## Principles of Polymer Synthesis Prof. Rajat K Das Material Science Centre Indian Institute of Technology, Kharagpur

## Lecture - 27 Design of Chemical Reactors (Contd.)

Welcome back we have been continuing to talking about designing principle of chemical reactors. And so, today's topic is also going to be continuation of the same, and if you recall we discussed one problem of CSTR conversion in the previous class, we will continue and we will discuss some more problems. So, let us then define another problem here.

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We are going to consider this particular reaction liquid phase reaction and this is following an elementary rate law which means minus rA equals to k into CA into CB. So, both are first order now to do this reaction what we have is 2 reactors that are available, either you can do this reaction in a 800 decimetre cube plough flow reactor, which is operated at temperature of 300 kelvin so this is the volume of the plough flow reactor or you can do this reaction in a 200-decimetre cube CSTR which is operated at temperature of 350 kelvin.

Now for both this flow reactors either you do the reaction in this or you do the reaction in this basic basically the inlet us stream. The inlet stream is consisting of 2 separate feeds of A and B they come together and they form one single inlet stream and, in this inlet, stream you have basically equal molar A and B.

And the total volumetric flow rate is 10 decimetre cube per minute and also what is given is you know that there are 2 separate streams that are coming for A and B they are coming together and forming one inlet stream that is entering your reactor. So, these 2 separate streams the concentration of A in one stream which is it is before mixing before mixing concentration of A is basically 2 moles per decimetre cube and concentration of B.

Before mixing is also 2 moles per decimetre cube and the volumetric flow rate of A is before mixing again is 5 decimetre cube per minute, the same as the volumetric flow rate of B and also since the reactors are operated at different temperatures you need to know the k value at these 2 different temperatures. So, what is given is the k value at 300 kelvin is 0.07 decimetre cube per mole per minute and the activation energy is 85000 joule per mole kelvin.

So, all these things are given now the question is which of the 2 reactors is going to give you a higher conversion. So, if you want to pictorially depict these thing at least for CSTR we will depict you can do this for PFR after, it is something like this. So, you have the CSTR this is the starrer and this is the reaction mixture where it is operating. So, one stream is coming for A separately another stream is coming for B separately and they are coming together, they are coming together and entering into the reactor.

And whatever is coming out has a separate molar flow rate out for A say and for B also and X is the conversion X is what we are going to find out. And so, this is CSTR we are talking about so CSTR is operated at 350 kelvin and the volume is 200 decimetre cube for the CSTR. And also, so this is a so separate stream of a is coming and separate stream of B is coming and then they are coming together and in this you have equal molar A and B and here also if your looking at CA 0 am just putting small b.

So, CA 0 before mixing is 2 moles per decimetre cube. So, from here and from here CB 0 before mixing is 2 moles per decimetre cube and this is so the molar flow rate in FA 0 here it is FA 0. So, when they mix up; obviously, the concentration for A becomes half and concentration of B becomes half and the total volumetric flow rate is basically 10 decimetre cube per minute. So, volumetric flow rate of A here is 5 decimetre cube per

minute and here it is they are coming at 5 decimetre cube per minutes. So, when they come together it is 10 decimetre cube per minute.

So, at least this is the definition of problem for CSTR the same thing you could do for PFR also, this is just to pictorially depict the meanings of this things because you might be confused what do you mean by before mixing what do you mean by before mixing and after mixing. So, that this is the situation so this is before mixing and then they mix and then they enter. Now let us try to solve this problem.

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CET

So, what is the mole balance of CSTR is FA 0 into XA divided by minus rA and what is the mole balance? Actually, this is the design equation that we get from the mole balance. So, what is the design equation of V PFR or PFR it is sorry you will get the V PFR from there. So, that is FA 0 into 0 to XA 0 to XA dXA divided by minus rA. Rate law we already know this is the rate law.

So, stoichiometry now step by step let us proceed will go to the stoichiometry concentration of A is equals to FA into 1 minus X divided by small v and that is equal to so this is FA 0 into 1 minus X by small v is equal to CA0 into 1 minus X this is appearing everywhere. So, am not going to explain again how these appears even before when we construct the stoichiometric table we had discussed about this. So, this is the expression so if you put this value this here in V CSTR this expression in V CSTR it will be equal to FA 0 into XA divided by remember minus rA is equals to k into CA into

CB ,and CA and CB are the same CA 0 and CB 0 are the same we have already talked about that. So, this would be FA 0 and XA divided by k CA 0 square into 1 minus X square.

So, CA is this CB is also this equal molar flow what about V PFR it is FA 0 into integral 0 to dXA express write the expression of rA minus rA it will be a same expression as this into 1 minus X whole square am just putting X it will be a XA actually X subscript A. Now this case is 0 square is constant they come out so on and so forth. What is the value of small k at 350? Now remember the value of small k at 300 is given rate constant, at 300 it is given and the energy of activation is also given.

So, if you know the temperature at which we have to find out the rate constant. So, in this case 350, then what you do your expression of k is nothing but A into e to the power minus E by RT. So, this is the value of k at that particular temperature now this value is given at 300 A we do not know we do not need to know also in this case T we know for 300 degrees it will be T 300 and E.

We know activation energy that does not change with temperature. So, what we can do is that we can take a ratio of k 350 to k 300, then this a will cancel out and we will get what you what you have has e to the power E by R into 1 by 300 minus 1 by 350, and these value is already known this value is 0.07 decimetre cube per mole per minute.

So, if you solve for these just take the ratio between the 2 this A is cancel it out you do not need to know the A you know the value of e. So, from here your k 350; that means, rate constant 350 kelvin is 8.447 decimetre cube per mole per minute. Now let us solve these expression because we want to know V CSTR is already known, we want to find the value of X, that is what we want to find. Now remember that this reaction is done at 350 kelvin 350-degree kelvin at 350-degree kelvin in the CSTR.

So, the value of k that you are going to use is that 350. So, that is the value of k that you are going to use. So, if you now put those expressions here what you will get CSTR at 350, this is 1 minus X whole square these 1 minus 2 X plus X square these equals to this term is FA 0 into XA divided by k CA 0 square this V comes here. These this into X just putting X here this is the term if you re arrange this what you will get this is a quadratic expression in terms of X FA 0 divided by k CA 0 square into V plus 2 into X plus 1 equal

to 0 this the quadratic expression that you will get. So, this you can easily solve remember this value is also known

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m mal min. 10 mol/min 20.9

What is this value? This value is nothing but FA 0 divided by k CA 0 square into V, this value is nothing but FA 0 is 10 mole per minute already it was defined in the in the problem that your inlet stream what is the volumetric flow rate in that is 10 decimetre cube per minute the total volumetric flow rate in, but if you look at

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The picture here, your CA 0 which is the concentration before mixing these 2 moles per decimetre cube and what is the volume of volumetric flow rate in of A? It is 5 decimetre cube per minute. So, you have to multiply these 2 and if you multiply these 2 you get the value of FA 0 which is 10 the value of FA 0 is 10 moles per minute. So, here is 10 moles per minute small k is 8.447 we already determined the value of small k 350 kelvin.

So, 8.447 decimetre cube by mole minute. CA 0 is basically 1 moles per decimetre cube that whole square 1 because ultimately the entering concentration has become halved it was 2 moles per decimetre cube here 2 moles per decimetre cube here for A and B. So, when they came together the CA 0 at the entrance is basically 1 moles per decimetre cube. So, CA 0 is 1 moles per decimetre cube and v is already known what is the volume 200 decimetre cube.

So, if you solved this thing and if you put here in this expression. So, remember this expression is you're a quadratic ax square plus bx plus c equals to 0. If you have quadratic equation like this ax square plus bx plus c equals to 0 then your X will have 2 solutions it will be minus b plus minus root over b square minus 4 ac divided by 2 a and both solutions may be visible or only one solution may be visible depending on if it is physically possible or not here only one solution is visible now work it out and the value of X from here that you will get because you know this value will be equal to 0.926; so 92.6 percent conversion. So, what will be the corresponding?

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Value for PFR for PFR remember your expression was FA 0 divided by k CA 0 square into integral 0 to XA dXA by 1 minus X square; I mean am using X or XA interchangeably you can understand that both have the same. So, if you do the integration and do a rearrangement. Basically, what you will get is k CA 0 square into FA 0.

So, this goes here divided by FA 0 into V which is a V PFR that becomes equal to if you do the integration here it will be X divided by 1 minus XA divided by 1 minus XA. So, if you put all the values you know this values of CA you know the value of FA and you know the value of k. Now this value of k has to be the value of k at 300 because of PFR is the operated at 300 kelvin.

So, if you put all these things here your X becomes or XA becomes 0.84. So, basically if your using a CSTR you will get higher yield around 93 percent yield and if your using a PFR you will get a lower yield. So, that is the solution of the problem. Now let us solve a problem a simple problem for polymerization. Let us say we are going to use a batch reactor and we are polymerising pure styrene at 60-degree celsius with benzoyl peroxide as the initiator.

And we are going to use a batch reactor and we are aiming for 10 percent conversion this is our aim and what is given is the initiator efficiency is one let us assume that the value of kp square by kt this is the propagation rate constant this is the timing rate constant this value is equal to 0.95 into 10 to the power minus 3 litre per mole per second and the concentration of the initiator is given as 4 into 10 to the power minus 3 moles per litre and dissociation constant is given as 1.92 into 10 to the power minus 6 per second.

So, the question is what is the time required to obtain 10 percent conversion in a batch reactor.

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CET LLT. KGP male balance

Now, remember that we had solved for a first order reaction previously we had solved this expression, this was the expression that we had got if the reaction is first order in order to get this kind of conversion we have this expression. Now if you replace this X the conversion remember conversion is nothing but initial concentration minus final concentration divided by initial concentration of the monomer, if you are looking at a polymerisation you replace that here.

Then let us just assume in this case it is a first order reaction for styling. So, if you put that value here what you will get is nothing but now we are solving the problem solution, we will get ln M by M0 equals to minus kt. So, we are basically getting from here by replacing XA with this value. Now this k is basically equal to kp into fkd divided by kt to the power half into I to the power half, if you recall the expressions of the rate that we had. Then this is the value of k. So, if you recall what we discussed about in the during the classes on the analysis of kinetics for the rates of polymerisation so on and so forth.

So, then you can realise this k will be something like this. So, if you take the square of that will be kp square divided by kt into f kd into I. Now you know all this values you know value of kp square by kt put that value here kp square by kt is this f is 1 I is already known kd is also known

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So, what you will get is 7.30 into 10 to the power minus 12 second square inverse so; that means, your value of k will be square root of that which will be 2.70 into 10 to the power minus 6 per second.

So, you now put this values here. Then what is the expression you will get from here if you put the value there you will get ln M by M0 equal to 2.7 into 10 to the power minus 6 into t now there is a minus term here. So, if you do the calculation here what you will get you will get from here, let say you are going for 10 percent conversion then your M0 minus M divided by M0 is 0.1.

So, from there you can get this expression of M by M0. So, if you use this expression here in this term it will come out as 0.9. So, it will be basically ln of 0.9 will be equal to minus 2.7 into 10 to the power minus 6 into t and if you solve that t is your 8.1 hour. So, that is how you get your final expression. In this case we assumed data styling polymerisation is first order and a low conversion of course, alright. So, we have done one simple problem for polymerisation here. Now what we are going to do is the following, let us try to probably workout one more problem let us try to work out one more problem.

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Phildellene

Say let us say we are trying to appear polyethylene and you are using a tubular reactor it could be a plough flow reactor also and the dimension here is 2.5 centimetre diameter and 1 metre long tube and the reaction is done at 250 degree celsius and 2500 atmospheric ratio and conversion per pass is 30 percent, also what is given is the flow rate the flow rate is 40000kg per hour and also you assume that the polymerisation is the first order in ethylene concentration.

So, all these things are given then the question that is being asked is you should try to estimate the value of the rate constant for propagation. So, it is a polyethylene polymer ethylene polymerisation is being done. So, product is polyethylene ethylene polymerisation is being done at tubular reactor of these dimension this is diameter this is the length at 250-degree celsius and 2500 atmospheric ratio at high pressure under high pressure and the conversion per pass is 30 percent flow rate is this. So, what is the value of kp this is the question.

So, remember that this was the plough flow reactor that we talked about and then if you have a small element small volume element here say mass M is going in and M plus dM that is going out. The molar flow rate in molar flow rate out so on and so forth and this dV is the volume of this. Now if you write down the design equation here if you remember it is FA 0 into integral 0 to X XA dXA divided by minus rA this was design

equation. So, from here what you can actually write down is the following FA 0 dXA dV is equals to rA FA 0 dXA dV is equals to rA.

And rom these expression your FA 0 which is the molar flow rate in this is the molar flow rate in right. So, this molar flow rate in is basically equal to volumetric flow rate in into the mass that is going in the concentration let us say concentration in this case the concentration of the ethylene, that is a go that is going in this dXA dV equals to the rate of polymerisation basically this is the rate of polymerisation rate of propagation is a rate of polymerisation now if you do an integration here.

So, let us say you do an integration from volume 0 to V and you do the corresponding you know re arrangement it will be dV by v small v into M0 equals to 0 to XA dXA divided by Rp. So, this is the expression that you will get here. So, this is dV by v M0 and then this this side dXA divided by Rp that will come. Now if you remember your value of XA is nothing but M0 minus M divided by M0 this is the value of XA. So, what is the value of dXA the value of dXA is nothing but minus dM by M0. So, you replace the value here and then what you will get is the following.

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0 to V dV by small v into M0 small v into M0 equals to now you are replacing XA by m. So, this integral will also change from initial value of M to final value of M.

So, if you put those values here that will be M0 to M minus dM divided by M0 into Rp. So, finally, what you will get is this this small v comes out and when you integrate froM0 to V the dV it becomes capital V by small v this equal to integral M0 to M0 goes here. So, this is minus dM divided by Rp.

This is the final expression that you have now your Rp your Rp is nothing but kp into M Rp is nothing but kp into M. So, if your putting that expression here what you will get here is capital V by small v equals to minus integral M0 to M into dM by M, because remember that the reaction is first order in ethylene that is what we talked about this kp value here we are trying to find out. So, if you put that value here. So, Rp is replaced by Rp is replaced by kp into M.

So, that will be kp into M. So, if you put the kp out of the equation that will be kp capital V by small v equals to minus integral M0 to M dM by M, this is your residence time capital V by small v. So, it will be kp into tau equals to minus integral M0 to M dM by M and if you do integration ultimately what you will get is M equals to M0 into e to the power minus kp tau and also what you will get is the value of kp which will be nothing but minus 1 by tau into ln M by M0.

You are trying to find out a value of kp here now M by M0 you already know it is 0.7 because you are look looking for 30 percent conversion so that is M by M0. So, if you put that value here then it will be minus 1 by tau ln 0.7. Now what you have to find out the problem has been reduced to finding now the value of tau the residence time then you can find out value of the kp or the propagation constant or the rate of polymerisation corresponding the rate constant.

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CET LLT. KGP 

So, tau is equals to capital V by small v, capital V is the reactor volume. So, the reactor volume is nothing but a cylindrical reactor pi r square into L, r is the radius, L is the length. So, if you put those values and because r is in centimetre.

So, you have to go to the metre into one metre is the length. So, ultimately what you will get is 4.9 into 10 to the power minus 4 metre cube then this capital V. Now what is the small v volumetric flow rate in what is a small v? Let us say ethylene is in if it is in gaseous state you can use this ideal gas equation v equals to nRT. So, here v equals to nRT divided by p, p being the pressure.

Now n is the number of moles right and the question actually mentioned the problem mentioned that the flow rate is 40000 kg per hour. So, correspondingly if you change that to number of moles what you will get? N equals to 40000 divided by number of molecular rules that will be this moles per hour this I have calculated as 1.43 this calculation is already done we are trying to explain the principle moles per hour. Now this R is known universal gas constant and your t is also known 250-degree celsius.

So, you just convert that to kelvin and you put all the values here then your small v is nothing but 1.43 into 10 to the power 3 moles per hour and the value of R is basically the 0.8206 metre cube atmosphere mole kelvin into your temperature is 523 kelvin you can convert to kelvin divided by p which is 2500 atmosphere.

If you can work this out it will come out as something like this 24.55 metre cube per hour. So, this is what you revised down to. So, what will be the residence time then because residence time is what you have to find out. So, that you can put into this expression. So, if you write down the residence time capital V by small v. Capital V is already known this is the capital V 4.9 into 10 to the power minus 4-meter cube divided by small v is 24.55 metre cube per hour.

So, if you work this out this 2 into 10 to the power minus 5 hour. Then the expression of k is nothing but minus 1 by tau into ln 0.7 if you put that value there the value of k will come out as 4. 95 per second. So, that is how we solve this thing. So, what we will do is we will stop here today and tomorrow we are going to go to a different concept we are going to because we have talked about mole balance until now, we will now go in to a topic of what we call as energy balance and then we will talk about different kind of a steady state conditions for reactors especially for CSTR so until then.

Thank you and good bye.