

Advanced Ceramics for Strategic Applications
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Lecture - 42
Mechanical Properties of Ceramic Materials

Welcome. Our today's topic is little bit different from what we have discussed earlier. In our earlier discussions, we have been primarily concerned with the electronic structure, the electronic mobility of the charge carrier mobility, and the electronic transitions and so on. Today we will not be discussing about the electronic structure, electronic transition or electronic energies, but in slightly macroscopic scale or more in atomistic scale, the property what we are going to discuss today is mechanical property. And there it is the atomic bonding, the energy, the surface energy terms, and macroscopic properties are more important. So, our topic of discussion today is mechanical properties of ceramic materials.

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Introduction

- Ceramic materials are characterized by their high hardness, high compressive strength, high Young's modulus and extreme brittleness (low fracture toughness).
- Understanding the mechanism of fracture and enhancement of fracture toughness of the ceramic materials, in general, are the two most challenging tasks before the ceramic scientists and engineers.

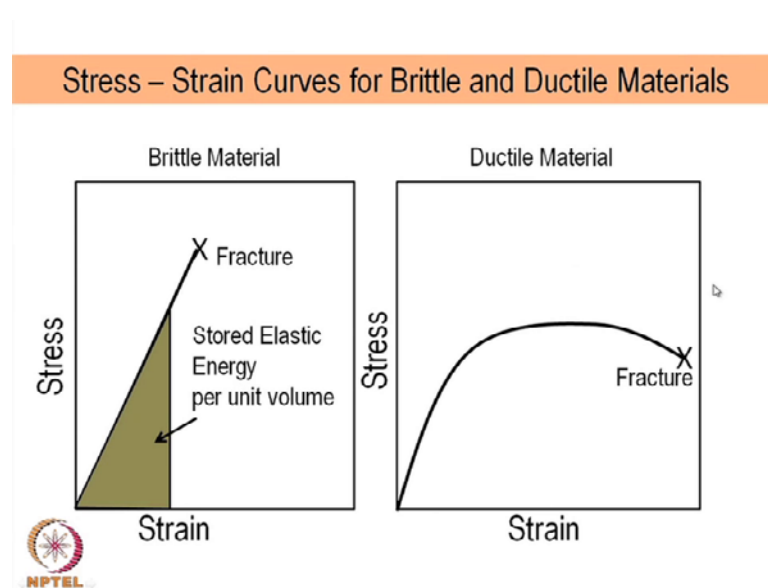


Well to start with, as all of us know that ceramic materials are characterized by their high hardness, high compressive strength, high Young's modulus, and of course the most characteristics of ceramic materials so far as the mechanical property is concerned is extreme brittleness, or in other words we also talk about low fracture toughness or even low toughness sometimes. Now understanding the mechanism of fracture and

enhancement of fracture toughness of the ceramic materials in general are the two most challenging task before the ceramic scientists and engineers, because the material is brittle, it has a high-strength.

But many a times we are not able to exploit that high strength because of the brittleness or low fracture toughness; that means the materials are more prone, highly prone to fracture and once some fracture starts there is no way to stop it. So, it is almost an instantaneous process and it does not give any indication where, when and on what conditions the fracture will occur. So, that is one of the major uncertainties so far as the mechanical properties of ceramic materials are concerned, and in our discussion we will primarily focus our discussion will be basically based on some of the aspects which we will be considering in this discussion.

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Now the basic concept as we have said the brittle materials, the ceramic materials are brittle, and therefore there is a significant difference between the stressed N curves of a brittle material and a ductile material; I think it is very fundamental to mechanical properties of ceramic material, and this is how we compare. This is a brittle solid like ceramics, and this is a ductile solid representative of which is metals and alloys. They have a stressed N curve which is of this nature where there is this considerable amount of strain or deformation before the fracture takes place. Fracture takes place here, but you have always two distinct regions, one is what we call the elastic region; that means when

you release the stress the strain comes back; that means material comes back to the original shape and size whereas there is a plastic region, in this it is a permanent deformation; it is a permanent deformation at higher stress level.

So, that does not come back when the stress is released or stress is withdrawn, whereas if you compare that curve with that of brittle solid or brittle material you will find it is mostly the elastic strength. So, it has a straight line behavior like here, and it goes up to a higher level of stress, and then suddenly fractured. There is no this region is missing there; that means there is no plastic deformation. So, before it fractures there are no plastic deformation and therefore, there is no gradual fracture process. The fracture is actually be almost instantaneous once it starts it is instantaneous; that is one of the major difficulty with brittle material and that is the characteristics of also for the brittle material.

So, there is no indication when the material is going to fracture until and unless you have some prior knowledge what will be the strength or fracture strength. This area under the curve whether this one or this one is always the stored elastic energy within the elastic limits and total area here is the total energy required to fracture a material and that is sometimes called the toughness. So the energy, the area under the curve is a major of the toughness; here also it is more or less same, but it is a better way to represent it is a store elastic energy per unit volume. So, that is the distinctive features between the ductile material and the brittle material so far as the fracture is concerned.

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Theory of Brittle Fracture (I)

- Calculation of Theoretical Cohesive Strength:

- From the first principle fracture of a solid means rupture of inter-atomic bond and consequent creation of new surfaces and enhancement of surface energy. Based on these considerations, theoretical cohesive strength (σ_{th}) may be expressed as

$$\sigma_{th} = \left(\frac{E\gamma}{a_0} \right)^{1/2}$$

- where E is the Young's modulus, γ is the surface energy and a_0 inter-atomic spacing.



Now let us try to understand or try to calculate from the brittle solids or brittle fracture how much is the theoretical stress required to initiate or to fracture a material, calculation of theoretical cohesive strength. It is basically the fracture is basically you are separating the two atoms or two layer of atoms, and the bond energy comes into picture, say, from the very first principal fracture of a solid means a rupture of inter atomic bonds because without the separation of the cohesive force or the inter atomic bond you cannot separate two parts of the solid. So, basically it is inter atomic bond or the rupture of inter atomic bond which leads to the fracture and consequent creation of a new surface.

So, whenever there is a fracture not only the bond is broken, but also you create a new surface because where there is no surface solid to solid surface energy was missing, but now you have two different surfaces; that means solid to vapor surfaces. So, two additional vapor surfaces solid to vapor surfaces are created and that those are associated with some surface energy terms. So, based on these considerations the theoretical cohesive strength what we can say the theoretical strength of a material that is the energy stress required to separate the two layers of atoms. This can be expressed by a formula like this; this is the Young's modulus that is the slope of the stressed N curved, gamma is the surface energy, and a_0 is the inter-atomic spacing.

So, E is the Young's modulus that is primarily the slope of this stressed N curved; this is the E the Young's modulus, and then you have surface energy because you are creating a

additional surface and a a_0 is a inter-atomic spacing, because this fracture is taking place in the atomic level. So, the two atoms having an inter-atomic spacing of a a_0 are now getting separated. So, this is an expression of course, there is detail calculation of that, but I am not going to that part of it. So, this is the theoretical calculation or theoretical strength of any solid for the matter is not only for brittle material but for any material.

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Theory of Brittle Fracture (II)

Assuming $E = 10\text{GPa}$, $\gamma = 1\text{ J/m}^2$ and

$$a_0 = 3 \times 10^{-10}\text{ m} \quad \sigma_{th} \approx 10 - 15\text{ Gpa.}$$

This value is an order of magnitude higher than that experimentally obtained.

Stress concentration at the crack tip provides an answer.



Assuming well an order of magnitude calculation can be done, what is expected theoretical strength of that of a material assuming that E is of the order of 10 GPa Giga Pascal, and surface energy this is γ ; surface energy is approximately one joule per meter square, and a_0 is the order of inter-atomic spacing with some angstroms about three angstroms taken here. So, it is about 3 into 10 to the power minus 10 meter and if you use these numerical values from the earlier equation you can get σ_{th} theoretical of the order of 10 to 15 Giga Pascal for any material. Now that is the theoretical energy value expected. We have seen earlier in one of earlier lectures that this theoretical value available for any material is always less than the experimental value. In our earlier situation where we are considering dislocations we said that the experimental value is lower because there is a dislocation.

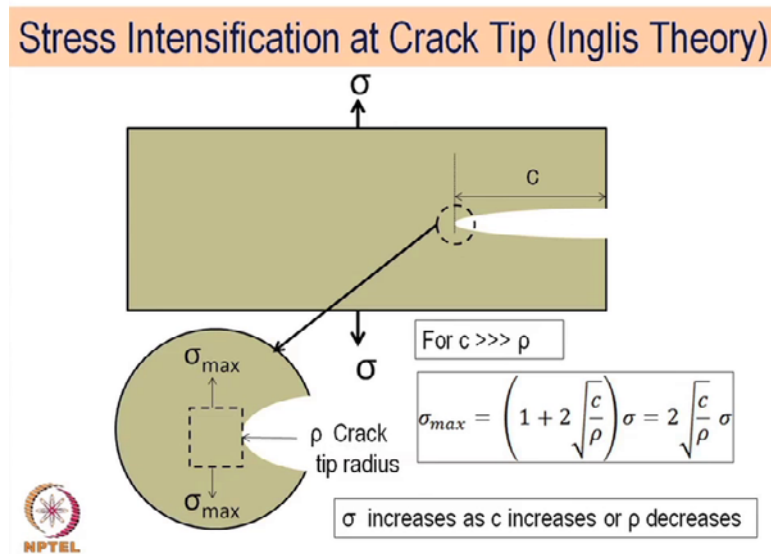
So, presence of a dislocation or a kind of line defect a defect in the arrangement of atoms that leads to a lower stress or requirement of a lower stress for movement of the atoms or that is of course, it was not a fracture; it was primarily the deformation the plastic

deformation. So, presence of a dislocation provides you that kind of explanation why the stress required for plastic deformation is much less than the theoretical value. Here you are considering a more or less similar situation, but it is a fracture stress. It is not a deformation stress required for theoretical stress required for deformation, but we are talking about a theoretical stress required for fracture or separation of the two portions of the solid.

So, here also we find that this theoretical value is order of magnitude higher than what we normally experimentally measure, and there must be an explanation for that. So, we are trying to look at what is the reason why the experimental value is much less than that of the theoretical value, and we will discuss much more about it, but just a statement here stress concentration at the crack tip provides an answer. What is crack tip and what we mean by a really crack; that means we are initiating a crack that was a stress required for initiating a crack. There was no crack initially, and then by stretching applying a stress we have the cohesive energy or the bond energy is broken. So, a crack is initiated whereas we will see later that actually we were not initiating a crack.

We are basically trying to expand or propagate a crack, and that is the reason crack is already there. There crack has been generated in the samples because of many several other reasons. So, before we do any experiment with the measurement of the strength, measurement of the stress required for fracture, we assume that there are already cracks available, and those cracks are actually getting propagated, and that is how the fracture is taking place. So, it is the concentration at the crack tip which is of concern, and that is our point of discussion in the next few slides.

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Well, we were trying to look at what we call the stress intensification at crack tip, or sometimes called the Inglis theory; we will see that was proposed by the particular scientists and therefore, we look at it what exactly it means. We have a solid, and there is crack already has been initiated; that means this part has been separated. Initially this was same, now because of some reason it may be stressed, it maybe some other stress generated by some other means not applying external stress, but internal stress can be also generated by different ways, we will look in to that. So, a stress a crack is already there, and the crack is normally of this nature an ellipsoidal nature, and this part is having a much sharper end than the rest of it. So, there will be two sharp ends on the two sides.

So, we are looking at one of these sharp ends. These have been magnified here; this part has been magnified this particular circle. So, we are applying a stress a kind of tensile stress in this case. We are trying to separate this or expand this crack, and look at the magnified scale or magnified image here. We will see in this part of the solid there is a different amount of stress is being experienced or generated than this part, why that it so? We can find out the expression is, well before that we define this geometry by this. The rho is the crack tip radius, this radius or the radius of curvature here is rho, and the total length is half length in fact, the total half length of the crack is c. So, the total length is 2 c and if c this area or the curve c is much, much greater than rho; that means this length is much larger than the radius of curvature here. So, there is a sharp end.

So, ρ is very, very small compared to c , and then we will find the stress experienced at this region in terms of the externally applied stress σ is an expression like this. So, σ_{\max} equal to $1 + 2\sqrt{c/\rho}$ into σ . So, this σ is this σ or is externally applied stress, and this σ_{\max} is basically the σ experienced by this portion of the solid just ahead of what we call the crack tip. So, the crack tip here is a very sharp tip, and because of this sharpness and that sharpness is defined by the curvature radius of curvature ρ and the length half length is c , then you have an expression like that. So, there is a multiplying factor, there is a multiplying factor $2\sqrt{c/\rho}$.

So, the σ_{\max} is not exactly equal to σ applied from the external sources. So, there is a multiplying factor, and that multiplying factor is dependent on the length of the crack as well as the radius of curvature at the crack tip, and from this you can also simplify it is like this. So, where it is this term is more important than the other term. So, it can be two into particularly for this condition where c is very, very large than ρ . So, this becomes equal to this. So, $2\sqrt{c/\rho}$ into σ and that is actually the multiplying factor. So, this value is different from σ and one can see smaller is the ρ this multiplying factor is large and longer is the c this multiplying factor is also large; that means smaller is this radius of curvature or it is the finer is the tip, finer is the tip the multiplying factor is larger. So σ_{\max} , \max is much larger than σ .

So, because of this very fine tip and the radius of curvature being small you have a multiplying factor and σ_{\max} is quite different, and in fact it is much larger than the applied stress. So, σ_{\max} increases as c increases or ρ decreases. So, this is very very important for us, because if you have a sharp tip there the stress experienced by that sharp tip is much larger. So, there is a multiplying action or multiplying situation. So, that is what we call the stress intensification at the crack tip. Here of course there will be no multiplication, because it is more a flattered region. So, flattered region will have more or less the same stress here, but the stress will be maximized at the crack tip and which is more critical, because this crack is trying to grow in this direction and higher is the stress here; obviously, the extension of the propagation of the crack will be faster will be easier.

So, that is one of the major reasons why once a crack is generated and a sharper crack is formed in a brittle solid it propagates almost instantaneously, and there is hardly any

plastic deformation which is lead by this crack propagation. This is unlike dislocation where dislocation does not need to crack immediately when large number of dislocation pile up together and the stress at that particular point increases then only the crack gets generated. So, before that only few dislocations or a single dislocation does not lead you a crack. So, that is one thing have to be understood that in a ductile solid it is the dislocation movement gives you the plastic deformation whereas in a brittle solid is is the propagation of the crack which leads to the complete fracture.

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Stress Intensification at Crack Tip (Inglis Theory)

$$\sigma_{\max} \gg \sigma$$

For fracture to take place $\sigma_{\max} = \sigma_{\text{th}}$

$$2\sigma \sqrt{\frac{c}{\rho}} = \left(\frac{E\gamma}{a_0}\right)^{1/2} \Rightarrow \sigma_c = \left(\frac{E\gamma\rho}{4a_0c}\right)^{1/2}$$

Assuming $\rho = a_0$

$$\sigma_{\text{inglis}} = \left(\frac{E\gamma}{4c}\right)^{1/2}$$



If sigma max is greater than sigma for fracture to take place this is of course the conditions we have seen earlier sigma max is actually much, much larger than sigma and for fracture to take place; that means for the propagation of the crack sigma max must be equal with equal to sigma theoretical. So, that is possible. Sigma max is much larger than the applied stress, and if sigma max is equal to the theoretical stress which is required for the separation of the two layers of atoms, then of course the facture will automatically take place. And since there is stress intensity or intensifying action at the crack tip then it is not difficult to reach a theoretical value even without applying a much smaller stress from the outside. So, applying a much smaller stress from the outside that may also lead to a fracture, because at the tip of the crack the value is much larger and that value may exceed the theoretical value.

So, that is the cause why the major value of fracture is much smaller than the theoretical value. Now we have seen this equation earlier; that is this is the theoretical value, and this is what we have calculated as σ_{max} . This is what we have calculated as σ_{max} , and earlier we have seen the theoretical value or theoretical expression is like this, right. So, the condition of fracture is the theoretical value must be equal to the σ_{max} which is getting generated. So, this is your theoretical value and this is sorry this is the σ_{max} for the local stress which is getting experienced by the crack, and these are the theoretical values. And that leads you a critical stress, a critical stress σ_c which is E or the young's modulus surface energy and ρ the radius of curvature at the crack tip and then 4 is the multiplying factor, and then a_0 the inter-atomic spacing and c is the distance half distance of the crack half length of the crack.

So, σ_c the critical stress which is required for the fracture then it becomes this one this expression. Assuming that the ρ which is the radius of curvature at that tip is almost equal to the inter-atomic spacing, because we are basically separating the two layers of atoms or two neighboring layers of the atoms, and that distance is approximately the inter-atomic spacing. So, we can assume that the curvature radius of curvature is almost equal to the inter-atomic spacing a_0 . Then the stress required stress required for fracture sometimes it is designated as σ_{Inglis} is equal to the $\sigma_{Young's}$ modulus E , the surface energy γ and then this and this cancels. So, ultimately it becomes $4 \sigma_c$.

So, this is an important expression, the stress required for fracture and of course, the c term is still there, because ρ and a_0 has cancelled, the c term is still there. So, it will depend on the crack length only, right, not so much on the radius of curvature, because you have assumed it to be equal to zero which it may or may not be under different conditions; under certain conditions it is true, but in other conditions it may not be true; we will look into those things later on. So, this only gives you a stress required for fracture is inversely proportional to square inversely proportional to square root in fact, the square root of the length of the crack. So, higher is the length of the crack lower will be the stress required.

So, that is it is the critical stress; what is the critical stress required for the fracture? Critical stress required, well the actual applied stress is more than the critical stress then the fracture will take place that we will see later, right. So, that is the critical stress

minimum stress required for the fracture to take place, and for that larger is the crack length smaller is the critical stress required for the fracture. So, at lower stress; that means larger or the longer is the crack at a lower stress the fracture will takes place. So, that also is an important criteria or important point to note that it is not the cracked tip radius but ultimately it is the crack length also important here; although the stress intensity factor when we are talking about stress intensity factor this crack length has a less importance. So, this is how one can calculate the critical stress required for the fracture.


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Griffith's Theory (I)

- This is by far the most well accepted theory of brittle fracture. It considers the change of energy during fracture.:
 - 1) strain energy and 2) surface energy
- Elastic energy stored may be obtained from the area under the stress-strain curve (referring to the figure in the next slide)

$$U_{elas} = \frac{1}{2} \varepsilon \sigma_{app} = \frac{1}{2} \frac{\sigma_{app}^2}{E}$$

$$U = U_0 + V_0 U_{elas} = U_0 + \frac{V_0 \sigma_{app}^2}{2E}$$


 Where, U_0 is the energy in absence of stress and V_0 is the volume of the material under stress.

So, with this background let us go to a theory which is very well known and is very universally accepted theory for the fracture or brittle solids. This is by far the most well accepted theory for brittle fracture. It considers the change of energy during fracture, the overall change of energy we have earlier considered the surface energy term, but here we will also consider the two different terms, one is the strain energy other is the surface energy. So, by applying a stress you are actually increasing the internal strain energy of the system, and then also if there is a fracture and a new surface is created you have a surface energy term which is coming up.

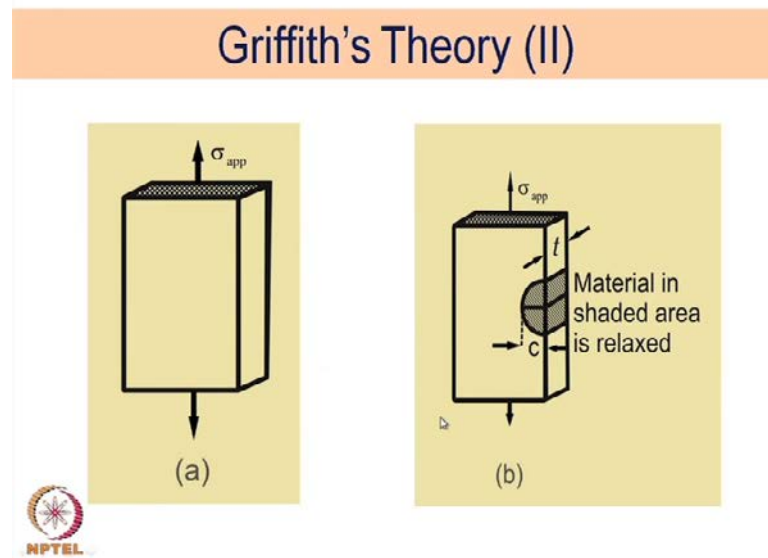
So, there are two different terms; one is the strain energy terms and that is the total energy during fracture change of energy during fracture, because you are applying external stress and that stress it generating some internal strain energy. Even if the

fracture is not there the strain energy term will be there because you are trying to pull the atoms, so the layer of the atoms are actually pulled from, the neighboring atoms are being pulled from one another and therefore, the bond length increases. So therefore, there is an extension of the bond length. So, the distance between atoms is increasing even if they are still attached to each other, but there will be a strain energy generator inside it. Only when the atoms or the cohesive energy is broken the bond is broken then you have a surface energy term as well.

So, the elastic energy store may be obtained from the area under the stress strain curve referring to the figure in the next slide, is it in the next slide? Yes. In fact, it was shown earlier also very first slide let me see. So, that is the store elastic energy whenever you are applying a stress the fracture has not taken place, but the area under this curve gives you the store energy. So coming back, right, so this is the expression for store energy half this is epsilon strain into sigma placed; it is the area under that curve. So, we just have the area of that particular diagram. So, this is elastic energy and if you replace the strain energy strain here by sigma applied by E is the Young's modulus once again that is the slope of that curve. So, we have half epsilon sigma applied square by E.

So, that is your that is your elastic energy term, and then you can also write because this is the incremental energy; this is the incremental energy and if you have originally U_0 is the energy in absence of the stress. There is already an energy available even if you do not apply a stress then the total elastic energy becomes this, so which is quite an obviously thing. So, V_0 of course you have to multiply by v_0 because this energy terms is per unit volume. So, since this is a per unit volume and if you know the volume V_0 is the volume of the material under stress then this becomes the total energy elastic energy. So, U_0 plus V_0 terms will come here sigma applied square by $2E$. So, this is the elastic energy term which we have indicated here.

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Well this is a geometric representation of what is happening in the sample or in the solid. This is a rectangular sample where each σ applied is the externally applied stress and it is under tension here, and as a result of that you are generating a cracked region. This crack was not there originally, and because of this tensile force you have a crack that has been generated, and there is a strain region here. We have seen it earlier also where we are considering that σ_{max} , here also a crack has been generated and surrounding that crack we have a strain region where the atoms are not exactly in their equilibrium sides, but it has been displaced from the equilibrium sides, so that is a strain region. So, a strained field has been generated surrounding the crack.

So, the material in shaded area is relaxed of course; that means there is a strain generated whatever stress has been applied because of the stress the atoms have got relaxed, so that the strain is released. But it is basically a strain region, and you have an increase in strain energy. Now this is the crack length in this case, earlier we added two-dimensional pictures. So, you are basically looking at the crack length half crack length here, and this is the t , t is the thickness over which the crack is generating. So, as if a uniform crack has generated over the thickness. So, this is the thickness, and this is the crack length, and this is a kind of semicircular region where you have strain energy has been generated.

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Griffith's Theory (III)

$$U_{strain} = U_0 + \frac{V_0 \sigma_{app}^2}{2E} - \frac{\sigma_{app}^2}{2E} \left[\frac{\pi c^2 t}{2} \right]$$

Surface Energy: $U_{surf} = 2\gamma ct$

Total Energy:

$$U_{tot} = U_0 + \frac{V_0 \sigma_{app}^2}{2E} - \frac{\sigma_{app}^2}{2E} \left[\frac{\pi c^2 t}{2} \right] + 2\gamma ct$$

At equilibrium $\frac{\partial U_{tot}}{\partial c} = 0$; $\sigma_{app} = \sigma_f$ and $c = c_{crit}$

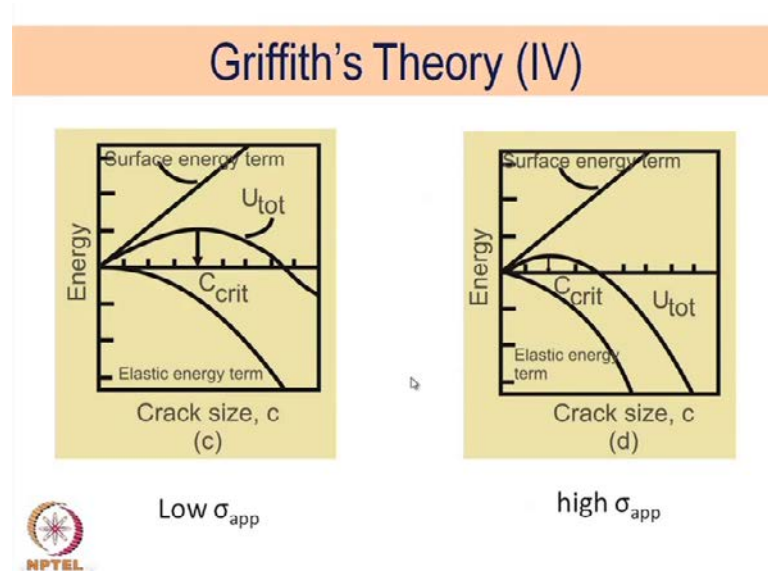


So, this is already we have seen. Now we have strain energy is this one min this part I think we have seen there. This is the expression of the elastic energy, and there has been because of the crack initiation of the crack there has been a release of some of the energy. Some energy has already been relieved and therefore, this term comes here. This is in the form of geometry. This c term is not here, because it is before the crack has been generated; this is the energy before the crack has been generated. Here after the crack has been generated you have this term also comes in. So, p zero in fact, this is the volume this is the volume in some sense the volume of that pi r square by 2 into t; that is the volume.

So, this becomes the total strain energy after the crack has already been generated; this was before the generation of the crack. At the same time you have created an additional surface that we have already considered that the surface energy is actually 2 gamma c t, c is the length and t is the thickness. So, that is the surface energy term. So, adding these two together you have total energy total elastic energy here is u 0 this minus this into this; that is this term and plus the surface energy. So, this is an additive term; this energy is also additive. So, you have this is the strain energy, this part is the strain energy and this part is the surface energy. Now if you want to equilibrate or under an equilibrium situation you must have the lowest energy, so the differential of this becomes zero has to be zero. Before that we can look at it; you can look how it can be represented

geometrically. This is the way as a function of crack size, there are two, here is also crack size c and here is also crack size c , this is square and this is linear.

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So, this is your elastic energy term which is decreasing as a function of crack length whereas the surface energy term is increasing linearly, this is a square term inverse square term and this is a linear term. So, it is an increasing function, and this is a decreasing function. So, the total energy becomes an addition of this two, and it goes through a peak. The total energy as a function of the crack size is not a monotonically changing function, but it is initially it increase because of the surface energy term is greater, and then this becomes predominating term. So, it decreases, and that is a very important concept and important criteria for understanding the crack growth. So, for a equilibrium situation where this energy is the maximum. So, that is what it says at equilibrium the slope of that curve should be zero, and that is what it looks like from the geometry also or the diagram.

And if you apply that then the sigma applied externally applied stress becomes the fracture stress; that is the stress required for the fracture because up to this the energy is increasing, and after that there will be automatic extension because the energy will decrease. So, there is a lowering of energy beyond this critical crack length. So, beyond this critical crack length this energy is decreasing and that is almost a spontaneous change whereas up to this the energy is increasing, so you need external stress to

increase the energy. So, as you increase the energy and increase the external stress the energy of the system increases and after a particular crack length of the crack you do not need any further energy for lowering of the energy.

You do not need any further external stress for the lowering of this system energy, and it is a spontaneous growth of the curve a growth of the crack, and that is the most important concept of provided by Griffith's theory and that is what is a very critical concept or important concept to understand the brittle fracture of the brittle solids or the fracture of brittle solids. So, coming back to the earlier page when the differential is zero that is at the maximum of the of energy curve σ applied becomes σ fracture and c becomes c critical; that means this is the minimum crack which is needed which will automatically expand or automatically propagate without any further application of the external field and so these two are very important for our concepts or for understanding, and this is more or less a situation what we have seen or we see in a phase transformation nucleation and growth phenomena.

In a phase transformation also a very similar kind of situation exists. There is a nuclear size which is called the critical nuclear size which automatically will grow and if the nuclear size is less than it, it becomes embryo and that embryo gets collapsed. So, that part does not grow, because here the energy goes back energy increases or energy decreases when the size decreases whereas on this side the energy decreases when the size increases. So, this part will automatically increase on its own but it automatically decreases whether it is embryo or a critical nuclei or a crack. So, and very analogous situation happens like that of a nucleation and growth theory of phase transformation. These two curves are more or less same except that this is a low applied stress, and this is a high applied stress.

If the applied stress is high then we will see the critical crack length is small. So, this is much smaller than this length. So, if you have doubled this thing the applied stress you will find a root c a root two change in the critical length critical crack length. So, there are many implications of this particular concept. One is as we have said that there is a critical fracture stress or critical applied stress controlled by the fracture, fracture stress. If it is stress is applied more than that the fracture will automatically take place, and what will be the size, what will be the size of that critical crack length? That is also

determined by the applied stress. So, higher the applied stress the critical length of the crack is smaller, and that is also very important for our physical understanding.


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Griffith's Theory (V)

$$c_{crit} = \frac{4\gamma E}{\pi \sigma_f^2} \quad \sigma_f = 2 \sqrt{\frac{\gamma E}{\pi c_{crit}}}$$

Fracture takes place when

$$\sigma_{app} \geq \sigma_f \quad \text{or} \quad c \geq c_{crit}$$

$$\Rightarrow \sigma_f \sqrt{\pi c_{crit}} \geq 2 \sqrt{\gamma E}$$


Using that condition of differentiation or the differential become zero one can find out the critical crack length is equal to $4 \gamma E / \pi \sigma_f^2$; σ_f^2 is the fracture stress, σ_f is the fracture stress and where σ_f is given by this. So, there is a relationship between the critical crack length and fracture stress here and same also is there. So, for fracture to take place as I have already discussed the applied stress must be greater than or equal to the fracture stress; fracture stress is given by this, and that is again dependent on the size of the critical crack length. So, σ or either this is satisfied or the critical crack or the crack length is more than the critical crack length.

As I have said that higher the crack length or any crack greater than the critical crack length will automatically grow; that does not need any increasing stress. These two things give you a parameter which is $\sigma_f \sqrt{\pi c_{crit}}$. Now these two terms or these conditions lead to this. So, σ_f the fracture stress multiplied by the square root of πc_{crit} is greater than equal to $2 \sqrt{\gamma E}$. Now these are the material parameters; these are the two material parameters, what is γ the surface energy, and E is the Young's modulus. These two are the material parameter of any material, and these are the actual the critical conditions of the stress and the critical crack length.

So, these are they are related in this; that means when this multiplication or this product, this product is greater than equal to this product, then the fracture will take place. So, that is what the Griffith theory says, and these two terms this particular product appears in many different calculations whenever we are considering different forms of mechanical energy or mechanical strength or mechanical property, and that is what is given here; why if they are important, and what is there definition? The term sigma root over phi c that is what we have seen earlier. This is of course a particular value of fracture at fracture whatever is happening.


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Fracture Toughness (I)

The term $\sigma\sqrt{\pi c}$ appears very often in the analysis of fracture behaviour of brittle solid like ceramics and it is termed as

K_I = Stress Intensity factor (Unit: Mpa.m^{1/2})

$K_{Ic} = \sigma_f\sqrt{\pi c_{crit}} = \text{Critical Stress Intensity Factor}$
= Fracture Toughness (Mpa.m^{1/2})



But otherwise in general this product the applied stress and multiplied by the crack length root over or the square root of crack length appears very often in the analysis of the fracture behavior of brittle solids like ceramics, and it is termed as this particular term is called the stress intensity factor. This is the stress intensity factor K_I with the unit of mega Pascal meter to the power half, and the critical value when this stress is sigma f the particular value f at which the crack initiates or the critical length of the crack that is what we have seen earlier, this particular term, this particular term is the critical stress intensity factor. So, this is both of them are critical; that means specific value for that particular material, and this is critical stress intensity factor.

This is only stress intensity factor in general at any stress level or any crack length but if you have a specific value of the stress and the specific value of crack length which has

been defined by the Griffith theory that becomes your critical stress intensity factor, or in other words it is called the fracture toughness. So, that is the definition of the fracture toughness how we define fracture toughness from the consideration of the Griffith theory. So, the crack the existence of crack is not only very important in brittle solids, but how it behaves under applied stress that is also of critical importance. So, that is a term the fracture toughness which we will use very often. In fact, it is used very often to understand not only the crack behavior, but basically the mechanical strength of a ceramic material; fracture toughness is one of the most critical parameters we normally consider, how this material will be reliable, how it can behave under different stress fields?

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Fracture Toughness (II)

More generalized expression for Stress Intensity factor


$$K = Y\sigma\sqrt{\pi c}$$

Where, Y is a factor depended on the location and orientation of the crack and also on the loading pattern.

For materials with appreciable plastic deformation

$$K_{Ic} = \sqrt{EW_c}$$

Where, E is the Young's modulus and W_c is the toughness of the material in joules per square meter.


 $E \uparrow, W_c \uparrow \implies K_{Ic} \uparrow$

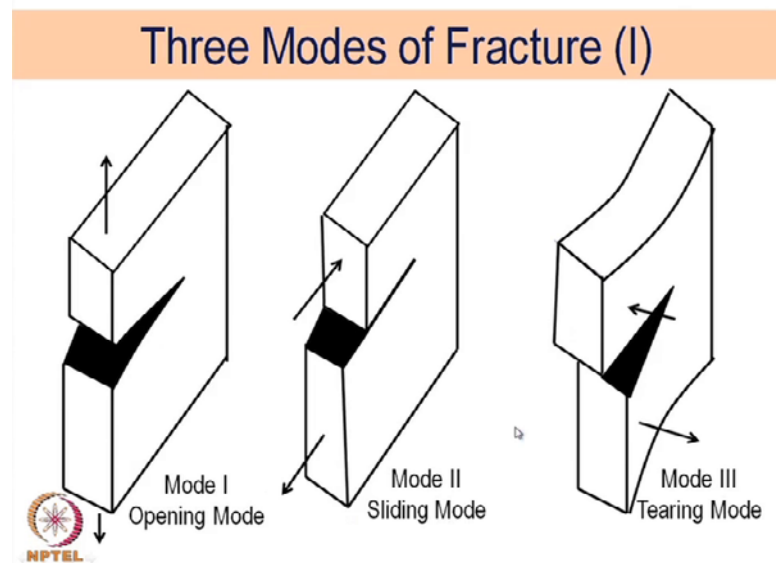
More generalized expression of stress intensity factor that was a simplified factor, this is the stress intensity factor where there are only two term sigma and c; however, a more rigorous analysis shows that there is another multiplying factor in addition to this two terms there is an another multiplying factor y. Y is a factor dependent on the location and the orientation of the crack and also on the loading pattern. So, many of the other complications have not been considered in the Griffith theory or Griffith's analysis what we have presented just now, but a more rigorous analysis including what is the orientation of the crack, what is kind of loading pattern? Is it a point loading or distributed loading whether it is or the location of the crack so far as the volume of the material is concerned, and then many other variations may be there.

So, depending on that there will be always a multiplying factor. So, y is a multiplying factor which ultimately gives you the stress intensity factor to the applied stress and the crack length. So, in addition to the applied stress and crack length you have another multiplying factor y which is dependent on the various different nature of the crack or the loading pattern. For materials with appreciable plastic deformation K_{IC} also gives you an idea of the fracture toughness, not the fracture toughness, the toughness the total toughness. Fracture toughness you must also distinguish between fracture toughness and toughness; they are not exactly the same, the terminology are not exactly same, the definition are different. So, fracture toughness is different and toughness is different.

So, the material with toughness is normally we define for more plastic material or ductile material where the plastic deformation is large, and the area under the curve is large. Whereas in case of brittle material the area under the curve is much lower and the more critical parameter is fracture toughness that the stress require to initiate the crack, and that is more important than what is the area under the curve, the total energy. This total energy is not that important, but it is the stress intensity factor which is more important. So, this is how they are related, this fracture toughness K_{IC} is equal to root over E into W_c is the toughness of the material in joules per square meter, the total energy and here it is stress.

So, Young's modulus E is there and W_c is the toughness of the material in joules per square meter. So, from this equation if E increases or W_c increases the K_{IC} also increase. So, there is a relationship but between the toughness Young's modulus and K_{IC} the fracture toughness. So, if young's modulus high, obviously, the fracture toughness will be high. If toughness is high the K_{IC} the fracture toughness will also be high. So, there is a relationship between these, but the two terms are completely different, not completely different they are related, but they are different, so one needs to distinguish between these two.

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So, I think, next we will be the three different modes of fracture, maybe will take it up in the next class for the time being; that is our end of discussion. So, we have introduced the basic property of the fracture or the brittle material the fracture how it takes place and stress intensity factor, the maximum stress experience by the crack t is quite different than what we apply from the outside. And then there is a critical length of the crack beyond which the crack automatically propagates, because there is a lowering of energy, and that is a most critical concept we have introduced so far.

Thank you; Thank you for your attention.