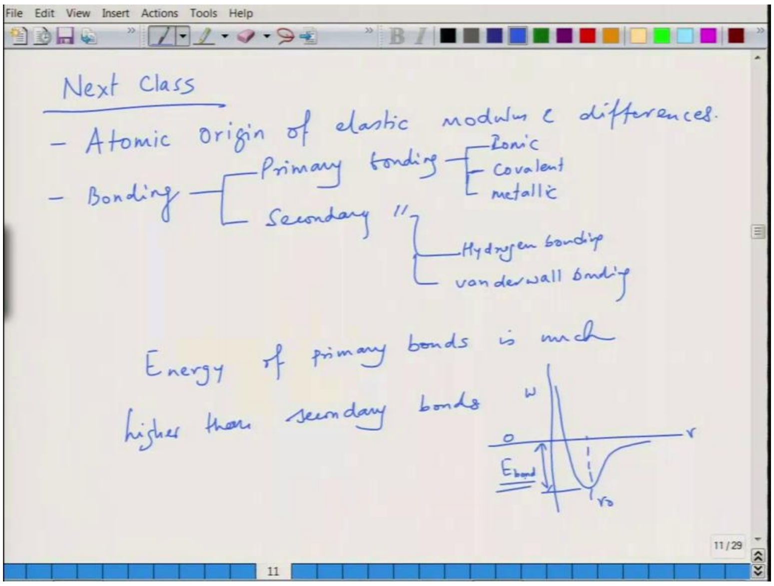

So in the next lecture what we will, now, what we will do is that, we will look at the atomic origin of elastic modulus and differences which is basically related to bonding. So we know that materials are basically you have primary bonding and you have secondary bonding. In primary bonding, we have ionic bond, we have covalent bond and we have metallic bond. And in secondary bonding, we have hydrogen bonding, van der walls bonding and so there are some other secondary bondings.

In general, the energy of, is much higher than secondary bonds. So we know when you plot the potential energy, the potential energy goes something like this. So this is w, this is r. This is the separation between the atoms. And this is the equilibrium separation, let us say r naught and this y axis distance from minima, the distance of minima from the 0 is essentially you can say E bond.

So in ceramic materials, generally this E bond is very high and the energy, the potential energy curve is much more shallow as compared to than in soft metals and polymers. So we will see that the materials with higher bond energy and narrower energy, the potential energy wells, they have higher modulus as compared to the materials with smaller bond energy and broader potential energy wells.

So this is where we will stop today. We will continue this atomic origin discussion in the next lecture. Thank you.