Analysis and Modelling of Welding
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Lecture - 07
Thermal Modelling - Part 2

Welcome to the part-2 of the lesson on Thermal Modelling, as part of the Analysis and Modelling of Welding course - online course from NPTEL.

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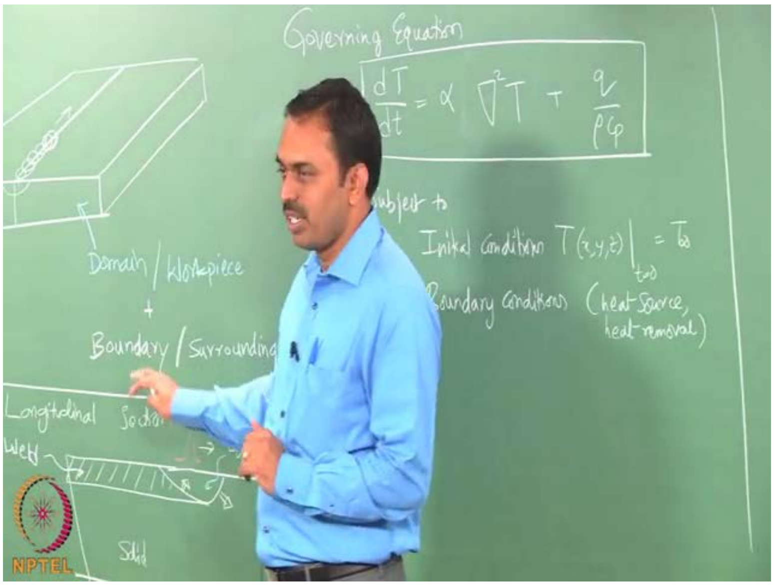

We have a small recap of what we discussed in the part 1. So, we were discussing that there is something called domain and then there is a boundary. So, in our case, the workpiece itself is the domain; and the surroundings will be coming under boundary, which means that the thermal modelling is to be done for the domain which is workpiece, and we are going to apply anything else that is happening around the workpiece in the boundary conditions. And here is the governing equation that we derived with very simple methodology of enthalpy balance.

And we have arrived at this equation, where alpha is the thermal diffusivity, q is the heat generation in the volume of the workpiece, which means that the current is going through it, you would have something there; otherwise normally, it can be taken as zero in solids that are isolated and having no reaction that are going on. Rho C p, are the properties of the solid phase for which we did this equation derivation; rho is a density and, C p is the heat capacity. And we then have posed the problem of thermal modelling as solving this governing equation subject to the initial and boundary conditions.

Now, the initial conditions are nothing but what all the temperature at every location in the domain; and at normally ambient temperature will be applied, but then there are situations where preheating is to be given. So, this temperature can be the preheat temperature for the initial condition. The boundary conditions are to be taken as two parts; one is the heat source, which is coming on the surface of the weldment; and then the heats removal processes through conduction, convection, and radiation and all the balls that are irrelevant.

So, that poses the problem a completely and we will be able to proceed to do the thermal modelling of the solid. Now if you want to proceed further, we need to make one analysis a namely whether this is applicable for every part of the domain. So, what to make that point clear what we do is that we look at the assumptions that are behind these equations.

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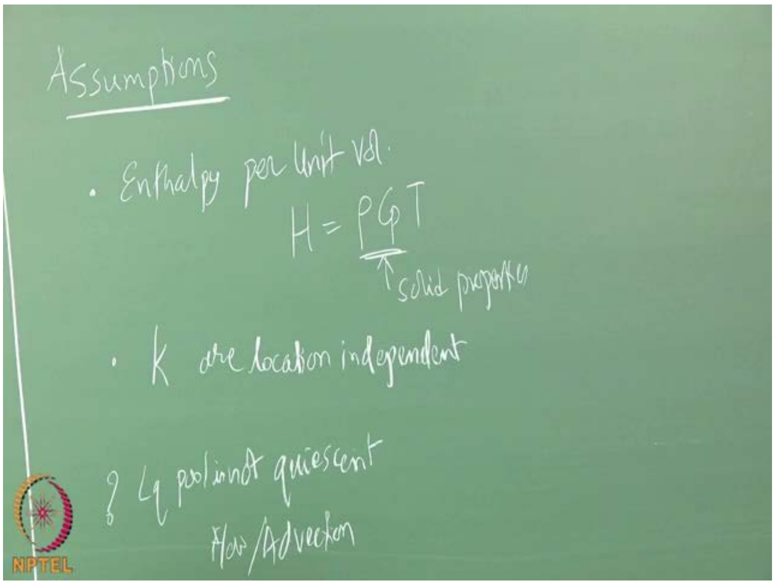

So, what are the assumptions, so that list of assumption will tell us what are the limitations. So, the first assumption is that originally we had while writing the thermal balance heat balance using the enthalpy for a control volume, we wrote saying that enthalpy per unit volume, and we wrote it as rho C p into T. And then the reference temperature is being ignored, because it is any way going to be differentiated later on.

And this means that these properties are to be taken for the solid. So, naturally this equation is valid only when this particular relationship is valid. So, if you want you can actually applied to the liquid domain also, but then you have to take liquid properties, but you cannot take both of them simultaneously; and we will see the reason, why we are having.

And another assumption is that it is valid only when the thermal conductivity, and the parameters that are there are location independent, which means that where we have a strong temperature dependents of this properties. And because temperature itself is dependent on the location as you solve, and then we need to then separate this term expand, and then not have alpha, but k there and rho C p it should be taken out. So, by and large, otherwise, in the domain you are set. And we would let know address this part by looking at the liquid domains.

So, what happens in the longitudinal section for the weld is clear here, we can see that as you have the heat source moving, you have the liquid melt pool that will be moving; and behind the heat source, the liquid melt pool will solidify to give the solid belt. And this is the rest of the base material that is untouched. And you can expand and shrink different regions to match the different welding conditions. So, in this region, what is happening is that in the front part, the melting process is taking place; and the back part this solidification process is taking place.

And this melt pool is not necessarily a crescent, and therefore, we can then put a question mark, what happens if the liquid pool is not that means, there is a flow. So, if there is a flow or advection then what to do. So, there is something that we need to address and we will come to that in a moment. So, we have made the problem definition, we have put the equation in the transient mode and then we have put the boundary conditions initial condition etcetera. So, the problem is (Refer Time: 05:20) made; this term is also called as the transient term.

Whenever that governing equation has the term, you can say that the thermal analysis that is being done in the transient analysis. So, what are the changes that we are needed to make, so that we can apply this for a liquid domain that is what we are discussing now; and that is where we will expand this term - applicable to the liquids.

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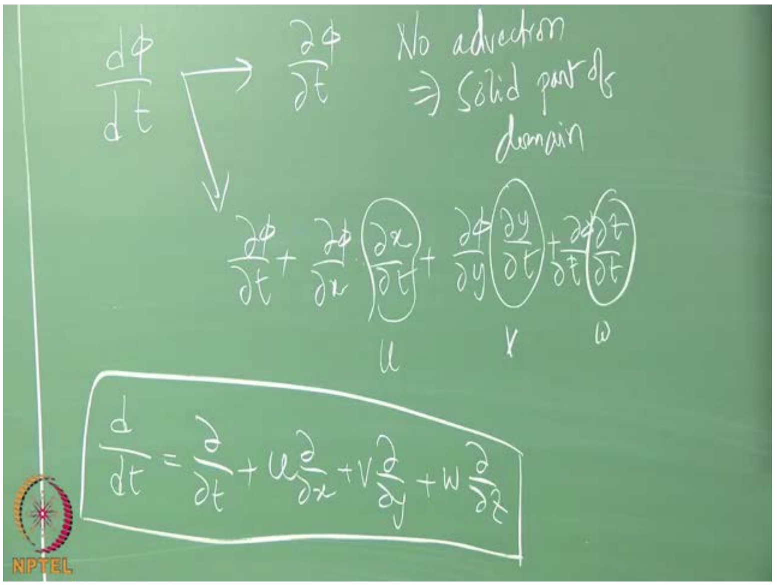

So, in that context, we need to introduce a concept called material derivatives. I am making it a little simplistic here, so that we can apply to welding in a very quick fashion. There is a regress way of deriving all these things as part of transport phenomena you could do that separately later on, if you want to understand how these things are coming about in the governing equation. So, the idea is as follows this operator is to be approximated with a partial differential; in case, you have basically no advection which implies it can be used for the solid part of the domain.

However, in the case, when there is a liquid domain then what happens is that the control volume is actually advecting, it is moving around. In other words, whenever you are looking at the time derivative at t plus delta t the location of the control volume has changed and that has to somehow coming to the differentiation, and we could do that by saying that this by using this chain rule as this and so on.

So, this would then tell us that the variation with respect to the allocation can also be taken into account. And this can be for any variable, and we could just put phi there and so which means that you could then look at this, and substitute what is known for these things. This is nothing but the u velocity along velocity along the x-direction, in the melt pool; this is the v velocity along the-y direction in the melt pool; and this is the w component, which means that you could write this fellow as these. So, I would put this here. So, this is a complete derivative or material derivative that is applicable whenever the control volume is getting advected with in the domain. And it is definitely relevant for the liquid part of the domain, where the melt pool is there.

And this is a general form because in case the velocities are 0, then u, v, w setting to 0 would recover the partial derivative with respect to the time and that is what is normally seen in most of the equation, whenever we are look at only the solid domain. So, this is what is called as the material derivative, and this is what is to be substituted on the left hand side, so that the equation is now relevant for the liquid domain as well as the solid domain. So, we would just add that term there, so that will make it comfortable and compact then you would see also that this term can be simplified by a vectorial notation as follows.

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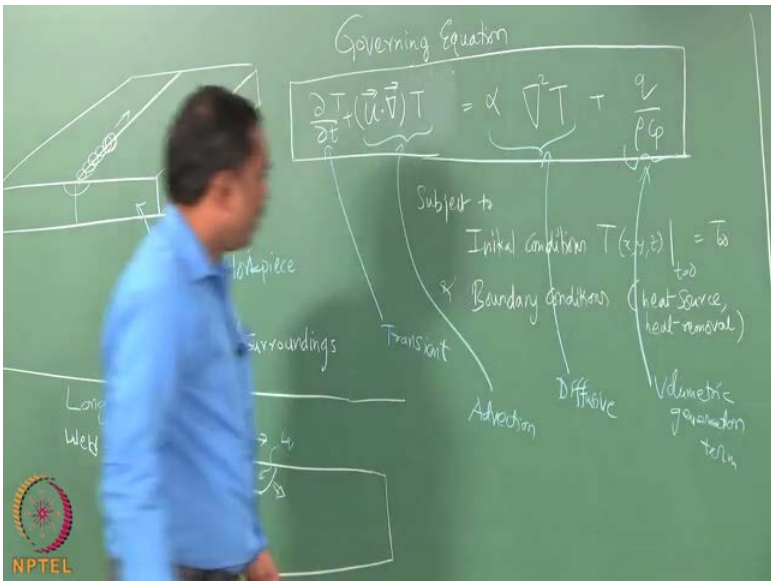

So, I would do that here. So, the vectorial notation would be this way. So, now, you can see that there is an additional term that is coming up, and that takes cares of the advection within the melt pool for the temperature distribution. And if it is not there, that means, it is a fully solid otherwise equation is remaining same. So, now, this equation is valid for the liquid domain also.

So, we have got both the domains covered and we could them separately. So, this is the generalized equation with advection and you could look at the terms and tell what they are and we will designate them with some names. So, this is called as transient term, because it has a time derivative. And this is called as the advection term, advection or convection term. And this is given a name, it is diffusive term. And you can see why because in the derivation of this, we have got k into the temperature gradient which is basically the heat flux and there is what is causing the heat deficient to takes place. So, that origin you keep in mind to know this diffusive term. And this is basically volume generation term, volumetric generation term.

With this equation, and the relevant parameters that are available, you would be able to solve the thermal field within the liquid domain as well as in the solid domain. And one could ask where is the velocity of the torch is it the v that we have written here, it is very important to know that u, v, w are the x, y and z components of the velocity of the liquid pool, and they are not related to the velocity of the torch. Velocity of the torch is actually embedded in the heat source description itself; and you can refer to the lesson on heat sources, where the origin of the heat source is made to move with respective time and that are where the velocity will come. So, it does not that is the reason why the velocity is not appearing explicitly anywhere.

Now, you can see that there is no mention of latent heat anywhere; and latent heat is definitely a large amount of heat that actually place a role in the weldment, because you are going to give that much of heat to melt, and that much of heat is also released when it is solidifies. So, that also helps in making these weld pool having a shape that leads to the trailing effects also.

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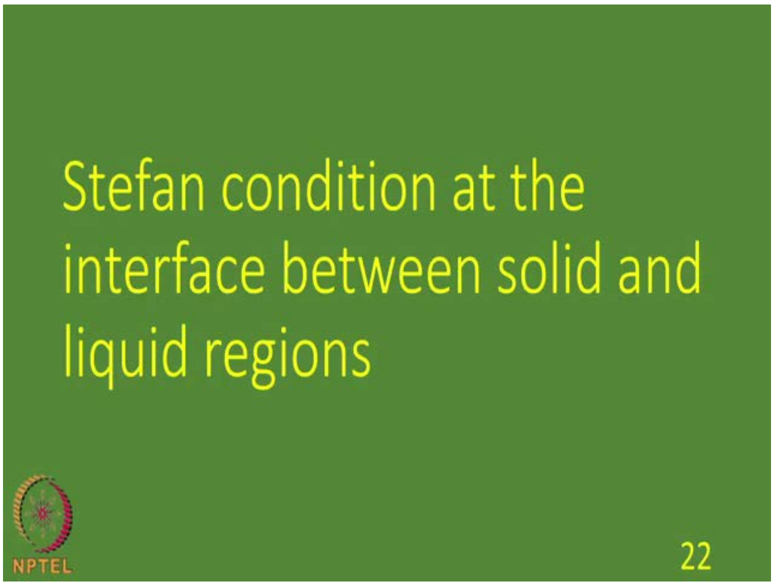

So, you need to see whether it can come in anywhere and that is where we come to the relationship where we need to understand how the solid liquid boundary condition has to be kept. So, while this equation is applicable for both solid domain and liquid domain, but for the internalize between them we need to have a condition and that condition I am going to now show you, and it is as follows. I would just erase this part.

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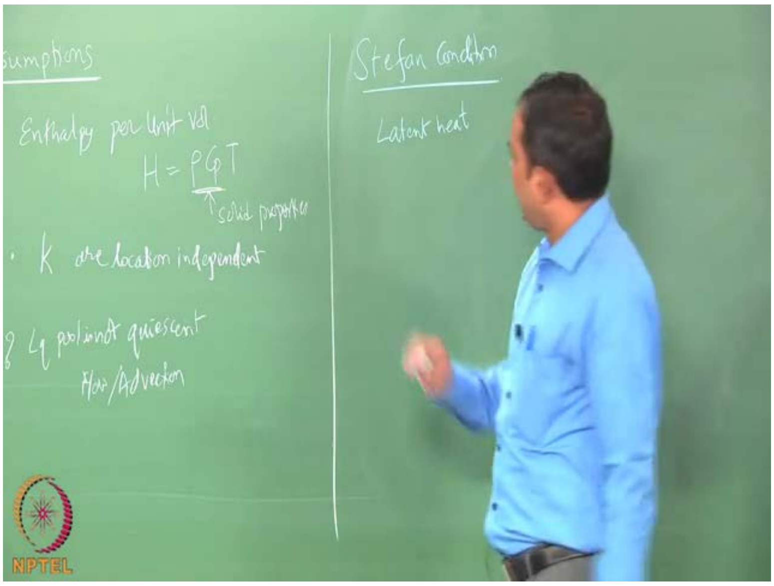

In the region, where you have solid and liquid you need to apply what is called in a Stefan condition. So, the idea as Stefan condition is as follows. The latent heat has to be removed if the liquid has to solidify or it has to be released if solid has to form liquid, so that has to be there. And to understand the how the equation is written, you could consider a small strip of the domain and that you could do it by looking at this domain and looking at a box and I will draw the box here.

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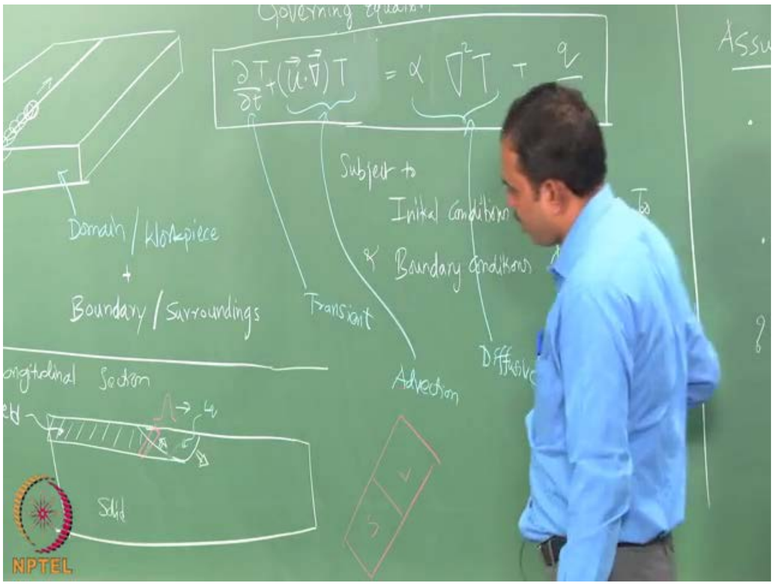

A box like that you can see and understand how the liquid is going to become solid. And that box I will expand and I will show you here to expand that box I am going expand like that. And you would see that this boundary is going to be here and this is a solid and this is a liquid. And as the weld torch is moving from left to right, this interface is going upwards.

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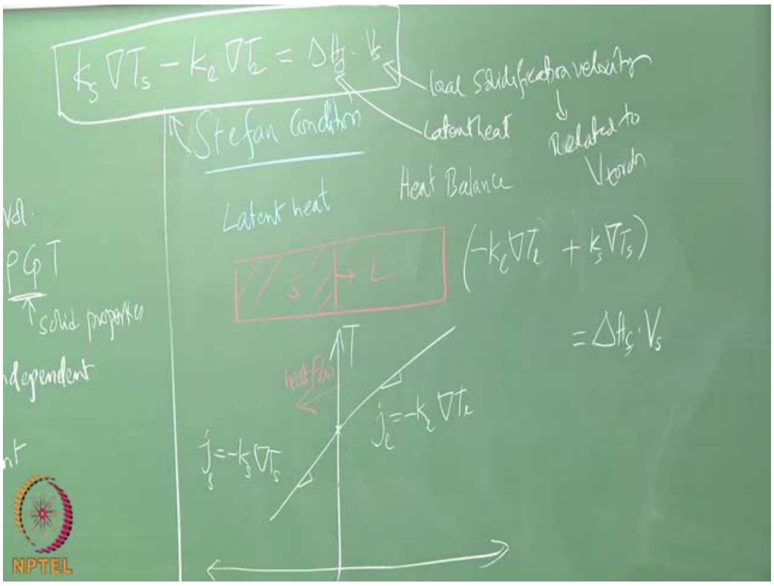

I will just rotate that domain here, and then show you how the boundary condition has to be applied. So, between the solid and liquid, we are going to apply the boundary condition. So, the idea is as follows; the heat has to be taken away from the weld pool so that it can solidify and that can happen only when there is a temperature gradient that is going in this direction. And the temperature at the interface is going to be melting point itself or the liquid as so that has to be identified here temperature and this is the distance.

And you would see that normally the center of the weld pool is much hotter than the melting point which means that you are going to have a temperature profile like that. And in the solid, you would have a temperature profile like that and therefore, the heat is going to be taken away in this direction that is the direction of heat flow. Now, one can question what are these flows and how are they related so that latent heat is taken or not. So, there must be a hear balance always at any interface and that is what we are going to use to write this condition.

The heat flux balance is as follows how much our heat comes in here in addition the latent heat has to taken away that is when the interface can move forward to create solid out of the liquid melt pool. So, which means that what is the amount of heat that is coming in this is the liquid part and this is the solid part. So, which means that you would write for this here minus k in the liquid part and this is the liquid part and this would write it as and the difference between them must be then equal to the way latent heat is taken away.

So, you would write them this minus that. So, you would write it as minus K l and minus K this is s delta T s is equal to the latent heat into the velocity of the interface that is solidification velocity. And this you can then modify by using the two signs and you would them make it simpler here, you put a symbol plus, so which means I can write it in a simpler fashion as follows. I could write it as here K s grad T s minus K l is equal to latent heat times the velocity of the interface; this is what is called the Stefan condition, which means that if you were to solve this equation in the liquid domain and solid domain between those two domains this condition must be made as valid.

And what are the parameters this is nothing but the latent heat, and this is the local solidification velocity, is it related, yes, it is related to torch velocity in a trigonometric fashion the reason being that these shapes are quasi steady state, they are remaining same as the torch is moving. And therefore, by looking at the orientation of the normal to this interface with respect to the velocity of the torch, velocity is this way, and the normal is this way. So, by looking at this angle in this angle theta then you can actually calculate what would be the V s.

So, in other words, velocity of the torch will come into the play here. And what would be the velocity of the interface at the bottom it is 0, because always the entire thickness of this layer is going to be liquid. And as you go upwards the velocity is coming closer and closer to the torch velocity, so that is how the variation of the solidification velocity will be happening from bottom to the top, and that is how that can be related to the last term. And these are the two gradients.

And you can analyze what would happened for the solidification part and liquid formation part, and you can see that which of the two gradients are to be larger etcetera, so this condition has to be applied, which means that we can now modify the situation as follows. You could say that this equation has to be solved subject to the initial conditions, boundary conditions and the Stefan condition in between which necessarily means that you need to treat the liquid domain separately from the solid domain.

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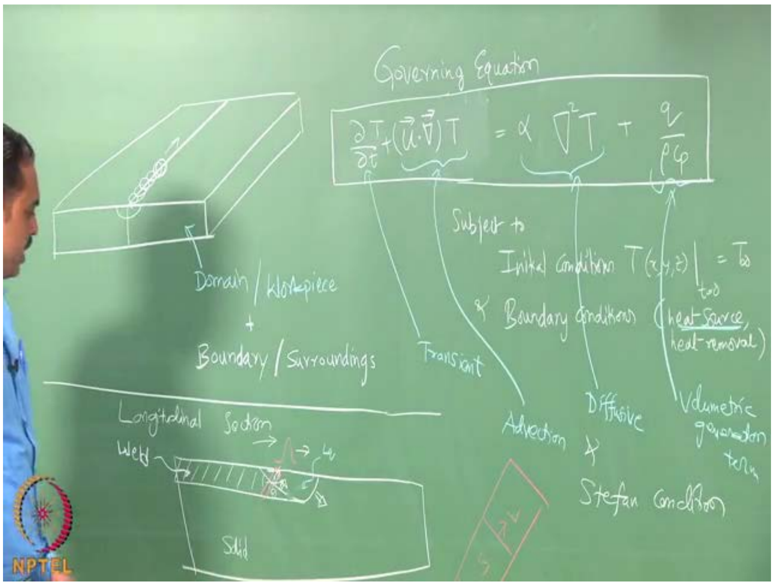

And where is the problem, there is a serious problem here, because if you want thermal modelling to predict, how much of the weld pool is going to form, what is the shape of the weld pool that is to be formed, then you have actually a close loop of a portion. And that closed loop I will just illustrate here.

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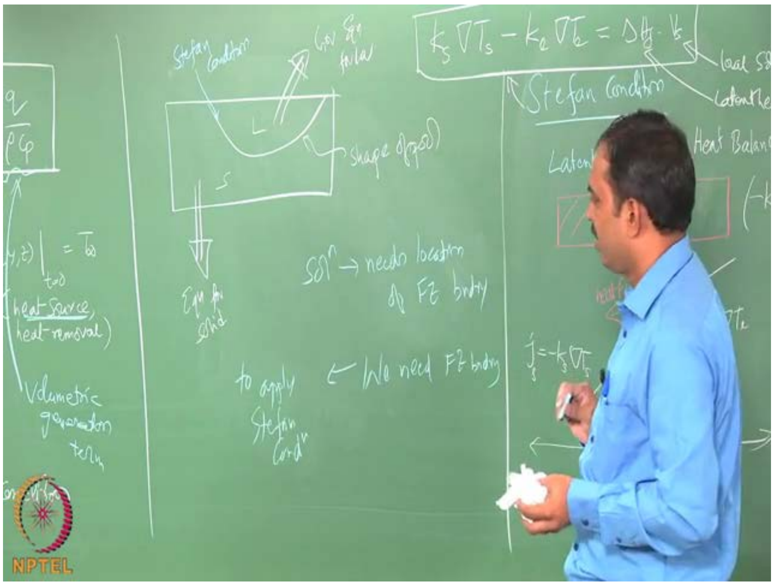

So, the situation is like this. If you want to predict this shape, you know that in this region, you can use the governing equation for the solid; and this can use the governing equation for the liquid. And at this boundary, you need to apply the Stefan condition. So, in other words, the solution needs location of the fusion zone boundary, but we need to the fusion zone boundary to apply the Stefan condition which means that you can see that you have loop.

Unless you know where the boundary is, you cannot apply the Stefan condition, and apply unless you apply the Stefan condition, you cannot actually solve to find out what is the thermal field. So, that is why you have difficulty there are ways by which go about normally you do know what would be the rough shape of the pool. So, you can actually approximate it, solve in a solver using a FEM or control volume method, this will be discussing later on.

The thermal field in both this domain separately applying the boundary condition, and then iterate it by changing the shape such that there is a total thermal balance. So that, how much of the heat is given by the heat source is equal to how much of the heat is lost to the environment and to create this kind of a liquid pool etcetera. And then you can then find out what would be the shape that is stable, so that one can do.

However, it is also very nice if you have an equation, which would be actually taking into account both the domains simultaneously. So, there is a need for a single domain equation; and single domain equation can be developed with a small variable that comes in addition from whatever we are doing; in addition, there is one more variable that we need to bring and that I will just illustrate now.

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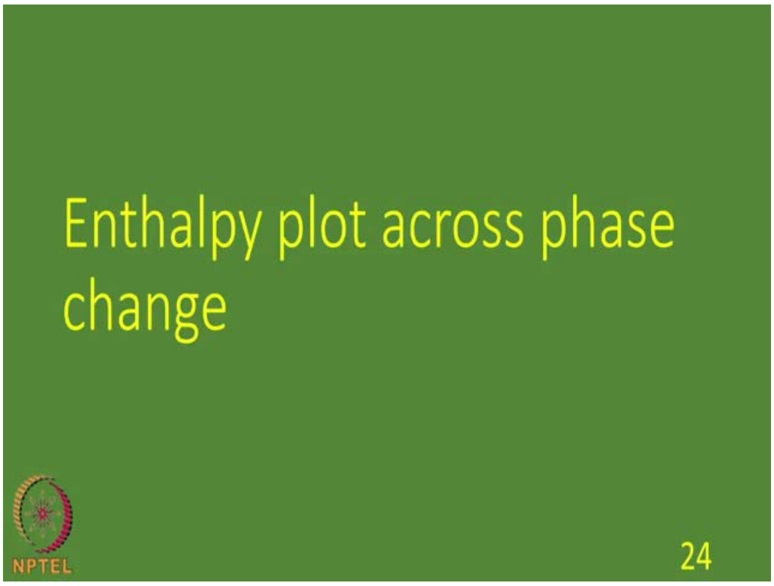

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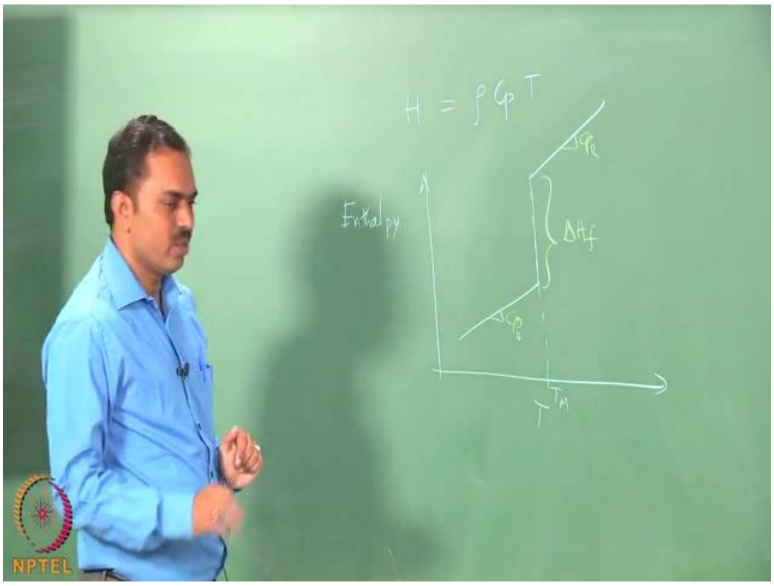

To write the single domain equation that will applicable for the entire you know workpiece of the weldment which is basically the fusion zone as well as base material irrespective of whether that material is a solid or liquid at the region, we need to inspect what would be the assumption that we brought in. We have made one assumption that the enthalpy per unit volume is written as rho C p into T minus T ref, T ref is ignored because it is going to be differentiated anyway.

And this actually applicable in this form only either in the solid or in the liquid, and the solid it can be taken as T reference to be as 298 Kelvin; in that liquid you have to take the melting point as the reference temperature. But if you want together then you cannot use the same reference and that is where you have basically this variation. You know how the variation of the enthalpy will be across this change. So, this is the temperature increase.

And let us say this is the melting point, what happens that to the enthalpy as a function of temperature if you notice, it would be going like that. And the slop here would be this slop is the C p, which is evaluated for the solid phase; and this is a C p which is for the liquid face. And this height difference is nothing but your latent heat of fusion or enthalpy of fusion. So, basically you can see that because of this nature, you can apply the linear relationship only here or here, but not across that is the reason why you have got the two different domains being written for two different you know manners. And the Stefan condition is basically handling how this delta h sudden jump is being taken care.

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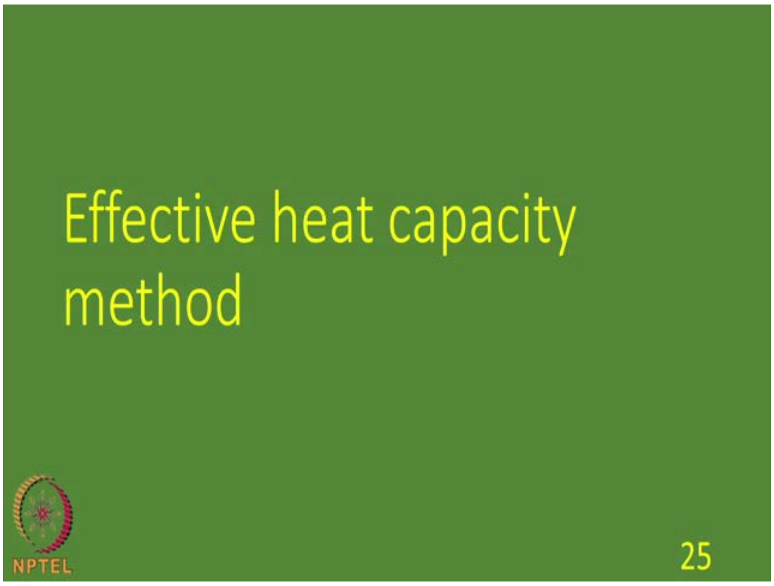

Now, there is a solution to do this and the first solution would be to basically smoothen in out that solution is basically called as the effective heat capacity method.

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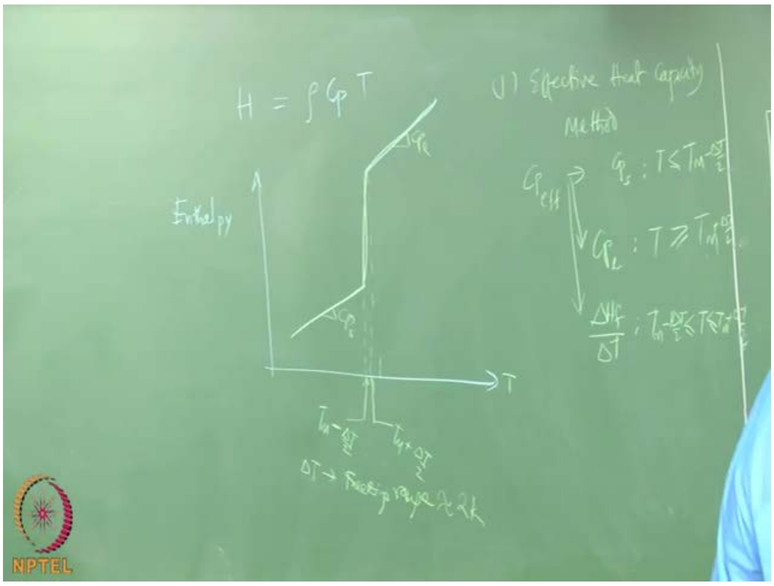

So, the first method to handle is called the effective heat capacity method. The idea is as follows. You defined a heat capacity which is effective in such a way that it takes the value of C p of the solid for temperatures less than the melting point, it takes the value of C p liquid for temperature that are much greater than a melting point.

And in between, if you notice this plot, you are suppose to actually take it as infinite because it goes vertically up. And you could then make that not so bad, you can actually spread it out over out a small interval you could write it like this. You could make this enthalpy change happen over a numerically small width and that width can be taken as let us say this will be let us say T m minus say delta T. And this can be taken as T m plus delta T by 2. Delta t is basically the freezing range you can say. And let us see you can take it as 2 Kelvin for pure metal so that in numerically you have a possibility to go ahead and do.

So, which means that this slope can then be not infinity but something finite and that would be basically delta H f divided by this temperature in difference which is basically when T m minus delta t 2 less than or equal to T less than equal to T m plus delta T by 2. So, you could also do this. So, you could make that kind of a arrangement and that why you can actually defined what is the effective heat capacity which means that effective heat capacity is nothing, but the slope of this smoothly varying function which takes a high value in between, but otherwise it will be taking the heat capacity on both the sides.

This method will basically mean that you do not need to take into the account what will be the heat what would be the latent heat and you do not need to worry about the Stefan condition also, you can actually apply to the entire domain. And wherever the phase change is happening automatically heat capacity will slow down or frozen in the heat transfer.

Now, the problem is that this is numerically not very stable the reasons being that these values are very high that the slope in this region is very high. So, suddenly when you change the heat capacity to very high values then you need to look at also numerically stability or the scheme that you are using for solution. And unless the time strips are very fine to capture this sudden jump, you may actually miss the phase change and that would cause some error near results to come out of your simulation. And therefore, this method has known numerical problems.

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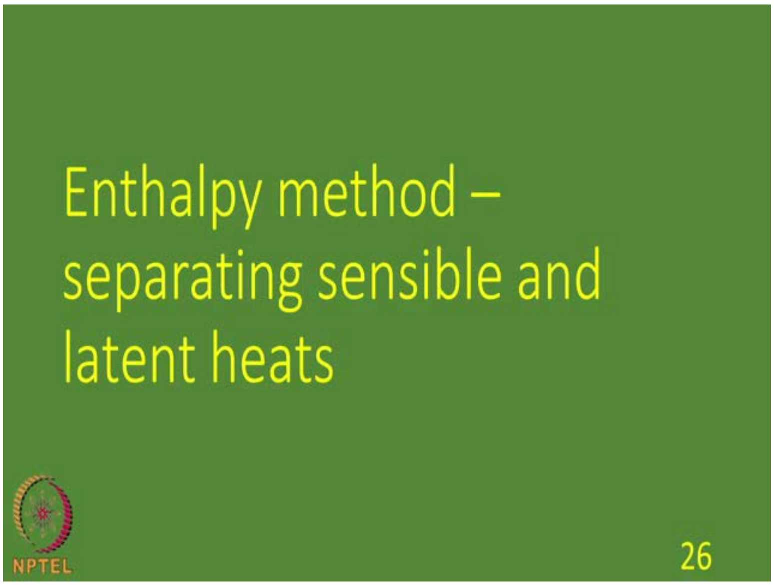

There is another alternative solution that is called as the enthalpy method. I will describe that now.

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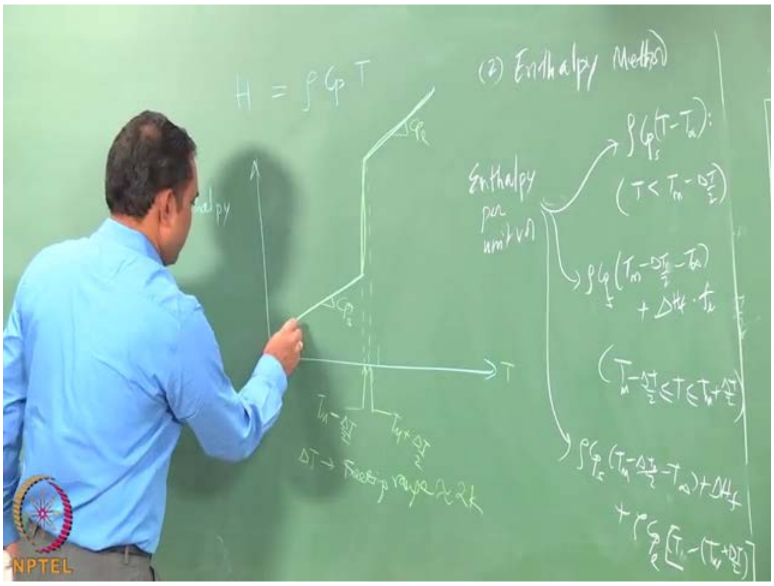

The alternate method is as follows. It is called the enthalpy method the idea is that we do not look at the equation with a temperature in it, we actually write this itself, so that we can just simply look at the enthalpy itself and we would write it as follows. We keep everything else same. And what we do is that the enthalpy of the control volume that is enthalpy per unit volume is written like this. It is written as rho C p in the solid into T minus let us say the ambient temperature, this is the enthalpy from the reference temperature T infinity, it is 298-Kelvin solids and this is the heat capacity of the solid.

And this is applicable in situations when temperature is less than T m minus delta T by 2. And the enthalpy per unit volume for intermediate regions will be given as this which will basically be rho C p into T m minus delta T by 2 minus T infinite plus delta H f into the fraction of the liquid. And this is for a temperatures valid for the temperatures, this is for the valid for the temperatures T m minus delta t by 2 less than equal to 2, less than it equal to T m plus delta T by 2 and for the liquid you would then see that it would be basically more than this and this.

So, you would come up like this solid-solid into T m minus delta t by 2 minus T infinity plus this delta H f, fraction of liquid is 1 and then plus rho C p into liquid T m minus T minus, the difference temperature would then be the melting point. So, this mathematically I have written, but essentially what you can see is as follows. This is the T infinity this is at T infinity and this fellow is T m minus delta T by 2, this is T m plus delta t by 2.

And this difference is the latent heat and this variation is done linearly with respect to the fraction of the liquid. So, fraction of the liquid is 0 at this point; and it goes to 1 at this temperature T m plus delta t by 2 and beyond this it may goes like this. So, you can see that the enthalpy is basically nothing but this plot itself without taking the slopes. Now you can actually plug this into equation which does not have the separation of enthalpy into the temperatures and you would see how you can write it. We would just look at that now.

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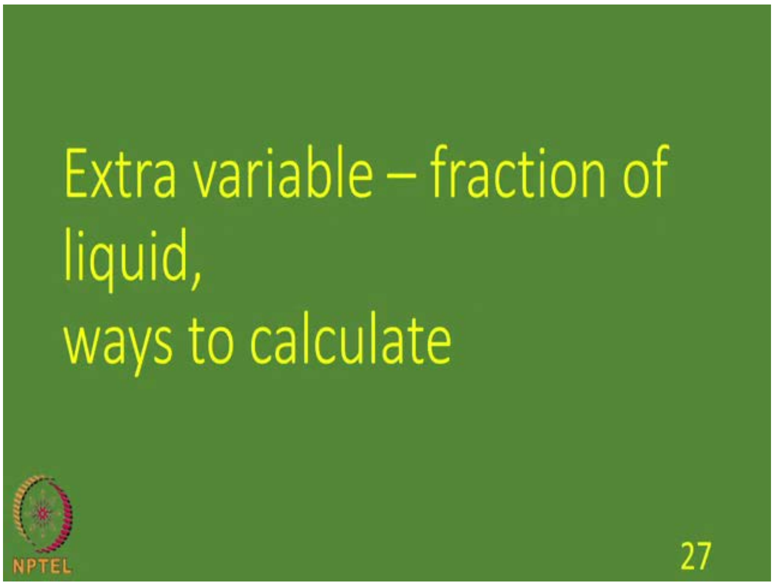

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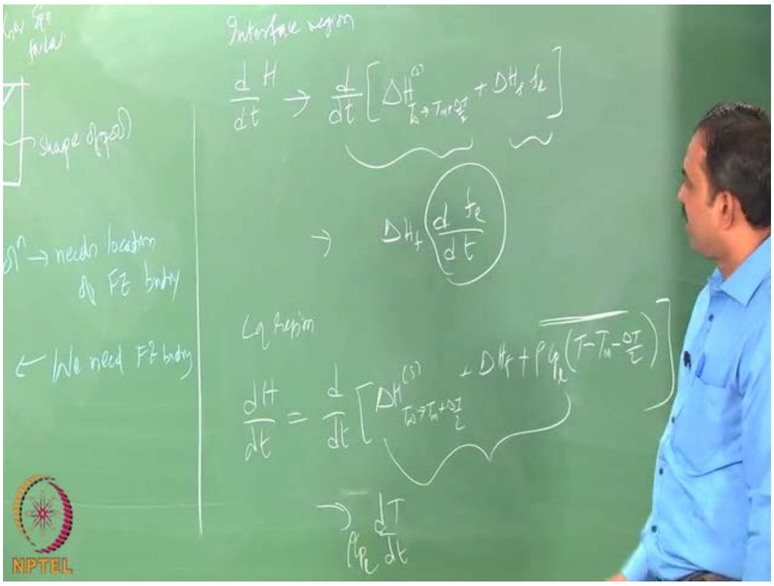

You have originally this term. This term you have and you would then expand this fellow as whatever you have written there, and you would write it as rho C p for the solid part rho C p into T minus T infinity, and this then will become only this. So which means that you are actually deriving what would be the term in the solid by the same approximation we have done earlier, and this term is coming out. And when you do the same thing for liquid what would happen is that you do not have the possibility of just taking it out you have additional terms that are coming in. So, I will just add those additional terms and you would see how that comes out.

So, this additional term is basically this entire thing this entire thing is nothing but the enthalpy of the solid up to the melting point plus the latent heat. So, that I will just give you one term delta H the sensible heat from the T infinity of the solid phase T infinity to T m plus delta t by 2 plus delta H f into the liquid fraction that is there. This will be the variation if you want to look at the region between the solid and liquid. So, you can say at the interface region. And you can see that this automatically implies that there is way by which you can actually expand this term with two parts; and the first part is basically constant and that will not be coming out; and the second part can be then taken as this.

So, you can see that in the intermediate region, when you expand it, when you would get term which is basically the change of the liquid fraction as the function of time. And when you do that for the fully liquid region, again you would see that this term will be coming as a constant that is entirely thing will be coming as a constant, and then you would see only the rho C p into d T by d time that would be coming out.

So, I would that here for the liquid reason you would get like this. This term is nothing but delta H for the solid phase, T infinity from there to T m plus delta T by 2 plus the latent heat itself plus the remaining part rho C p into T minus T m minus delta T by 2. Now you can see that this is the constant part and therefore, you would see only this part coming in picture and that means that this can be coming it out as, you can see that in the solid region and in the liquid region the expression comes to be what is known, but in the interface region, it comes to be one extra variable that is nothing but the liquid fraction as a function of time.

And what would be the value of the liquid fraction in the solid region, it will be 0; and the liquid fraction of the liquid region will be 1. And in the interface regions, it will be changing from 0 to 1, and the way it changes will affect the latent heat evolution. So, this how you can write the equation for the enthalpy balance left hand side the transient term in this faction, and therefore you will get this term and this term can be taken into the right hand side and then the employed. So, this is what is called as that the enthalpy method and they (Refer Time: 34:31) is by introducing a new variable called the liquid fraction. So, you have the way to do that.

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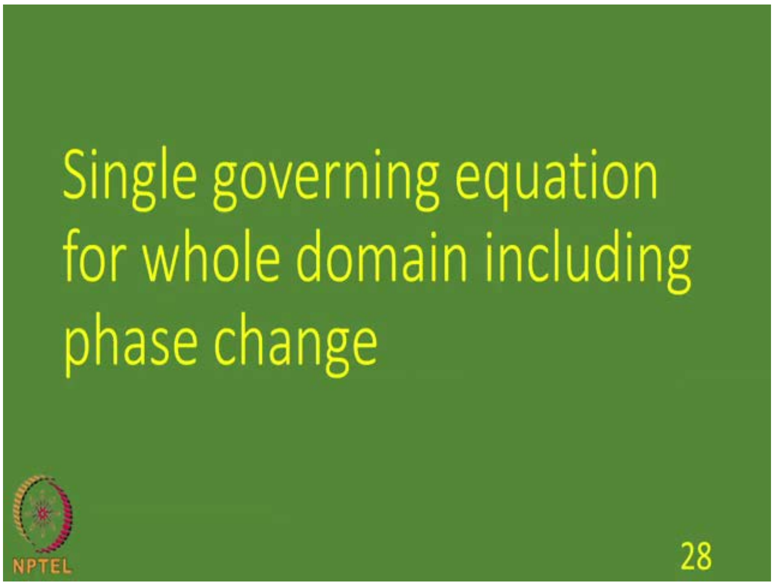

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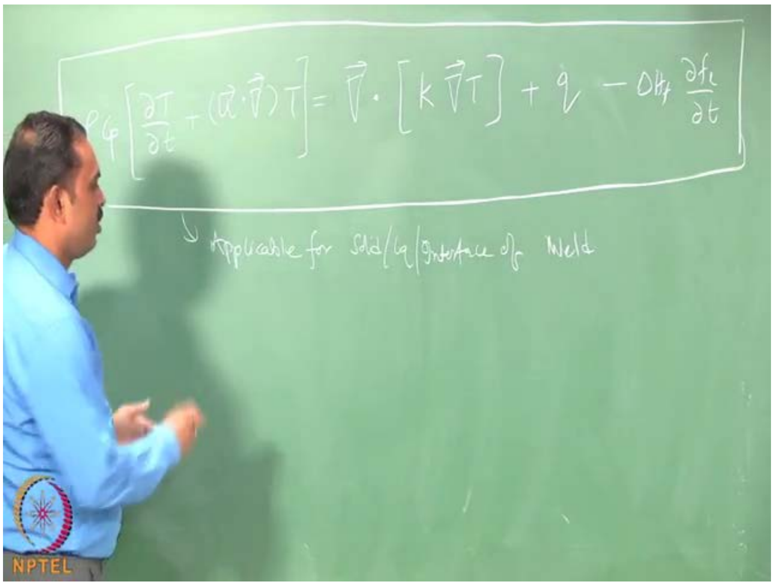

So, I would just write that generic equation now, so that it can combine all these thing together. It is equal to then you have got and you would get that in this and you would that the last one as this. So, you can get an equation of this kind of a nature, and what you would do is basically keep track of the properties at the different locations. If you are in the location where it is fully solid, you will apply the properties for the two quantities that are here, and this thermal conductivity, heat capacity and the density to be that of the solid. And in the case of solid, you also do not have the velocity within the domains, so that has to be 0. And then in the solid you also have the f l going to 0 this term also goes out, so that way you have one equation which would be valid for the solid domain.

In the case of liquid domain, what would happen is again rho and C p will take the values of the liquid, K will take the value of the thermal conductivity of the liquid. U, v, w will be the components of the liquid flow velocity, you will discuss about how the pool velocity can be calculated in a next lesson. And once those are there then you got these terms and the generation term will be there or will not be there depending up on the problem. And f l again is 1, unity at every location; and therefore, again this term will not be there.

So, this term will go away for the pure fully liquid region also. But what happens in between the solid and liquid regions for those locations in the domain which fall on the interface, what will happen is that you would take these properties which are averaged for the solid and liquid for this and this. And you would have basically this term because f l is neither 0 nor 1, but in between and it is changing with the time, so that is how you can then see that the latent heat evolution are the Stefan condition is in embedded within the governing equation.

So, you can say that some equation of this kind can be used for a single domain. So, for the entire domain this would be applicable for the solid part the liquid part and the interface part of a weld, so that you can use one set of equation for the entire domain and that will have boundaries and those boundary conditions can be then applied. So, this is how you would be able to write the thermal modelling governing equation for the entire domain using the enthalpy approach. And this is where the enthalpy approach is giving a new variable f l.

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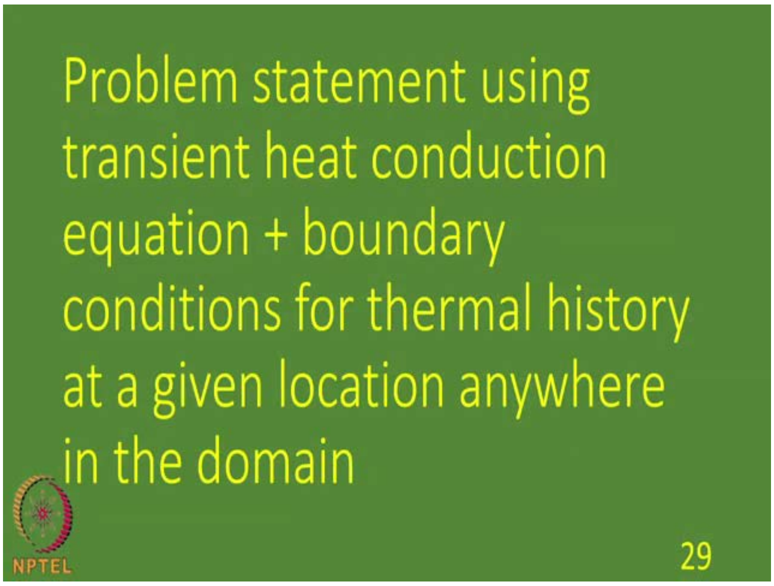

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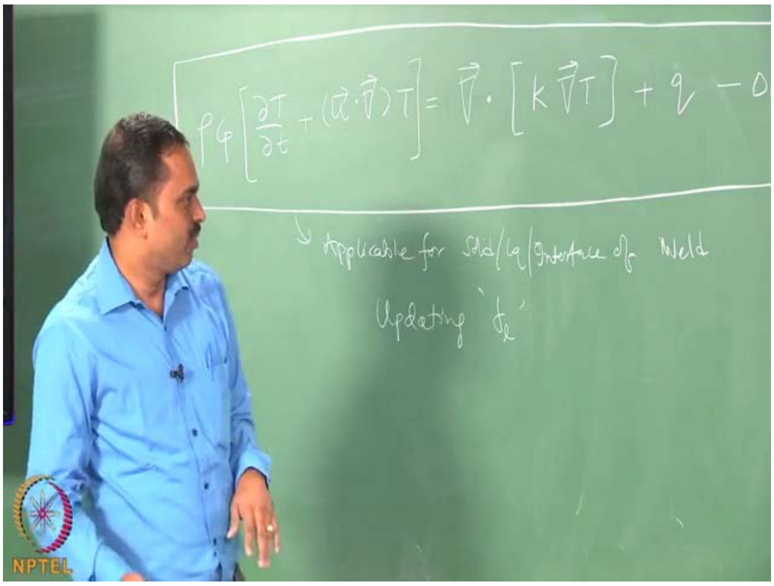

Now one comment on how this f l can be updated. So, this f l can be updated by looking at how much of enthalpy has come into any location, and that is known because you are evaluating this at every time step and you would know that how much of heat is coming in. And if the amount of heat that has coming in such that the temperature as reached the lower bound of the freezing range then you can start updating the f l.

And once the temperature has crossed upper bound of the freezing range then f l becomes 1, and then it does not get updated. So, there is a step that will be involved in the numerical scheme and which is basically updating the liquid fraction or updating the phase change that is happening in the entire domain. With this, we will able to take care of this thermal modelling, and we will look at some specific situations now.

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Now, what would happen when we have steady state, and spot welding and such things we can look up by looking at this equation; so, we already have seeing the names for these terms, we saw that this is what called as a transient term, this is advection term, this is a diffusion term, this is a heat generation term, and this is the latent heat evolution term and it has all the features that we need.

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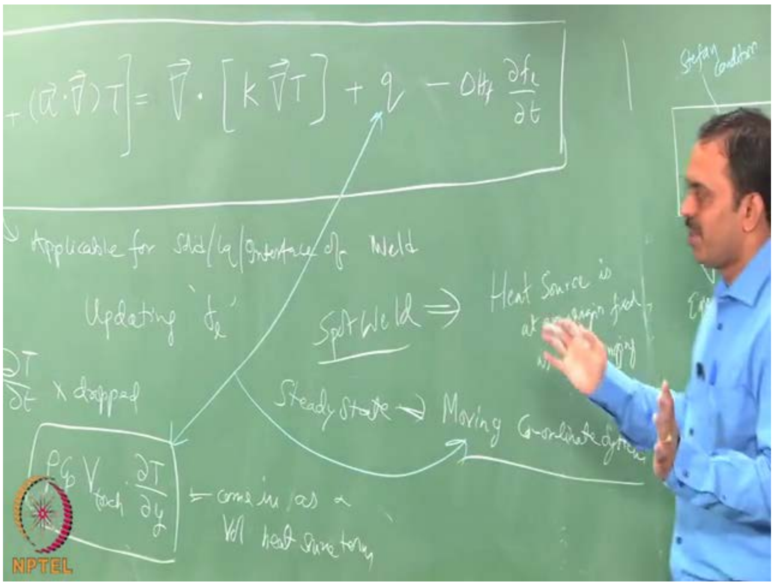

Now, in case the welding is such that it is the spot weld. So, what would I imply is; what does it imply? You know each of these conditions you have to see what does it imply; by saying that spot weld; spot weld also has a duration over which the entire process is happening only thing is that the heat source is not moving. So, we can say that spot weld is no different from continuous weld except for that the heat source is located at a given location and it is not moving. So, heat source is at an origin which is fixed as the time is changing that is all there is nothing else otherwise. So, in a sense that spot weld is no different from continuous weld except for the heat source description.

And very often when you look at a welding the bead is going to have the uniform width along the length of the bead which means that the conditions are quasi stationery or quasi steady state and that you can actually look at. So, you could also ask what happens if it is a steady state; steady state is nothing but long continuous weld and in such situations what is happening is that with time the weld pool depth shape does not change and that means, that time variation is very, very negligible which means that this term will not be there.

And that is not to be dropped upfront, but to be recognized when you start solving the problem and take into account the initial development of the pool and then noticing that

that solution will be working up. So, basically if you see any publication without the time term - transient term then you can say that it is for a steady state situation. And steady state situation is normally also when you do one transformation for the particular equation, and that is basically when you convert these on equation to be in a moving coordinate system that is the coordinate system is not fixed at one corner of your workpiece, but it is actually fixed with the heat source.

So, that as the heat source is moving, with respect to the location away from the center of the heat source, everything else is basically constant with time, the fusion zone dimensions are constant and the rest of the properties like you know the thermal gradient and such things are also constant as you proceed with the time as the bead is being laid. And which means that if you want to understand how the thermal modelling is done in a steady state for a long weld, you need to actually switch this equation to a moving coordinate system, and then look at the solution.

And moving coordinating system is nothing but basically you are expanding this term and ignoring the time term. And that is something that I would mention here, whenever you have the moving coordinating system what would happen is that this term will not be there, it will be dropped, but just because it is dropped does not mean it is handling what is happening in moving coordinating system, because there is a transient evolution that is also happening actually when you look at it from the stationery coherent system.

So, what happens is it instead of this term you must have a term that would be countering like this, let us say the torch is moving in the y-direction instead of this term, you must have this term, so that the coordinate system is changing from stationery to moving. So, that the time variable is replaced by y by the V torch, so that how much of distance is traveled divided by the velocity of the torch will give you the times that is elapsed. So, that is how it can be taken. So, this must come in.

And if you look at the weight is appearing it can actually come in on to the right hand side because everything else on the left hand side is well defined, and it can just get absorbed in the heat system. So, it can come in as a volumetric heat source term and anywhere rho C p is in the front of it. So, you can say that this term when you multiply.

So, you can think of like this. This would come on the right hand side into this and that would take care of the moving coordinate system. So, if you are going to use moving coordinate system that term will come and it would have a minus sign or a plus sign depending upon the direction of the velocity of the torch. So, it is going forward or backward accordingly the direction has to be done.

So, which means that if you are going to look at an equation in a general article, in which the transient term is not there, and there is additional term with velocity of the torch coming in the right hand side; that means, that particular problem is set in the context of a moving coordinate system. If one of them is missing, for example, the transient term is not there, at the same time this term is also not there then you must inspect what is the problem, because naturally it is not meant for a continuous weld. So, you cannot apply that kind of an equation for a continuous weld. So, you must read further to see if there is any problem.

So, with this, we have basically covered most of the aspects of the thermal modelling that is setting the equations, the boundary conditions, the terms etcetera for a thermal profile evaluation in a weldment. And by knowing how to solve these equations analytically or numerically, we will able to start drawing the contours or the plots of the temperature as a function of distance, time etcetera. We know what are the various ways solving these equations is something that we will be discussing in the next class.

So, at this moment, I would ah take a break, and we will have in the next lesson where were we discussing the simplified situations. I want also already alert you with some simplifications of this equation. This equation can be simplified with two-dimensional problem or three-dimensional problem as follows. You expand this and this term for two dimensions or three dimensions, and then you can see variety of equations coming up. You can expand this without the u and you see that the equation is meant for only laser heating or heating of a material with a torch and not with a (Refer Time: 46:08) etcetera.

And you can see that you could drop this term and only keep the remaining once and you would see that you are in a two domain kind of an approach. So, you can say that this equation covers multiple problems in one go, and everything else we see as far as the thermal modelling of the weldment in a rectangular geometry is concerned is like a special case of different, different kinds. And we will look at a summary of those special cases through a handout I will upload in the course websites. With that, I will close this lesson and will continue later on.

Thank you.