Analysis and Modelling of Welding
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Lecture – 06
Thermal Modelling – 1 by 2

Welcome to the part-1 of the lesson on Thermal Modelling, as a part of the NPTEL online course on Analysis and Modelling of Welding.

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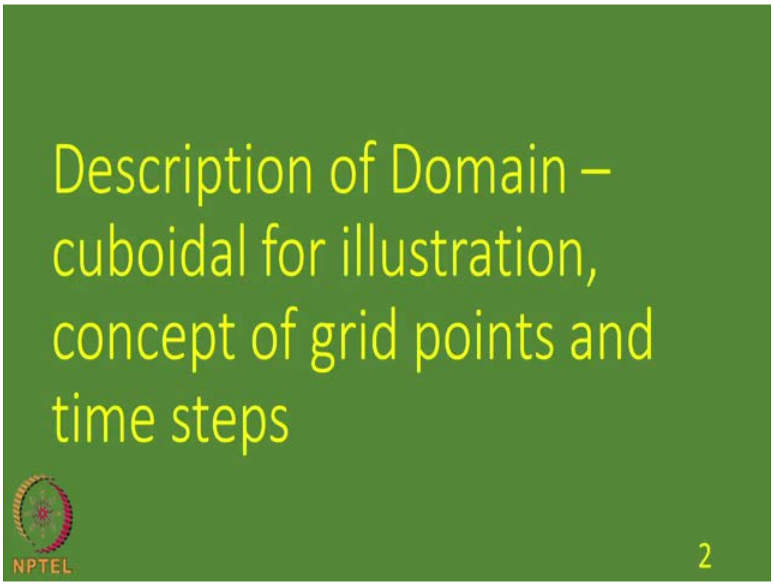

So, the first topic that we are going to look at is the description of domain and how we can identify the locations where we want to do the analysis.

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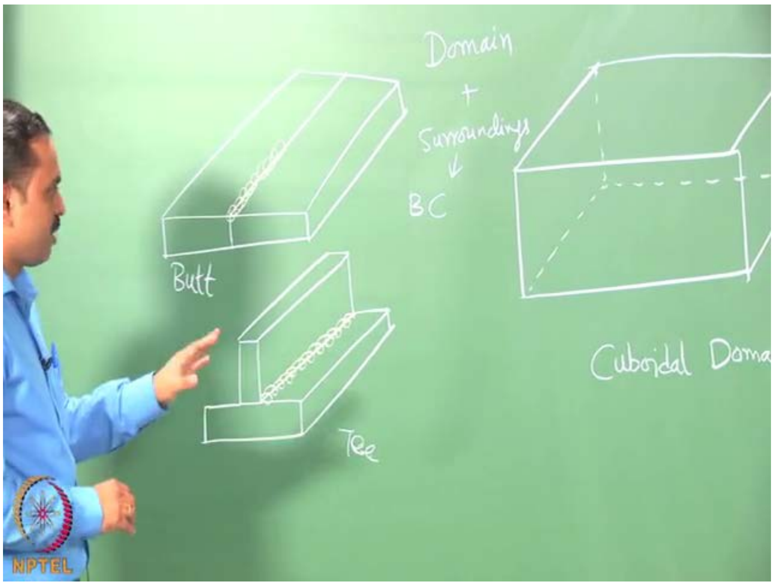

So, here I am just drawing Butt geometry of a weld and then a tee joint. As you can see that the welds are made at specific liquation's and we want to understand what would be the thermal profile in the entire weldment. So, what we mean by the word domain is this part, and what we mean bent by surroundings is already described in the previous lesson where we were discussing about the boundary conditions, heat source etcetera. So, everything else that is the heat source and that the removal of heat, the backing plate everything else is coming in the surroundings and those will basically be covered under the boundary conditions.

So, we are only now going to be bothered about the domain, which means that we are going to look at the actual weldment for our discussion. And for simplicity, we would take a cuboidal domain - the reason being that the analysis becomes easier as compared to complicated geometries, and you could always see that you can imagine a cuboidal domain as parts of various complicated shapes that we are going to look at. At what locations should we do the thermal analysis is also very important; we cannot do it at every single location in the domain.

So, if you look at a plain for example, here. We will then decide that in that plane we will have specific locations where we are going to look at the thermal field. So, which means that I am interested in temperatures at each of those intersection points and may be you could say that I want something like about at 25 different points on one plane. And then, at different widths we would take those planes and find out how many points we want to have the data.

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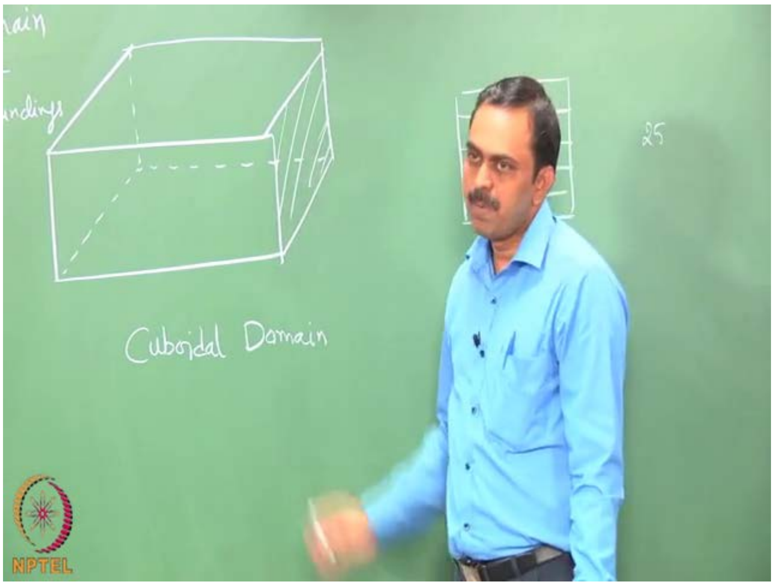

So, these points will be called as Grid locations and there are also other names for it you could also called as Nodes and Mesh points etcetera.

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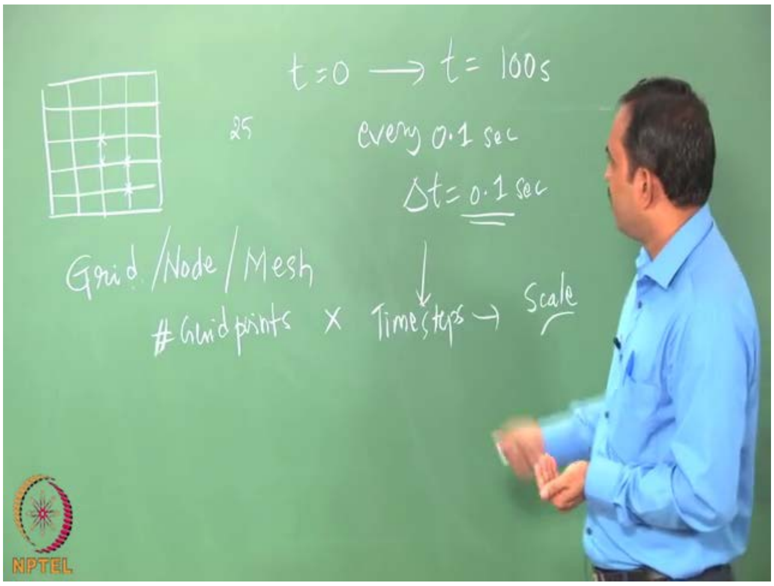

So, we cannot have the information at every location in the domain, but only at specific locations which are generally determined a-priori. There are also techniques where such locations can be adapted during the analysis itself, but we will restrict to a static analysis before we proceed. At what times are we looking at this particular information that is also to be considered; so, you have normally the base metals at you know a particular temperatures and starting at t is equal to 0 and the welding is over after certain amount of time. Let us say it is over in for example, 100 seconds. So, we may say that I want the thermal field every let us say 0.1 seconds. So, which means that I have the time step which is 0.1 seconds and that will tell you about 1000 different time steps for this particular analysis.

So, the number of grid points and into the number of time steps basically will tell you the scale of the problem. If that number is very large it means that the computer is going to spend the lot of time computing and storing the data, and one must also expect to see something like about a million grid points is possible for a complicated geometry. You may have situations where the time step is very small like about 10 milliseconds or something, and then you would come out with a huge number of data points and time steps where you have to do the analysis. So, it is very important to keep an eye on the locations and the time scales at which we want to do the analysis.

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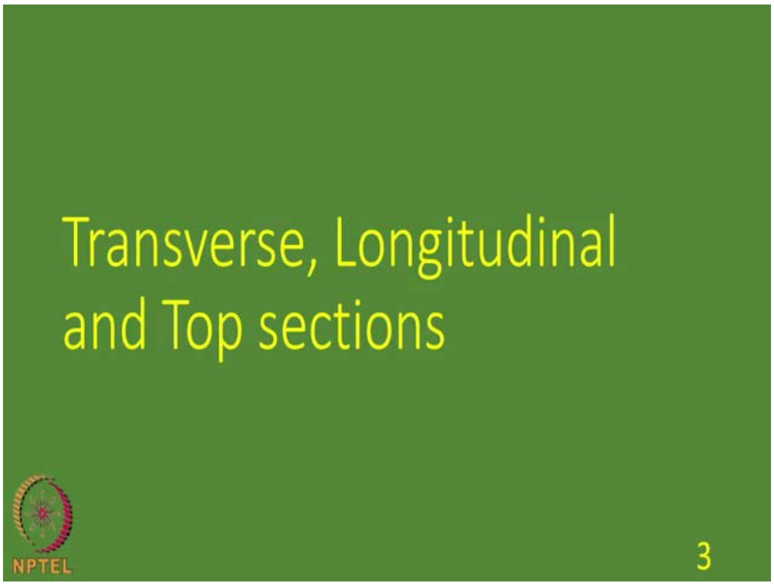

So, this analysis is going to be looked at by referring to certain sections and we must have the names of those sections popularised.

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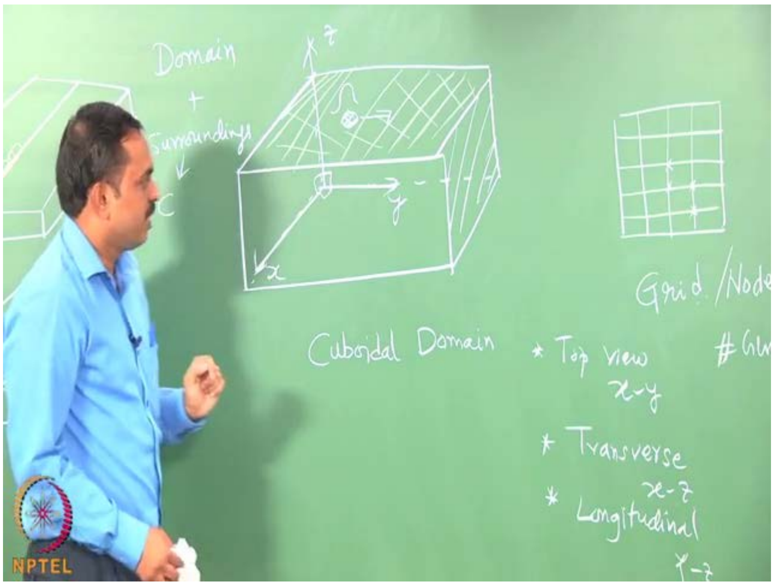

So, I am taking the same cuboidal geometry and let us take for example, the axis as follows x y and z. So, you have the 3 axis and we will see that there are names that can be given to the planes, and for simplicity we will just decide to have our heat source which is moving in this direction along the y direction. So, what we mean by the sections is as follows. So, we have those names that are coming - the first name is what is called the Top view or Top surface. So, top view is basically this fellow, this surface which means that it is the x-y surface. We have the hatched region here and that would be called as Transverse section, and transverse section would be having x and z. And what would be the name of the section that has the other two that would be called as longitudinal, y-z.

So, the information that is contained in each of these sections is as follows. In the top view you have basically the heat source, in the transverse view you have basically the section of the weld pool at any given instant of time and in the longitudinal section you have basically the vector that is containing the transverse motion of the heat source, along with the region that is molten and solidified.

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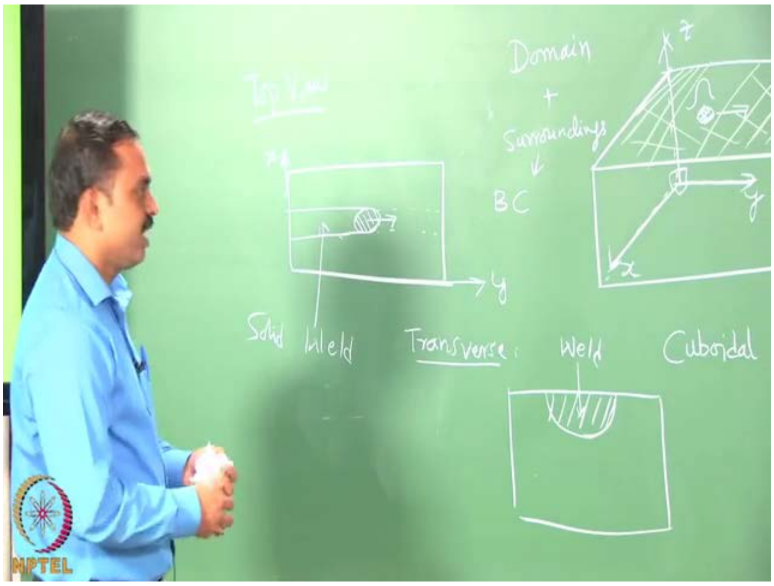

So, a quick schematic would show you how they would look like. So, let me erase this part then show you how those schematics would look like. So, I will show you the first the top view. Top view would look like this, and you have got the y direction and you have got the x direction and you would see that this is the weld pool and this region is the solidified weld, and I ahead of it is region that is to be molten and it is moving in this direction and transverse sections would be having this way the apparent.

So, the solidified weld is here if you take a section behind and if you take a section exactly the centre of the weld then it would be the molten pool, and when you take a section ahead then there will be no boundary that is possible, and this is the base material.

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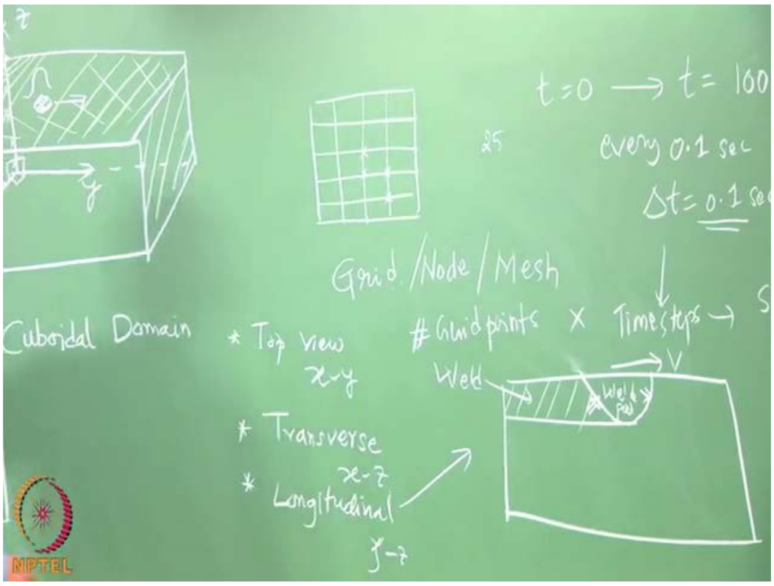

Longitudinal section would have further more useful information; it would have information of this nature. This is the solidified weld and this is the direction of the heat source moving, and you would have a possibility to see what is the plane that is melting and what is the plane that is actually solidifying.

You could see that each of these places would have different information's that is contained. If you are interested in the weld pool geometry, normally it is the transverse section that we are taking and if you are interested in for example, the ripple formation and what is happening to the maximum temperature etcetera then you would be looking at the top view. And if you are interested in the maximum revolution and how the solidification micro structure is evolving in the fusion weld, then you would look at the longitudinal section. We will be referring to these names as we go along in this course.

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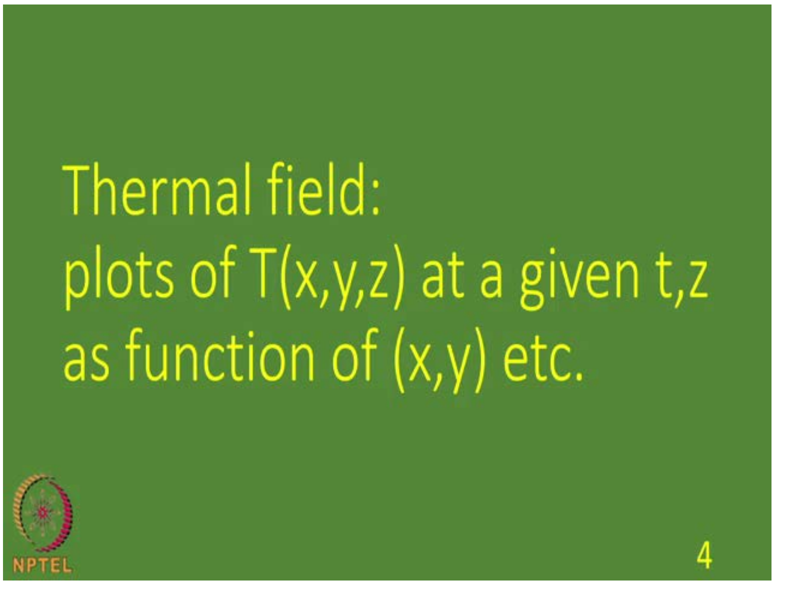

So, what we mean by somewhat terminology I will just mention here.

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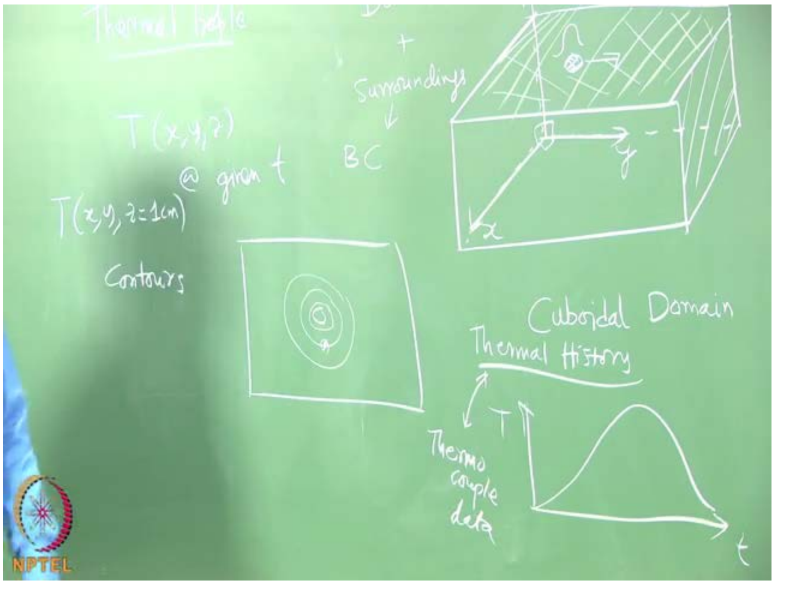

So, what we mean by thermal profile - is nothing but the temperature data at a given time and naturally, you can see that there is a volume data that is possible. There are 3 locations at which the temperature will be mentioned, which means that is there three dimensional data and such a data is available at each time step. So, how do we visualize? Normally we will have to take one of the three sections that we talked about.

So, let us take the top section or top view and you can make for example, at any given value of z and z is equal to some distance at which you are taking the section. And look at the contours of the temperature profile and such contours would look like that and at different-different values; the inner circle be at higher value of temperature, outer circle will be at lower value of the temperature. And such visualization is what we call basically as a thermal profile. Our objective in doing this thermal modelling is to be able to obtain this at the end of the exercise or at least to understand how such information is coming out.

Thermal history is something else; as a word indicates thermal history is basically the temporary variation of temperature at particular locations. So, we need to obviously identify the location at which we are interested in the history. So, you pick a temperature value and ask what would be the temperature at that particular location as a function of time; and you could see that when the torch was far away the temperature is low, when it is close the temperature is high and when the torch goes away the temperature is again low, which means that it would follow a temperature profile like that. So, this is basically the thermal history.

So, what this is important is because of the reason that we have the validation. The thermocouple data is what can be compared with the thermal history, and it is the IR visualization data that can be compared with the contours or the coloured maps of the thermal field. So, thermal field or thermal profile is like this, and thermal history is like this, where the temperature variation is there and one can actually have both of this combined in a videography of the thermal field.

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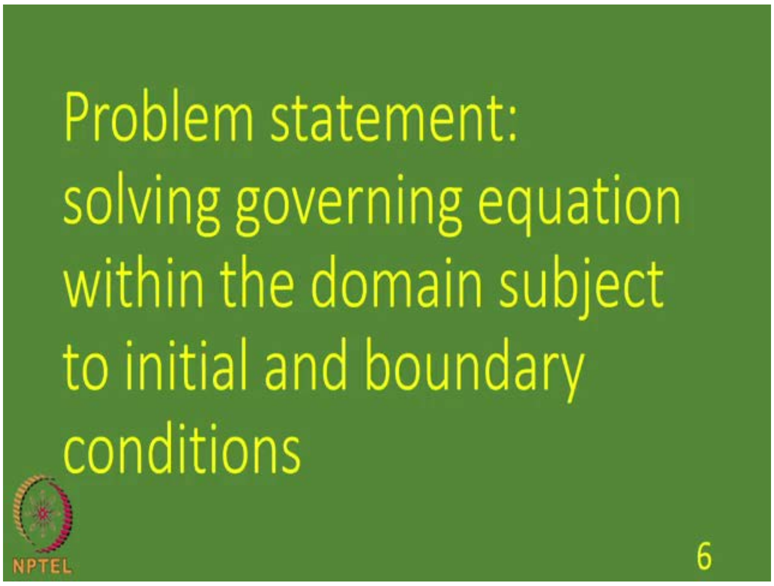

So, in order to have this information available to us through a computer model we will need to have information about the problem that we are going to solve. We then need to also see that the problem is well defined, because only well defined problems can be solved using a computer; ill defined once require assumptions etcetera.

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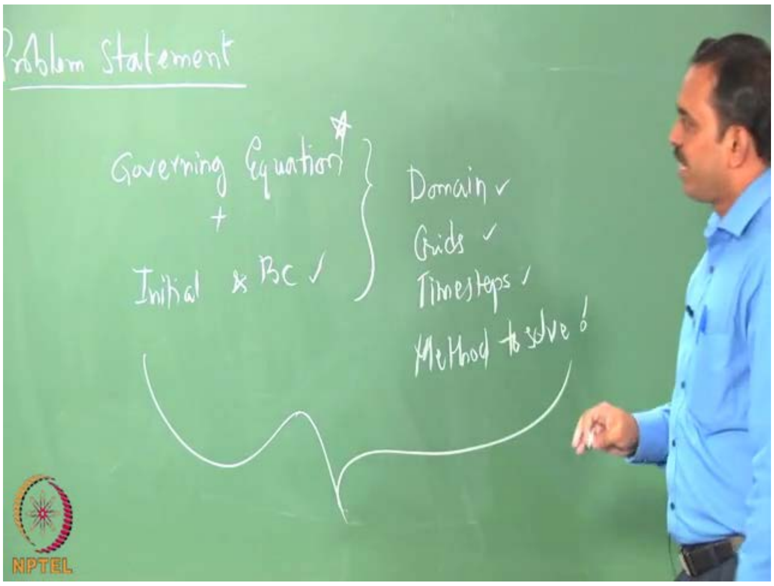

So, we now see what is called as a problem statement. What we mean by problem statement is basically a combination of various things for example; we need to first what is called the governing equation. What we mean by governing equation is; answer to the question, what are you solving if a computer program is going to give you the thermal field then how did it arrive at the data? How did it actually solve it? What is the equation that it is solving? So, that is the governing equation and governing equation combined with the initial and boundary conditions. Along with that you have information about the domain, the grids, the time steps, and any other assumptions that we need to do for example, the methodology to solve. So, this entire thing can be said that then is the problem well defined.

So, it is very important to have clarity on all of these. So, we have already discussed this in depth and we just mentioned about these just now and we will come to that shortly in later less and how to go about solving. So, we now need to get this guy understood because unless the governing equation is very clear to us then we cannot go about finding a how a welding software is doing this a thermal analysis for us, or how we are able to go about and understand how the thermal profile is evolving. So, governing equation is to be then clarified; how does it come about we will try to derive it in their simple manner.

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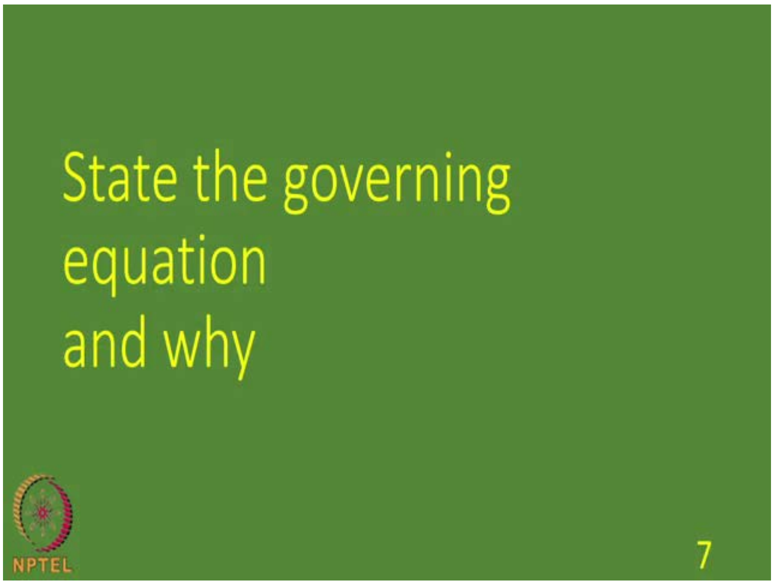

So, the governing equation is something that we need to remember with a name, very often with a name. Then we can look up, how it looks like for a given coordinate system. We do not have to by heart for every coordinate system how does it look like etcetera. So, there are names that are given to equations. The governing equation in the case of weld phenomena is nothing but the generalised Fourier Heat Conduction Equation.

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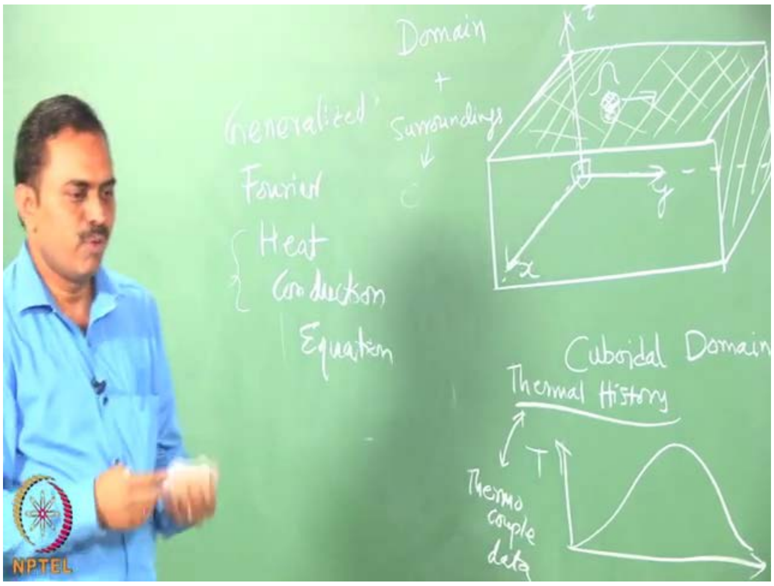

So, Fourier heat conduction equation is alone that is being done, and one may then wonder that during welding we are definitely passing gas on the surface of the weld, and then the cooling gas that is removing the heat and water cooling at the bottom of the plate to remove the heat etcetera.

So, may be is it not the fluid flow equation that we are solving; that is wrong, because we are actually treating all these things as a boundary condition only. And what we are solving in the domain is the conduction. Then one may also think the heat is actually released by radiative loss on the surface, and unless we take in to account how much of radiative loss is happening we are not able to proceed with the solution of the equation. So therefore, maybe it is a radiation equation we are going to solve; again, that is wrong because the radiation is going to be covered under the boundary condition and we are actually solving only the conduction equation within the domain.

The domain will have for example, where liquid is going to form - the liquid may be having some advection there may be fluid flow taking place and should not we be saying that fluid flow also been taking care, we are considering convection also to be part of convection.

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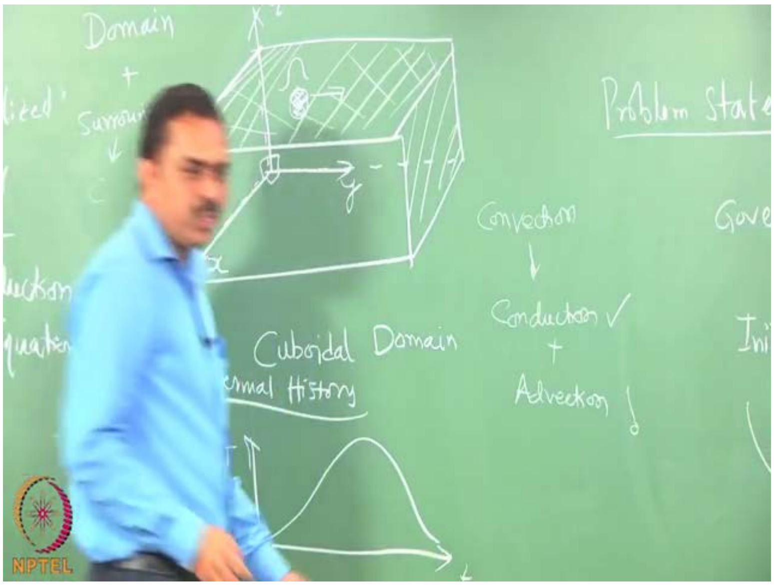

So, here I would remember a very nice way of understanding this word convection from one of my teachers, who said it is nothing but conduction plus advection. So, we still say that we are solving only the conduction equation and we may add advection term into the generalised Fourier heat conduction, later on to take into account that some region in the weld pool is actually molten. And therefore, you can still say that it is a very general equation of heat conduction equation that is being solved – So, how does it look like etcetera; is going to be the next few sessions, that we are going to do.

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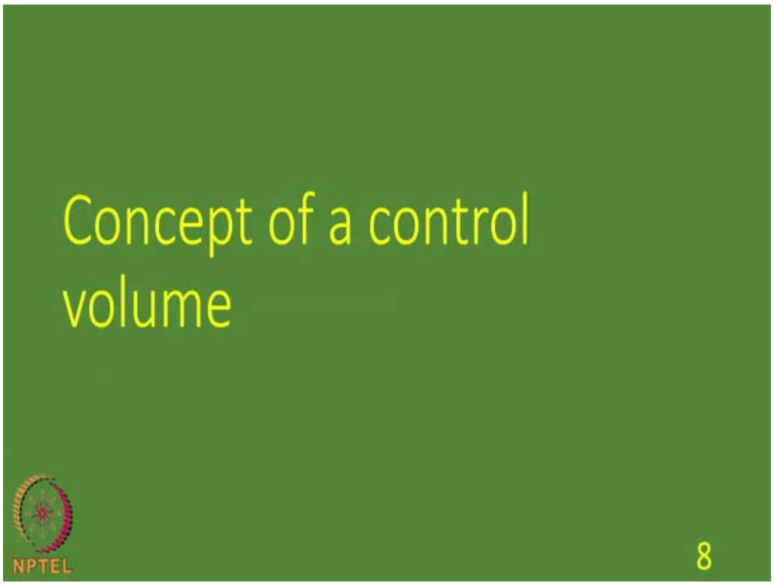

So, when we want to go ahead and make a small derivation of how the general Fourier heat conduction equation is going to look like; we need to understand one small concept that makes the derivation simpler and that is basically the concept of control volume. A control volume concept, I will just illustrate and I will just keep this domain here because we just need to contrast, control volume with that.

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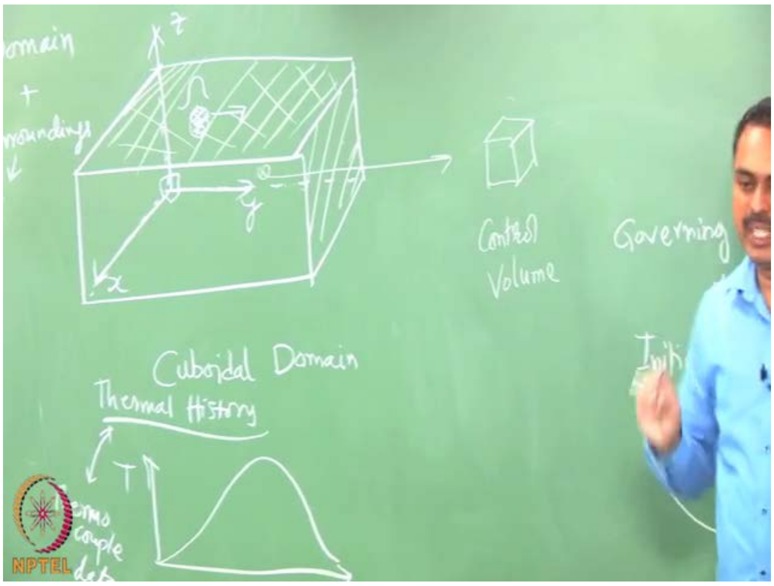

So, what we mean by control volume is as follows. Pick location somewhere and there look at the material as a small volume element. So, you can think of control volume as a small volume element that is present inside the domain and such elements are populated the entire domain, so that the entire domain is captured by a given number of control volumes. The centre of each control volume could be thought about as a grid or a mesh point etcetera. So, we can actually see how the equation is derived for a control volume, and then remove the integration of this control volume information in the entire domain and then we will see that the equation is available for application in the entire domain.

So, the idea is that we apply whatever we are going to do as a derivation of heat conduction for a control volume and then we are going to then transfer the information to the domain. So, when I am referring to control volume face I am not referring to the domain walls, I am not referring to the boundary conditions that have to be applied. So, we are actually very well within the domain, which means that a point that is within the domain is not going to see the boundary and therefore, the boundary conditions we talk about for the domain do not apply for the control volume. The control volume is going to only have the heat conduction that is happening inside the domain.

So, let us just look at how we can go about for the control volume approach. One of the reasons why I choose control volume approach is because it is very straight forward and simple to go ahead and it does not require you to go through minimization principles, and it has also a very natural way of showing you the effect of the balance and being in rectangular it is also possible for us to do it analytically in small number of steps.

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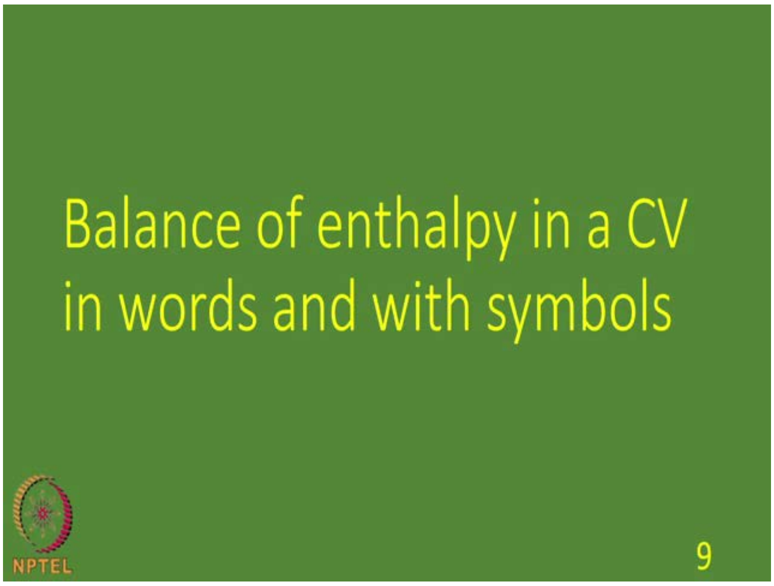

So, what is the meaning of this equation? Heat conduction equation, it is something basically balancing. So, we are balancing essentially energy, so sometimes we may be tempted to say that we are taking about conservation of energy conservation is actually very serious term we do not use that.

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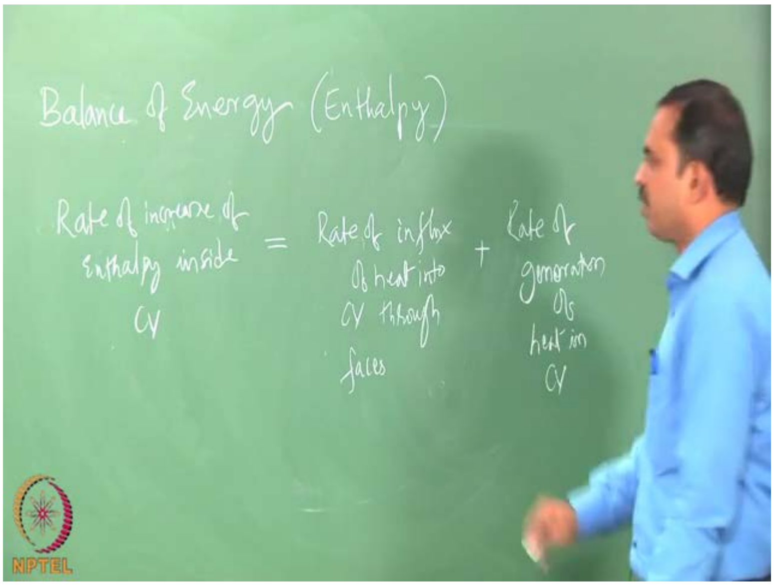

So, we basically are looking at balance of energy and there are lot of energy terms that we have already come across in other courses in materials, and one such term would be for example, internal energy. So, we are not talking about internal energy conservation or balance here. There are other energies like Helmholtz energy, Ziff energy and etcetera; we are not looking at those. What we are balancing here is - Enthalpy actually.

So, is the balance of enthalpy that we are going to do to derive the heat conduction equation for a control volume and then apply it to the entire domain, and that balance is basically a simple balance of looking at what is coming in and what is going out should be balanced.

So, what comes in as energy or heat should be balanced by what goes out and what is generated. So, we are going to write that in English, we can say that the rate of increase of the enthalpy inside a control volume is equal to the rate of input or influx - I could say influx of heat into the control volume through the faces, control volume faces plus the rate of generation of heat in this control volume. So, it is very straight forward kind of balance. The rate of increase of enthalpy in the control volume is equal the rate of influx what is coming in through the walls of the control volume and the rate of heat that is generated. So, in English language this is how the balance is going to look like and in mathematical terms we can then write it.

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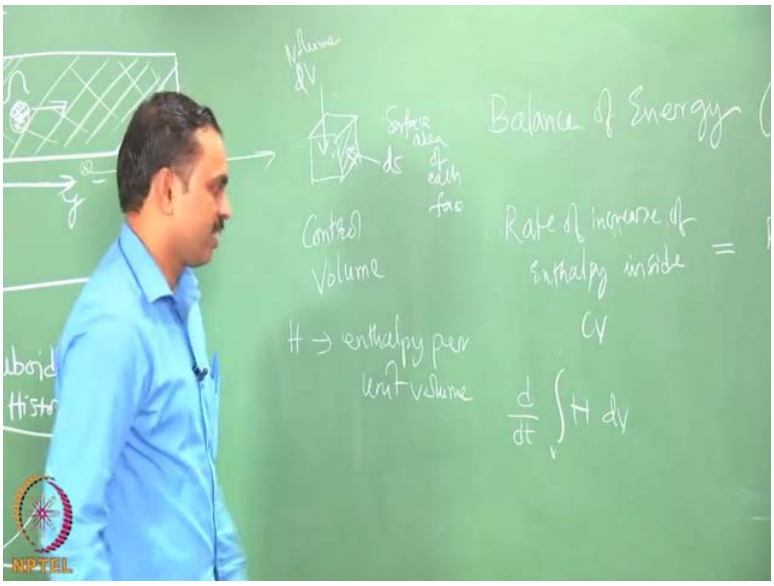

Enthalpy of the control volume can be then written, we could write it as enthalpy per unit volume. So, this guy H - we call it as enthalpy per unit volume, which means that into the volume of the control volume. So, it is volume is dv, the surface area of each face is ds, and this is the volume. So, this volume integral is going to be the rate of increase of the enthalpy; this is nothing but the enthalpy, this is the rate of increase.

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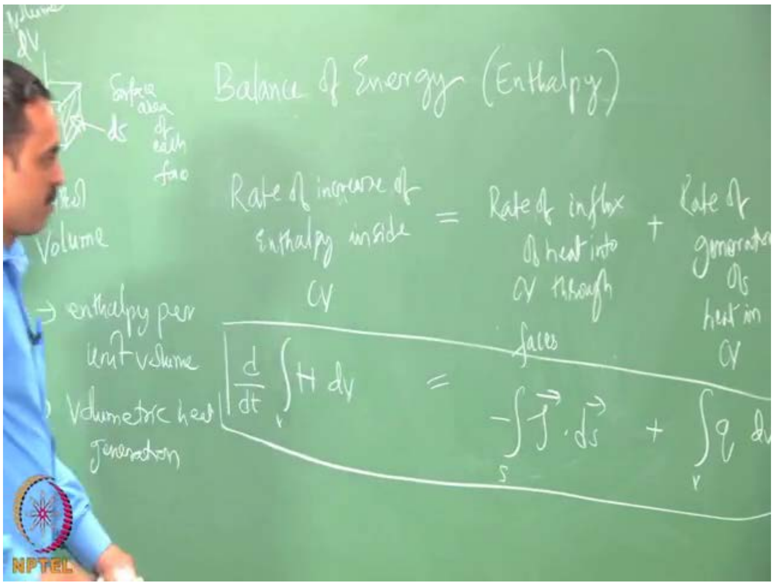

Rate of influx is suppose to be there and that we would come to it and we will say that if the flux that is coming in is to be given a symbol J and the face is given as ds we will see shortly that it is actually outward, so minus symbol to show you that is inward because J dot ds actually is a outward flux. Rate of generation of heat in the control volume the generation of heat can be given as g or may be q; let us not confuse with the gravity let us given it as q and we can explain the word q here as volumetric heat generation. So, volumetric heat generation does not exist in most of the welding situations except for example, when the current is passing and in the path because of a resistance the heat is generated otherwise, generally that term can be neglected. So, this is the equation that we are looking at.

This is the balance of enthalpy for a given control volume. So, let us not get scared of seeing the integral signs it just shows you that whatever is the per unit volume we should multiple with the volume element and whenever you put an elemental volume you must integrate it, so that you can get for any geometry that you are looking at. Eventually, we are going to drop these things and then solve the problem. So, let us not worry about that.

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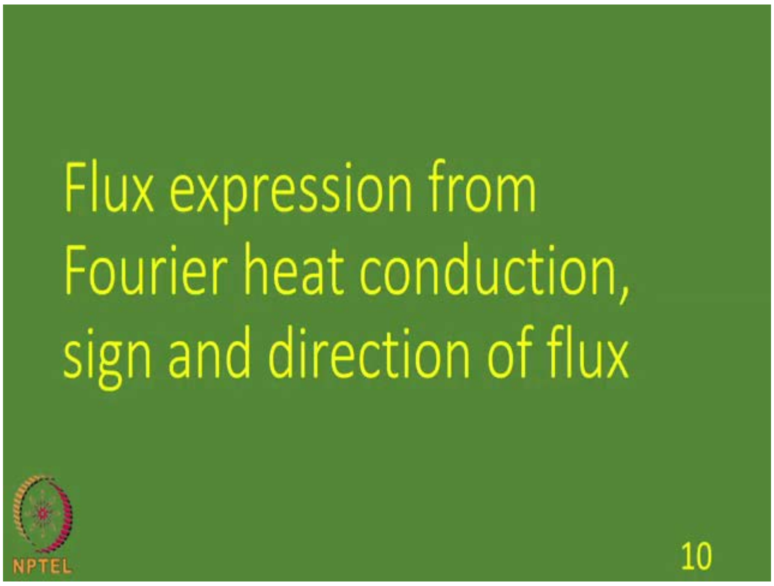

So, we will now need some expression for the flux and we will then refer to the constitute equation for flux that is due to Fourier and that is how the entire equation is going by his name Fourier heat conduction equation.

So, we will just have that expression written here and that is also referred to as the first law Fourier heat conduction.

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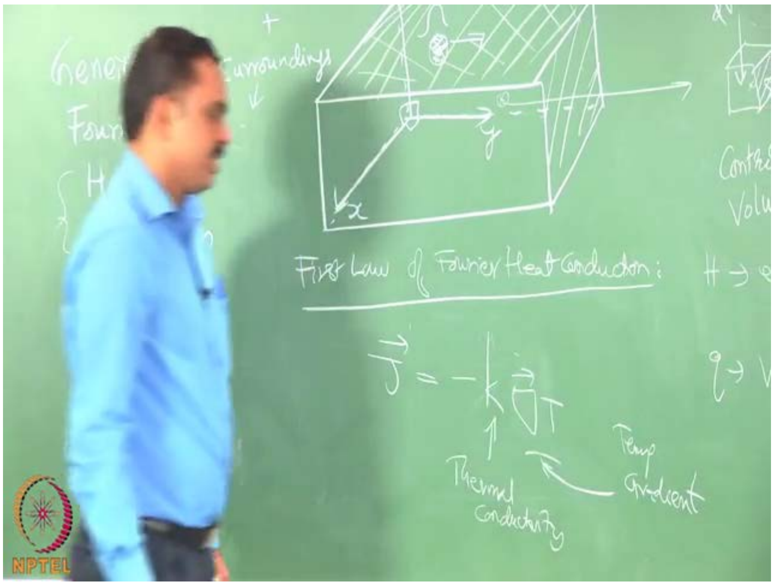

So, the first law of a Fourier heat conduction is given by the expression like this - J is equal to minus k dou t by dou x. So, in terms of English we could say that the heat flux is down the temperature gradient, it is down because it is minus symbol is here and this is the parameter we already know this is the thermal conductivity, this is the temperature gradient and J is the heat flux. And if you want you can replace this term in a vectorial notation so that it does not have any symbol for the particular coordinate system we are using. So, if you want you could replace it with vectorial symbol like this, which is nothing but the gradient operator or nabla as some people want to refer to it.

So, this is the equation that will tell you how the heat flux is going to be available. So, this fellow has to be now substituted into the expression that we have written here, so it has gone there. So, we can put it there and then see how we can change the equation.

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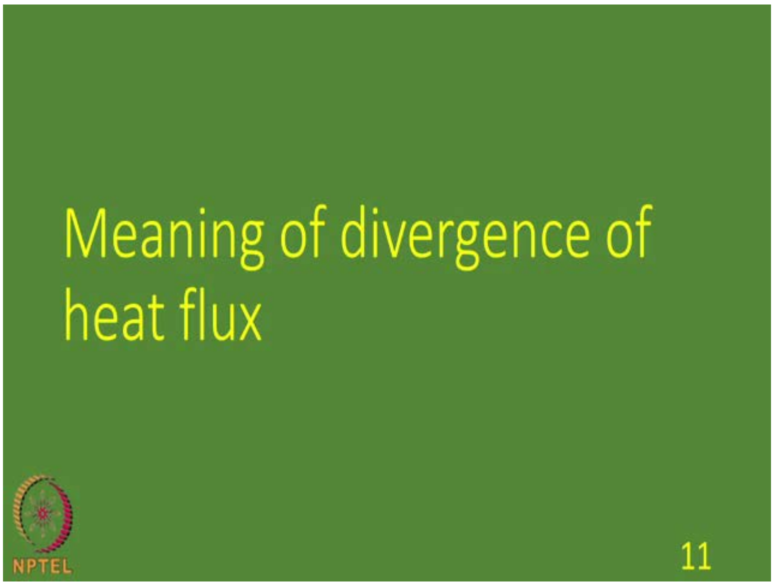

So, when we go there, we will be looking at the equation and we have some meaning for the terms that will be coming out and I want to explain that meaning before I do the manipulation for the mathematical expression that is there.

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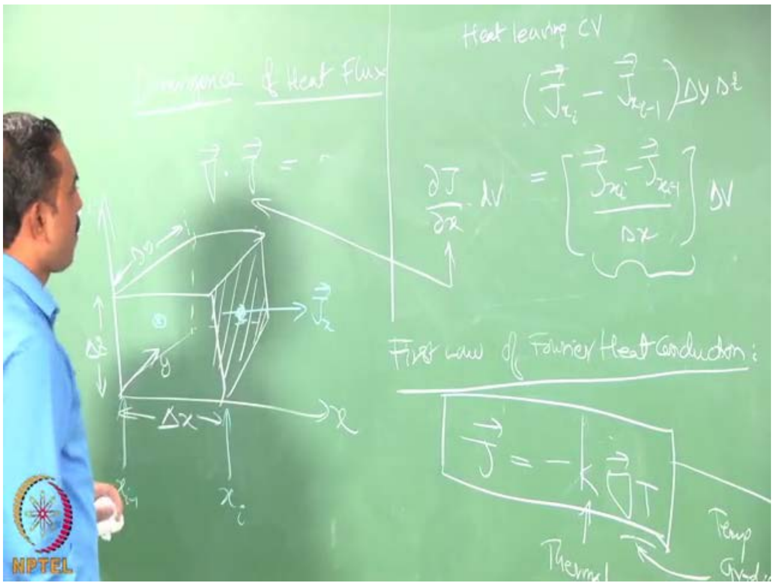

So, what we want to explain this is as follows. So, when I put it there, I will be getting an expression that would have what is called the Divergence. So, there is something called divergence of heat flux that will be coming across and I want to just tell you what it means. So, this is the divergence, and what does this mean? This will be evident, if you just see how the J is written for a simple coordinate system and we will just do it for a system like this, we will do it for this plane and this is x and this is y and then you will have z. So, each of this lengths of the control volume let us designate them as delta. So, so it will be delta x is this width and this would be for example, in the z direction delta z and in the y direction you have delta y as a distance.

Now, what we mean by this flux is something like this, we have this flux that is going through this face and it is evaluated at a location - let us say this location is x. So, Jx is a flux that is coming out. So, just see what would be the interpretation if you want to look at the difference of this flux here, persist here. So, what is the difference between the two? So, let us just expand it and then we will see how the difference would come.

So, you would see that Jx and minus the other one J at xi and J at xi minus 1 and you can then multiply this with the J - J is heat flux; that means, amount of heat per unit area. So, if you multiply the unit area then you will get the energy. So, this is the area and the area is nothing but you have delta x and delta y and delta z, so this is nothing but the energy that is coming out of the control volume and this term is energy that is coming in from the other side. So, this must be the amount of energy that is leaving the control volume.

So, then this if you want to expand, then you would see that you can multiply and divide. What is this? We have seen that, this is nothing but heat leaving the control volume because J is the heat flux heat and that is joules per meter square and heat per unit area into the area that is heat that is leaving out; minus of this that is coming in. So, the difference between them should be what is actually going out. If it is positive, that means, it is going out, so then what I will do is I just divide and multiply with delta x. So, I would write it as J xi minus J x minus 1 divided by delta x and here again I will put delta x delta y delta z. Now this is nothing but the delta v, which is the volume of the control volume, so delta v is there. This is nothing but this term, and what is this term? This is nothing but, you could write it as this is nothing but into dv.

Now, if you see that this is nothing but a one of the terms of this expansion in other words you could see that, the meaning of the divergence is nothing but amount of heat that is leaving control volume or if you put a minus sign, minus of divergence of J is nothing but the amount of heat that is coming in. And here, that is whatever we have taken. So, you could then use the divergence theorem and then change that and you would know the meaning, the meaning is that this will tell you how much of energy is coming in. So, we will do that manipulation just in a moment.

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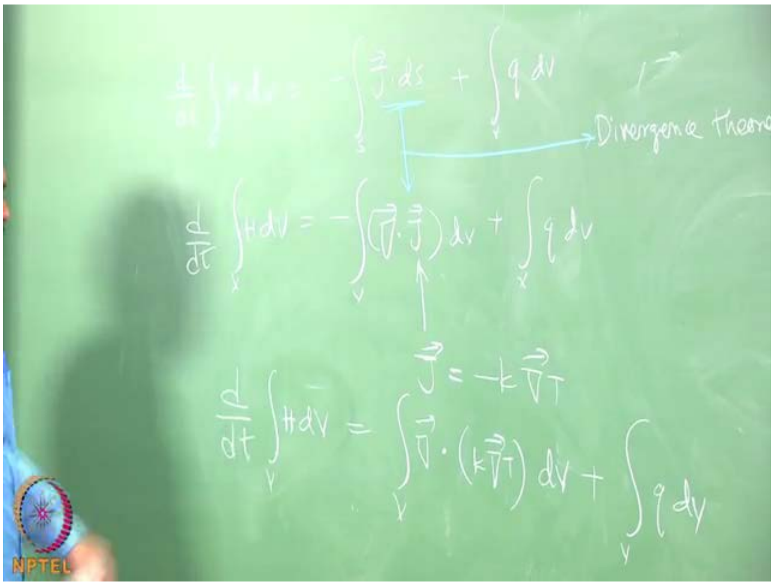

So, let us just look at the equation that we have written by using the divergence theorem and we have just now discovered what the meaning of divergence itself is. So, we will be able to look at the meaning of that. So, we have written it as minus of J dot ds s plus (Refer Time: 32:50) this is the rate of increase of enthalpy in the control volume is equal to the amount of heat that is coming into the control volume through the faces per unit time plus amount of heat there is generated inside the control volume because of any generation activity that is happening for example, current that is passing etcetera. So, this term can be then simplified as this is J dot ds can be written as, OK. And if you want now, you are ready from here to here you are using the gauss theorem.

So, I would use saying that the divergence theorem is used to go from here to here and then for the J expression we would write J is equal to minus k into gradient of temperature. So, that we will able to substitute is 1 minus here and this, another minus here. So, we just let us not mix them up and then we will just see how the equation would look like. This equation would then look like this.

So, all the integrals are volume integrals over the same control volume and we have got these terms, which means actually that we could remove them there is a small settle point can be take this term inside, before we remove and how does it effect, so that is something that we can discuss at a separate occasion, but believe me that it will not take too much of modifications to the equation that we are going to evaluate. So, therefore, we will ignore that valid is outside or inside and go ahead and do the derivation.

So, we can now remove the integral signs because they are having integration over the same control volume for all the terms. So, this is the equation which is basically the Fourier heat conduction equation, and it is now telling us how the enthalpy is going to vary at a given location because of the decrease in nature of a heat dissipating and the heat that is generated here.

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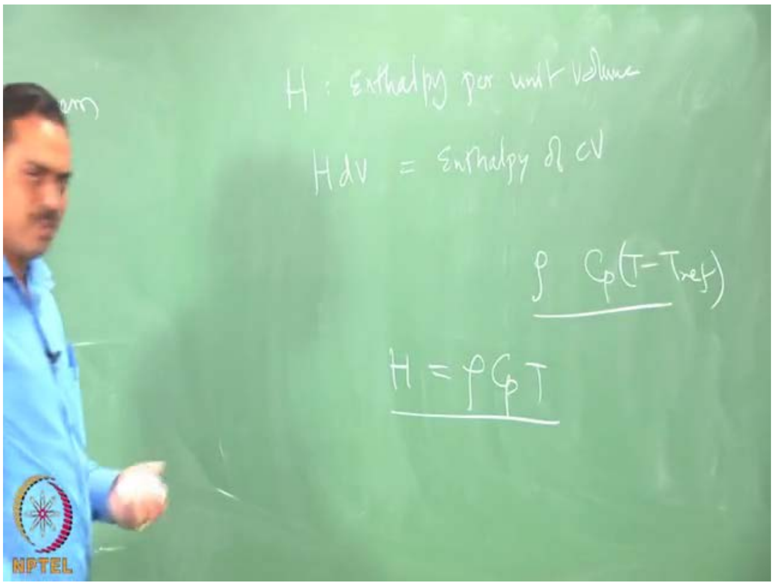

So, we have the enthalpy here. So, we need to expand it and we will do that now; so enthalpy per unit volume. So, let us expand the enthalpy per unit volume here. So, we said this is nothing but the enthalpy per unit volume which means that H into dv is nothing but the enthalpy of the control volume and we already know the enthalpy expression somewhere else that it would be m cp into T minus T reference temperature, that would be the enthalpy and m can be then thought of as rho into v and therefore, per unit volume if you want then it would be a rho cp into T minus T reference.

Now, the T reference temperature for the enthalpy estimates will be the same temperature 298 kelvin throughout the entire analysis and that is anyway going to be differentiated. So, therefore, we will not need to keep the T ref there, which means that this guy H is nothing but rho cp into T. So, enthalpy per unit volume is nothing but rho cp into T and that is what you can then substitute into the expression here and please note that this for a given face, which means for a solid and take this as the expression.

Therefore we can then substitute it and look at how the equation is going to look like and I would erase H and I put rho cp into T there. So, we can now then take the rho cp and keep it out and I will also take k out.

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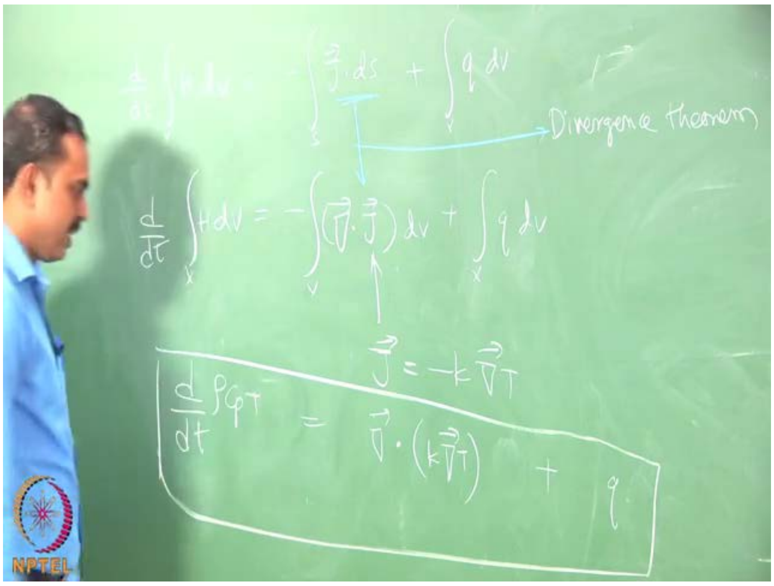

So, under what circumstances can I take the k out of the divergence here, is basically divergence is nothing but this is an operator. You can see that this is nothing but, it is an operator.

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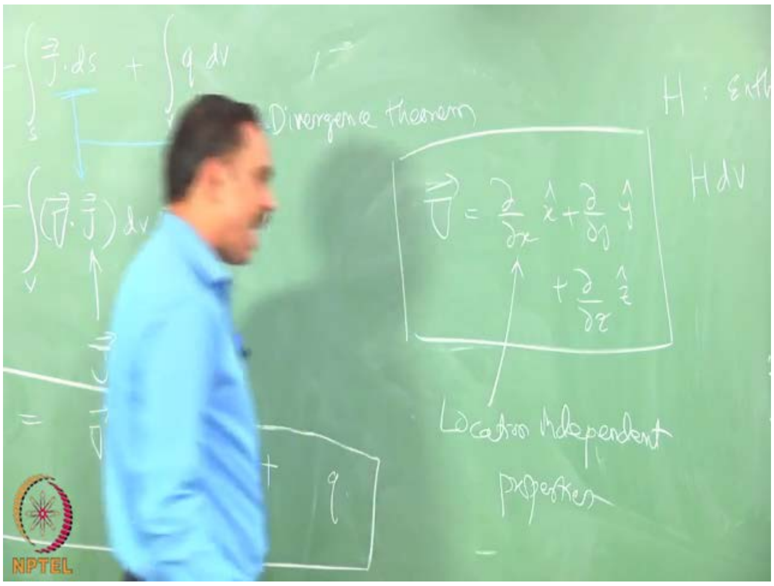

So, basically we are differentiating with respect to x, something that is there and we want to take k out, which means that k should not depend on x. So, which means that location independent properties. So, if you have location independent properties then you can take the k out. So, we will do that and we will take this thing out and then we will see how the equation would look like.

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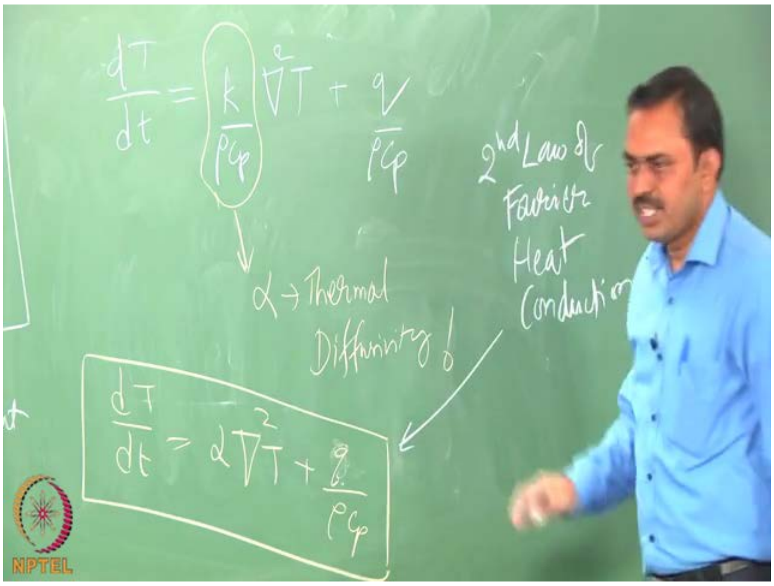

So, if you do that then you would see rho cp dT by dt is equal to, I will take the k out and here you would then get it as the (Refer Time: 38:94) and we will get a q. I will take the rho cp dou to make it simpler on the left hand side. So, we have now come across one combination of parameters, maybe that is possible for us to give a name and this is what we can give a name as alpha - which is basically thermal diffusivity. And this is the equation that we referred to as the generalised Fourier heat conduction equation, for a solid with location independent properties.

So, we will just write it down neatly – alpha, and in situations where we have the heat generation absent then this last term can be neglected and this equation also goes by the name that it is second law of Fourier heat conduction; so second law. So, first law is the expression showing you that the heat flux is downwards the temperature gradient with the definition of the thermal conductivity coming there, that is the first law. The second law is this expression which will tell you basically the balance of enthalpy. So, what is there in the second law? It is nothing but the balance in the enthalpy.

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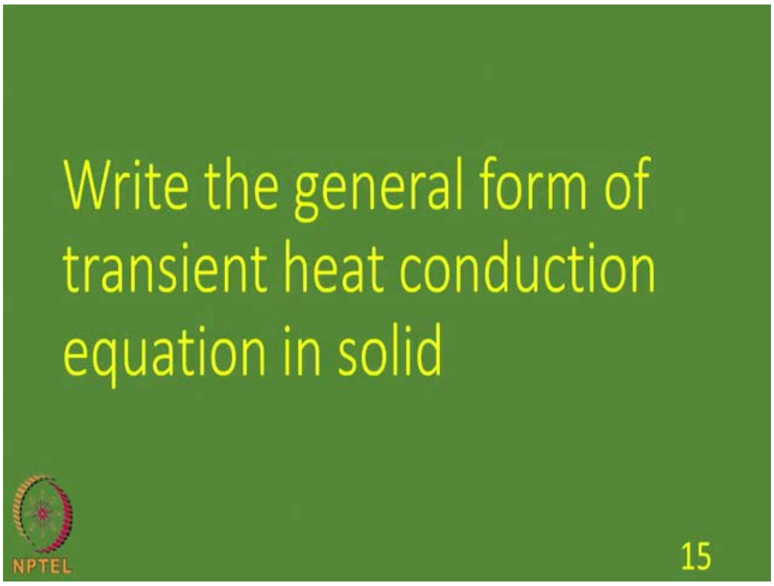

The most general form is here, we could also have expressions that can make it look little bit expanded and I can just write it for you, so that you can recognise the expression when you see it in any paper later on. And that is nothing but expanding the divergence and the gradient term; that is here, you could then write it in this manner.

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So, this is a general form that we are looking at. And solving this equation within the domain is basically solving the heat conduction within the domain for welding.

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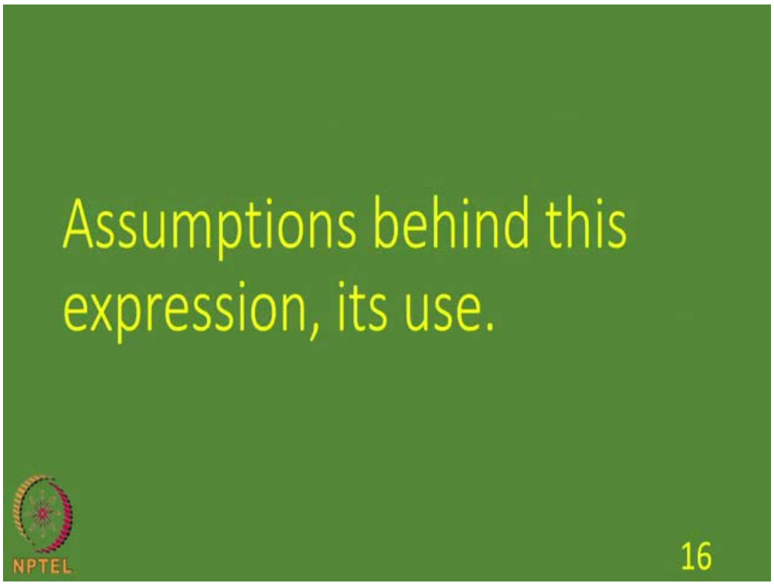

So, what are the assumptions? We have gone through the derivation. So, let us just look at the assumption. The assumption is that there is a location independence of the properties then the assumption also is that the enthalpy is given by rho cp T, which means that it is only for a single face - solid for example. And any other assumption we have seen that it is a rectangular geometry and of course, the vectorial form allows us to use it in any other geometry also. So, that way it is fairly generic.

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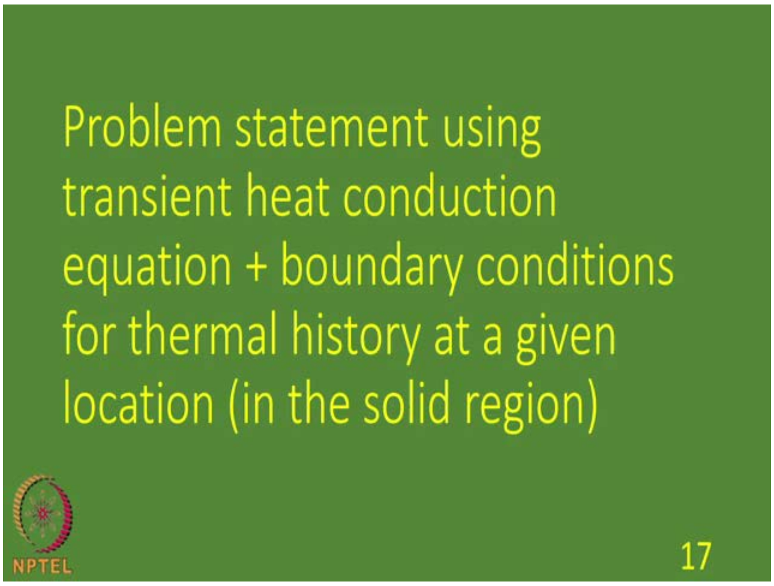

So, you could then say that problem statement can then be given like this.

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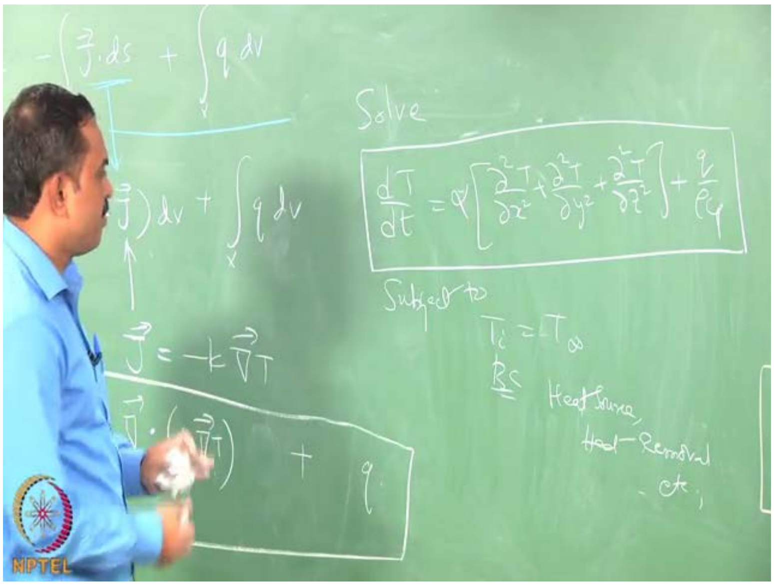

So, this can be a governing equation I will say, solve this equation subjected to the initial conditions, and the boundary conditions which will tell you that there must be a heat flux on the surfaces etcetera, so heat source - heat removal. So, that would then become a problem statement. Problem statement for solid heating, laser heating, laser welding in the solid state would just be only this much.

So, we will at this point take a break and then we will come back to the second part of this lesson in a moment.